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Coherent X- and gamma rays from Compton (Thomson) backscattering by a polaritonic pulse

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Abstract

It is shown that Compton (Thomson) backscattering by polaritonic pulses of electrons accelerated with relativistic velocities by laser beams focused in a rarefied plasma may produce coherent X- and gamma rays, as a consequence of the quasi-rigidity of the electrons inside the polaritonic pulses and their relatively large number. The classical results of the Compton scattering are re-examined in this context, the energy of the scattered photons and their cross-section are analyzed, especially for backscattering, the great enhancement of the scattered flux of X- or gamma rays due to the coherence effect is highlighted and numerical estimates are given for some typical situations.

Key words: laser accelerated electrons; plasma polaritonic pulses; Compton backscattering; coherent X- and gamma rays

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It is well known that laser pulses focused in a rarefied plasma can accelerate electrons up to considerable relativistic energies in the range of MeV's or even GeV's .[1]-[13] Various models, both analytical and numerical, in particular the particle-in-cell simulations for plasma electron "bubbles",[14]-[18] point toward the basic role played by plasmons and polaritons in laser-driven electron acceleration, as suggested long time ago.[19] It is widely agreed that the propagation of the laser radiation in plasma is governed by polaritonic excitations, arising from electrons interacting with the electromagnetic radiation. The well-known polaritonic dispersion equation is given by $\omega_1 = \sqrt{\omega_p^2 + \omega^2}$, where $\omega_p = 4\pi ne^2/m$ is the plasma frequency (n being the plasma density, $-e$ - the electron charge and m - the electron mass) and $\omega = ck$ is the frequency of the laser electromagnetic wave (where $\mathbf{k}$ is the wavevector and c denotes the light velocity). Polaritonic pulses propagating with the group velocity $\mathbf{v} = c^2\mathbf{k}/\omega_1$ can be formed by a superposition of plane waves. Such a superposition can be obtaiend by taking $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$, where $\mathbf{k}_0$ is the wavevector of the laser radiation (frequency $\omega_0 = ck_0$, wavelength $\lambda_0 = 2\pi/k_0$) and the $\mathbf{q}$'s are restricted to $q < q_c \ll k_0$. A wavepacket of linear size $d \simeq 1/q_c \gg \lambda_0$ is then obtained, propagating with the group velocity $v = c\omega_0/\sqrt{\omega_p^2 + \omega_0^2}$. In the particular case of a sufficiently rarefied plasma $\omega_p \ll \omega_0$ this group velocity can be written as $v \simeq c(1 - \omega_p^2/2\omega_0^2)$ and the mobile electrons are transported with the energy

$$E_{el} = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \simeq mc^2\frac{\omega_0}{\omega_p} \gg mc^2 \ , \tag{1}$$

which may acquire values much higher than the electron rest energy $mc^2 = 0.5MeV$. For typical values $\hbar\omega_0 = 1eV$ ($\lambda_0 = 2\pi c/\omega_0 \simeq 1\mu m$ and $\hbar$ is Planck's constant) and an electron density $n = 10^{18}cm^{-3}$ we get $\hbar\omega_p = 3\times 10^{-2}eV$ and $E_{el} \simeq 17MeV$.

We consider the well known plasma model consisting of electrons moving in a neutralizing, rigid (or quasi-rigid) background of positive ions. Let $\mathbf{u}(\mathbf{r},t)$ be a displacement field in electron positions, such as to create a small volume density imbalance $\delta n = -ndiv\mathbf{u}$. We have, therefore, a charge density $\rho = endiv\mathbf{u}$ and a current density $\mathbf{j} = -en\dot{\mathbf{u}}$. The polarization electric ($\mathbf{E}$) and magnetic ($\mathbf{H}$) fields obey the Maxwell equations

$$\begin{gathered} div\mathbf{E} = 4\pi endiv\mathbf{u}\ ,\ div\mathbf{H} = 0\ , \\ curl\mathbf{E} = -\tfrac{1}{c}\tfrac{\partial\mathbf{H}}{\partial t}\ ,\ curl\mathbf{H} = \tfrac{1}{c}\tfrac{\partial\mathbf{E}}{\partial t} - \tfrac{4\pi en}{c}\tfrac{\partial\mathbf{u}}{\partial t}\ , \end{gathered} \tag{2}$$

where we assume a non-magnetic plasma (*i.e.* the magnetization is zero, and the magnetic field is equal with the magnetic induction). It is easy to see that equations (2) lead to

$$\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \Delta\mathbf{E} = -4\pi engrad\cdot div\mathbf{u} + \frac{4\pi en}{c^2}\frac{\partial^2\mathbf{u}}{\partial t^2}\ . \tag{3}$$

We assume that the effect of the pulsed electromagnetic fields on the electron motion is non-relativistic, as a consequence of the high polarization field which may compensate to a large extent the original laser field. We assume therefore the Newton's law for the electron motion

$$m\ddot{\mathbf{u}} = -e\mathbf{E} - e\mathbf{E}_0 \tag{4}$$

under the action of the electric field, where $\mathbf{E}_0$ is the external electric field of the laser pulse. We note in equation (4) the absence of the Lorentz force and the approximation of the total time derivative with the partial time derivative, as for non-relativistic motion.

Making use of Fourier transforms of the type

$$\mathbf{u}(\mathbf{r},t) = \frac{1}{(2\pi)^4}\int d\mathbf{k}d\omega\mathbf{u}(\mathbf{k},\omega)e^{i(\mathbf{kr}-i\omega t)}\ , \tag{5}$$

we get easily from equations (3) and (4)

$$\omega^2(\omega^2-\omega_p^2-c^2k^2)\mathbf{u} + \omega_p^2c^2\mathbf{k}(\mathbf{ku}) = \frac{e}{m}(\omega^2-c^2k^2)\mathbf{E}_0\ ; \tag{6}$$

hence, we get

$$\omega^2(\omega^2-\omega_p^2-c^2k^2)\mathbf{u} = -\frac{e\omega_p^2c^2}{m}\mathbf{k}\frac{\mathbf{kE}_0}{\omega^2-\omega_p^2} + \frac{e}{m}(\omega^2-c^2k^2)\mathbf{E}_0\ , \tag{7}$$

where we can read the two well-known branches of elementary excitations: longitudinal plasmons, with frequency ω_p, and transverse polaritons, propagating with frequency $\omega_1 = \sqrt{\omega_p^2+c^2k^2}$.[19] The plasmons do not "propagate", in the sense that their group velocity is vanishing. We assume that the external field is transverse ($\mathbf{kE}_0 = 0$), and get rid of the plasmon term in equation (7). We get therefore $\mathbf{ku} = 0$ (and $\mathbf{kE} = 0$), *i.e.* a vanishing volume charge density, as expected, and a transverse displacement field given by

$$\mathbf{u} = \frac{e}{m}\frac{\omega^2-c^2k^2}{\omega^2(\omega^2-\omega_1^2)}\mathbf{E}_0\ . \tag{8}$$

It is convenient to introduce the vector potential $\mathbf{A}_0 = -\frac{ic}{\omega}\mathbf{E}_0$ in equation (8), where we perform first the inverse Fourier transform with respect to the frequency, and retain only the ω_1-contribution. The full inverse Fourier transform of equation (8) reads

$$\mathbf{u}(\mathbf{r},t) = -\frac{e\omega_p^2}{4mc}\frac{1}{(2\pi)^3}\int d\mathbf{k}\frac{1}{\omega_1^2}\mathbf{A}_0(\mathbf{k},\omega_1)e^{i(\mathbf{kr}-\omega_1 t)} \ . \tag{9}$$

We focus now, in equation (9), on a certain wavevector $\mathbf{k}_0$ and make a series expansion of ω_1 in powers of $\mathbf{q} = \mathbf{k} - \mathbf{k}_0$, where $0 < q < q_c$, the cutoff wavevector q_c being such that $q_c \ll k_0$. We assume an isotropic cutoff wavevector. As it is well known, we get an isotropic wave packet extending approximately over the length $d = 2\pi/q_c \gg \lambda_{10}$, where λ_{10} is the wavelength of the wave with frequency $\omega_{10} = \sqrt{\omega_p^2 + c^2k_0^2}$. This pulse is propagating with the group velocity $\mathbf{v} = \partial\omega_1/\partial\mathbf{k}$ for $\mathbf{k} = \mathbf{k}_0$, given by

$$\mathbf{v} = \frac{c^2\mathbf{k}_0}{\sqrt{\omega_p^2 + c^2k_0^2}} \ . \tag{10}$$

The displacement field $\mathbf{u}(\mathbf{r},t)$ given by equation (9) can be represented as

$$\mathbf{u}(\mathbf{r},t) \simeq -\frac{e\omega_p^2}{4mc\omega_{10}^2}\mathbf{A}_0(\mathbf{k}_0,\omega_{10})\delta(\mathbf{r}-\mathbf{v}t) \ . \tag{11}$$

It is worth emphasizing that the δ-function of the pulse is in fact a representation for a function of the type $\sim (\sin q_c x/x)(\sin q_c y/y)(\sin q_c z/z)$, along the propagation direction, where $x = r - vt$ is the coordinate along the pulse motion and y, z denote the transverse coordinates, perpendicular to the direction of motion. We can see that this function is localized over the volume $\sim d^3$, and has a peaked height $\sim q_c^3 \sim 1/d^3$.

Let us assume $\omega_p \ll \omega_0 = ck_0$. With this assumption the displacement in the pulse given by equation (11) can be written as

$$\mathbf{u}_0 \simeq -\frac{e\omega_p^2}{4mc\omega_0^2 d^3}\mathbf{A}_0(\mathbf{k}_0,\omega_0) \ . \tag{12}$$

It is easy to see that a similar pulse is obtained for the vector potential $\mathbf{A}_0$. We take it of the form

$$\mathbf{A}_0(\mathbf{r},t) = \mathbf{A}_0 d^3\delta(\mathbf{r}-\mathbf{v}t) \ , \tag{13}$$

where $\mathbf{A}_0$ is real. It consists of a superposition of frequencies in the range $\Delta\omega = cq_c = 2\pi c/d$, so we have approximately $\mathbf{A}_0(\mathbf{k}_0,\omega_0) \simeq \mathbf{A}_0 d^4/c$ and get finally

$$\mathbf{u}_0 \simeq -\frac{e\omega_p^2 d}{4mc^2\omega_0^2}\mathbf{A}_0 \ . \tag{14}$$

As we said above, the displacement $\mathbf{u}_0$ is transverse ($\mathbf{k}_0\mathbf{u}_0 = 0$), and there is no volume charge density in the pulse. The charge is distributed transversally toward the pulse surface. Let us assume that this distribution extends over a region of thickness l; then, we may take approximately $\delta n_0 = nu_0/l$ for the electron density imbalance, where l is of the order of the wavelength λ_0, for a perfect δ-pulse. We get the total number of electrons in the pulse

$$N \simeq \pi n d^3\frac{e\omega_p^2}{4mc^2\omega_0^2}A_0 \ , \tag{15}$$

where we can see that N does not depend on the thickness l. It is convenient to express the vector potential A_0 by means of the density of the field energy $w_0 = k_0^2 A_0^2/4\pi$. Making use of the notations $\varepsilon_p = \hbar\omega_p$, $\varepsilon_0 = \hbar\omega_0$ and $\varepsilon_{el} = e^2/d$, the later being the Coulomb energy of an electron localized in the pulse, we get

$$N = nd^2\lambda_0 \frac{\varepsilon_p^2}{4mc^2\varepsilon_0^2}\sqrt{\pi\varepsilon_{el}W_0} \ , \tag{16}$$

where W_0 is the total amount of field energy in the pulse ($W_0 = I_0 d^3/c$, where I_0 is the laser intensity). For typical values $I_0 = 10^{18} w/cm^2$, $d = 1mm$ ($W_0 = 10^{23}eV$ and $\varepsilon_{el} = 10^{-6}eV$), $n = 10^{18}cm^{-3}$ ($\varepsilon_p = 3\times 10^{-2}eV$), $\varepsilon_0 = 1eV$ ($\lambda_0 \simeq 1\mu$) and $mc^2 = 0.5MeV$ we get $N \simeq 10^{11}$ electrons in the pulse (transported with the energy $\simeq 17MeV$), wich is a relatively high flux of electrons. Their total energy is $W \simeq 10^{18}eV$, the remaining energy (up to $W_0 = 10^{23}eV$) being left in the polarized laser pulse. Numerical data from recent experimental measurements[11]-[13] seem to be in fair agreement with equations (1) and (16) given here.

The propagating polaritonic pulse is polarized, in the sense that the mobile electrons in the propagating pulse are displaced from their equilibrium positions with respect to the quasi-rigid background of positive ions, such that the polarization field compensates, practically, the laser field. The electrons inside the pulse accumulate on the surface of the pulse, along a direction which is transverse to the direction of pulse propagation (laser radiation is transverse), such as a new equibrium is reached, in the presence of the laser field. Using the same numerical values as above we can estimate the displacement given by equation (12) as $u_0 = N/\pi nd^2 \simeq 10^{-2}\mu m$, which is a very small displacement, as expected. It is worth noting that the pulsed fields acquire a very small frequency, arising from the factor $e^{i(\omega_{10}t - \mathbf{k}_0\mathbf{r})}$ which is omitted in equations (11) and (12). Since $\omega_{10} = \sqrt{\omega_p^2 + \omega_0^2}$, it is easy to see that this frequency is of the order of $\Omega = \omega_p^2/2\omega_0$, so the electron velocity in the pulse is of the order of $\Omega u_0 = \omega_p^2 u_0/2\omega_0$. This is a very small velocity in comparison with the velocity of light ($\simeq 10^{-4}c$), which justifies the non-relativistic approximation in treating the electron motion. The polarization charge oscillates slowly in the pulse, with a small phase velocity, $v_f = \Omega/k_0 = c\omega_p^2/\omega_0^2 \simeq 10^{-3}c$. It is the trapped motion carried along by the pulse that made the electrons to acquire relativistic velocities. This motion is decoupled from the displacement $\mathbf{u}$, it pertains to the pulse coordinate $\mathbf{r}$. The motion of the electrons as described here is an inertial transport. The charge is polarized by the external field E_0 and the electrons are kept inside the pulse by the polarization field E.

The positive ions in plasma are rigid (or quasi-rigid) in comparison with the electrons, which are mobile. While the latter are carried along by the pulse, the former will be depolarized by a wake field and an electron backflow, which give rise to plasma oscillations outside the pulse. This is the well-known picture of wakefield accelerated electrons, and the related bubble models.[1], [14]-[18] Therefore, the pulse energy is also spend for creating these depolarizing plasma oscillations in the sample, as expected. An unpolarized electron in the process of being accelerated by the pulse will experience an uncompensated field of the order of E_0 (or the compensating polarization field E). The energy gain E_{el} of an accelerated electron is therefore obtained by the work of the force eE_0 over a distance δ. With our numerical values used here we get $\delta \simeq 10\mu m$, which may give an estimate for the surface thickness l of the pulse (or the contrast thickness of the pulse).

The polarized electrons in the polaritonic pulse are practically quasi-rigid (though subjected to very slow density oscillations). The quasi-rigid electrons in the polaritonic pulse moving with relativistic velocities offer a unique opportunity of coherent Compton backscattering, which may produce coherent high-energetic X- or even gamma rays, *i.e.* an X-ray or gamma-ray laser.

We assume a head-on (unpolarized) electron-photon collision. With usual notations $p = (E, \mathbf{p})$ and $k = (\omega, \mathbf{k})$ are the electron and, respectively, photon 4-momenta, and we set $c = \hbar = 1$.

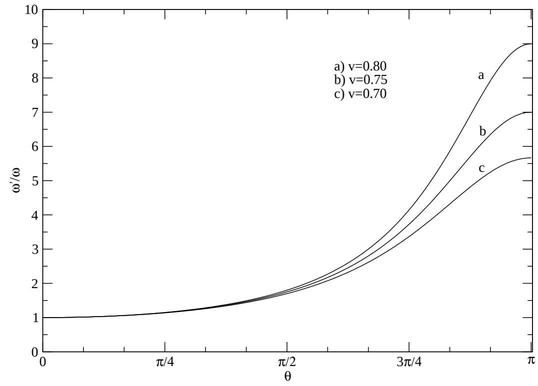

Figure 1: The ratio of the energy of the scattered photon to the energy of the incident photon *vs* scattering angle for a few values of the polariton (electron) velocity v (equation (18) for $1-v \gg 4\gamma^2(1+v)$).

$E = E_{el}$ denotes here the electron energy (not to be mistaken for the electric field). . From the momentum-energy conservation $p+k = p'+k'$, written as $p' = p+k-k'$, we get $pk - pk' - kk' = 0$, or, making use of $p^2 = p'^2 = m^2$, $k^2 = k'^2 = 0$,

$$\omega' = \omega \frac{E + |\mathbf{p}|}{E + |\mathbf{p}|\cos\theta + \omega(1-\cos\theta)} \ . \tag{17}$$

Since $|\mathbf{p}| = vE = mv/\sqrt{1-v^2}$, this equation can also be written as

$$\omega' = \omega \frac{1+v}{1 + v\cos\theta + \gamma\sqrt{1-v^2}(1-\cos\theta)} \ , \tag{18}$$

where $\gamma = \omega/m$ and v is the velociy of the electron (velocity of the polaritonic pulse). For all relevant situations (except ultrarelativistic limit) the inequality $2\gamma\sqrt{1+v} \ll \sqrt{1-v}$ is satisfied (Thomson scattering). The ratio ω'/ω given by equation (18) *vs* angle θ is shown in Fig. 1 in this case (for $4\gamma^2(1+v) \ll 1-v$) for a few values of the parameter v. The maximum value of the frequency ω' of the scattered photon is obtained for the scattering angle $\theta \simeq \pi$ (backscatering). This increase is sometimes assigned to a Doppler effect, which would introduce a relativistic factor $4/(1-v^2) \simeq (1+v)/(1-v)$ for $v \simeq 1$. For the typical parameter values used in this paper $1-v \simeq \omega_p^2/2\omega_0^2 \simeq 4.5\times10^{-4}$, which is much greater than $2\gamma\sqrt{1-v^2} \simeq 10^{-7}$ (we take the frequency of the incident photon $\omega = 1eV$, $\gamma \simeq 2\times10^{-6}$). Therefore, we may neglet the γ-term in equation (18), and get a maximum scattered frequency

$$\omega' \simeq \omega\frac{1+v}{1-v} \simeq 10keV \tag{19}$$

for the backscattering angle $\theta = \pi$. It is easy to see that an increase by an order of magnitude in the energy of the accelerated electrons ($E_{el} = E \simeq m\omega_0/\omega_p$) means a decrease by two orders of magnitude in $1-v$ ($1-v \simeq \omega_p^2/2\omega_0^2$), such that, by equation (19), we may get $\omega' \simeq 1MeV$ for the frequency of the backscattered gamma rays. Such high backscattering frequencies are concentrated around $\theta = \pi$ within a range $\Delta\theta \simeq \sqrt{2(1-v)/3v}$.

The well-known Compton cross-section can be written as[19]

$$\begin{aligned} d\sigma = 8\pi r_e^2 \tfrac{m^2 d(-t)}{(s-m^2)^2}[\left(\tfrac{m^2}{s-m^2} + \tfrac{m^2}{u-m^2}\right)^2 + \\ + \tfrac{m^2}{s-m^2} + \tfrac{m^2}{u-m^2} - \tfrac{1}{4}\left(\tfrac{s-m^2}{u-m^2} + \tfrac{u-m^2}{s-m^2}\right)] \ , \end{aligned} \tag{20}$$

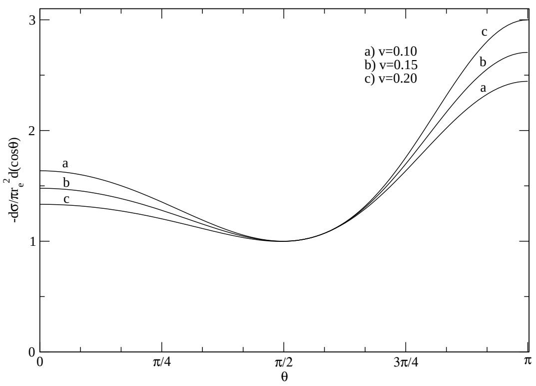

Figure 2: Compton cross-section *vs* scattering angle for a few values of the polariton (electron) velocity v (equation (23), $1-v \gg 4\gamma^2(1+v)$).

where $r_e = e^2/m$ is the classical electron radius and

$$s=(p+k)^2=m^2+2pk\ ,\ \ u=(p-k')^2=m^2-2pk'\ ,$$
$$t=(k'-k)^2=-2kk' \tag{21}$$

are the invariant kinematical variables. By straightforward calculations this expression can be put in the form

$$d\sigma=\pi r_e^2\frac{(1-v^2)\sin\theta d\theta}{\left[1+v\cos\theta+\gamma\sqrt{1-v^2}(1-\cos\theta)\right]^2}\times$$
$$\times\left[\left(\frac{v+\cos\theta}{1+v\cos\theta}\right)^2+\gamma\sqrt{1-v^2}\frac{1-\cos\theta}{1+v\cos\theta}+\frac{1+v\cos\theta}{1+v\cos\theta+\gamma\sqrt{1-v^2}(1-\cos\theta)}\right]\ , \tag{22}$$

where the transport velociy v is shown explicitly. Similarly, for the parameter values used here we may neglect the γ-terms in equation (22) (Thomson scattering), and get

$$d\sigma\simeq\pi r_e^2\frac{(1-v^2)}{(1+v\cos\theta)^2}\left[\left(\frac{v+\cos\theta}{1+v\cos\theta}\right)^2+1\right]\sin\theta d\theta\ . \tag{23}$$

This cross-section is shown in Fig. 2 for a few values of the parameter v. The total backscattering cross-section is given by

$$\sigma_b\simeq\pi r_e^2\frac{(1-v^2)}{(1+v\cos\theta)^2}\left[\left(\frac{v+\cos\theta}{1+v\cos\theta}\right)^2+1\right]\Big|_{\theta=\pi}\Delta(-\cos\theta)=$$
$$=\pi r_e^2\frac{1+v}{1-v}(\Delta\theta)^2\simeq 4\pi r_e^2/3 \tag{24}$$

and the rate of the bakscattered photons is $dN_{ph}/dt=c\sigma_b n_{ph}$, where n_{ph} is the photon density in the incident flux. The energy loss of the scattered (recoil) electron for backscatering is $\Delta E=\omega'-\omega\simeq 2\omega v/(1-v)$ ($\Delta E/E\simeq 2\gamma v\sqrt{(1+v)/(1-v)}\ll 1$), which is approximately equal with the energy of the scattered photon $\omega'\simeq\omega(1+v)/(1-v)$ given above for $v\simeq 1$ (since $\omega\ll\omega'$). The momentum transferred to the electron in the scattering process is very small, in comparison with the initial momentum of the electron. It is important to note that for a polaritonic pulse this momentum is transferred to the whole ensemble of electrons, as a consequence of the rigidity of the electrons in the polaritonic pulse. For the sake of the comparison, we note that the total cross-section is $8\pi r_e^2/3\simeq 2\sigma_b$, as it is well known.

The cross-section computed above refers to one electron (and one photon). The field bi-spinors in the interaction matrix element (the scattering amplitude) between the initial state and the final state are normalized to unity. If we have N electrons, then each of them contributes individually to the cross-section, which is multiplied by N (*i.e.* $\sigma_b \rightarrow N\sigma_b$). This is an incoherent scattering. For the electrons in the polaritonic pulse the situation is different. These electrons are not independent anymore (because of their rigidity inside the pulse), and they suffer the scattering collectively. This amounts to normalize the bi-spinors to N, such that each bi-spinor carries now a factor $\sqrt{N}$. Consequently, the scattering amplitude acquires an additional factor N and the cross-section acquires an additional factor N^2. In comparison with the incoherent scattering we get an additional factor N in the coherent scattering, which increases considerably the cross-section for large values of N.[20]-[24]

From the above estimations we can see that the energy of the backscattered photons is much higher than the energy of the incident photons. Therefore, in the following estimations we can neglect the energy of the incident photons. The energy of the scattered photons is produced at the expense of the energy of the electrons. By successive Compton scattering we may expect a certain limitation on the duration of the scattering process for electron pulses (beside the limitations caused by the pulse duration, both for the electrons and the incident photons). Such a limitation is more stringent for the coherent scattering (due to the occurrence of the factor N^2).

Making use of the rate $d^2N_{ph}/d\theta dt = c(d\sigma/d\theta)n_{ph}$ of the scattered photons we can write down the rate of the energy produced by Compton (Thomson) scattering

$$dE^{coh} = \left(\int d\theta\omega' dN_{ph}/d\theta\right) dt = N^2 cn_{ph}\left(\int \omega' d\sigma\right) dt \ . \tag{25}$$

The integral in equation (25) can be computed by using ω' given by equation (18) (with $\gamma = 0$) and the differential cross-section given by equation (23). The result is

$$dE^{coh} = \frac{8\pi}{3}N^2\omega c r_e^2 n_{ph}\frac{1}{1-v}dt \ . \tag{26}$$

This energy must be compared with the energy loss of the electrons in the polaritonic pulse,

$$-NdE = -Nmd\frac{1}{\sqrt{1-v^2}} \ . \tag{27}$$

Integrating the equation $dE^{coh} = -NdE$ with the new variable $x = m/E$, we get easily

$$\frac{8\pi}{3}N\omega cn_{ph}\Delta t = m\int_{x_0}^{1} dx\frac{1}{1+\sqrt{1-x^2}} \ , \tag{28}$$

where $x_0 = m/E_0 \ll 1$ corresponds to the initial energy of the polaritonic pulse. The integral in equation (28) can easily be estimated ($\simeq \pi/2 - 1$), so we get the duration Δt of the scattering

$$\Delta t \simeq (\pi/2 - 1)\frac{3mc}{8\pi N\hbar\omega r_e^2 n_{ph}} \ , \tag{29}$$

where we have re-established in full the universal constants.

We assume an incident flow of photons with intensity $I = 10^{14}w/cm^2$ focused on a spatial region of size $d = 1mm$ (picosecond pulses); the energy is $W = Id^3/c \simeq 3J$ and, for photon energy $\omega = 1eV$, we get a photon density $n_{ph} \simeq 5\times 10^{22}cm^{-3}$. For $N = 10^{11}$ given before for the polaritonic pulse (and $r_e = 2.8\times 10^{-13}cm$) we get $\Delta t \simeq 10^{-15}s$ (femtoseconds). This time is an estimate for the duration of the collision, and for the duration of emission of the backscattered photons. As we can see, it does not depend, practically, on the electron energy in the polaritonic pulse (E_0), for high, relativistic energies. It is expected that the polaritonic pulse is "stopped", and, in fact, destroyed, after the lapse of this time.

The total energy of the backscatterd photons can be estimated similarly, by using equation (25) and dt/dE from $dE^{coh} = -NdE$, where dE^{coh} is given by equation (26). Let us assume that we are interested in the photon bakscattering within an angle $\Delta\theta = \alpha\sqrt{2(1-v)/3v}$, with $\alpha \ll 1$. Then, we get easily

$$E_b^{coh} = \frac{1}{4}N\alpha^2 \int_m^{E_0} \frac{(1+v)^2}{v} dE \ , \tag{30}$$

and, folowing the same technique as above, we get $E_b^{coh} \simeq \alpha^2 N E_0$, where we can recognize the total energy of the polaritonic pulse $W_{el} = NE_0$. This result is valid for $\alpha \ll 1$. For high, relativistic velocities $\alpha \simeq 1$, and practically the whole polaritonic energy is recovered in the backscattering photons.

In conclusion, we may say that the polaritonic pulses of electrons transported by laser radiation focused in a rarefied plasma may serve as targets for coherent Compton backscattering in the X-rays or gamma rays energy range, therefore as a means for obtaining an X-ray or gamma ray laser. The coherent scattering, which enhances considerably the photon output and ensure its coherence, is due to the quasi-rigidity of the electrons in the propagating polaritonic pulse, which ensures (within certain limits) the stability of this interacting formation of matter and electromagnetic radiation. The energy and cross-section of the Compton (Thomson) backscattering was re-examined in this paper in the context of the coherent scattering by polaritonic pulses, and the (pulse) duration of the backscattering emission was also estimated. Similar ideas have been advanced recently, especially for laser-driven accelerated electron mirrors.[26]-[32]

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References

[1] T. Tajima and J. M. Dawson, "Laser electron accelerator," Phys. Rev. Lett. **43** 267-270 (1979).

[2] G. Mourou, T. Tajima and S. S. Bulanov, "Optics in the relativistic regime," Revs. Mod. Phys. **78** 309-371 (2006).

[3] E. Esarey, S. B. Schroeder and W. P. Leemans, "Physics of laser-driven plasma-based electron accelerators," Revs. Mod. Phys. **81** 1229-1285 (2009).

[4] S. P. D. Mangles, C. D. Murphy, Z. Najmudin, A. G. R. Thomas, J. R. Collier, A. E. Dangor, E. J. Divall, P. S. Foster, J. G. Gallacher, C. J. Hooker, D. A. Jaroszynski, A. J. Langley, W. B. Mori, P. A. Norreys, F. S. Tsung, R. Viskup, B. R. Walton and K. Krushelnick, "Monoenergetic beams of relativistic electrons from intense laser-plasma interactions," Nature **431** 535-538 (2004).

[5] C. G. R. Geddes, Cs. Toth, J. van Tilborg, E. Esarey, C. B. Schroeder, D. Bruhwiler, C. Nieter, J. Cary and W. P. Leemans, "High-quality electron beams from a laser wakefield accelerator using plasma-channel guiding," Nature **431** 538-541 (2004).

[6] J. Faure, Y. Glinec, A. Pukhov, S. Kiselev, S. Gordienko, E. Levebvre, J.-P. Rousseau, F. Burgy and V. Malka, "A laser-plasma accelerator producing monoenergetic electron beams," Nature **431** 541-544 (2004).

[7] W. P. Leemans, B. Nagler, A. J. Gonsalves, Cs. Toth, K. Nakamura, C. G. R. Geddes, E. Esarey, C. B. Schroeder and S. M. Hooker, "GeV electron beams from a centimetre-scale acelerator," Nature Phys. **2** 696-699 (2006).

[8] J. Faure, C. Rechatin, A. Norlin, A. Lifschitz, Y. Glinec and V. Malka, "Controlled injection and acceleration of electrons in plasma wakefields by colliding laser pulses," Nature **444** 737-739 (2006).

[9] C. G. R. Geddes, K. Nakamura, G. R. Plateau, Cs. Toth, E. Cormier-Michel, E. Esarey, C. B. Schroeder, J. R. Cary and W. P. Leemans, "Plasma-density-gradient injection of low absolute-momentum-spread electron bunches," Phys. Rev. Lett. **100** 215004 (2008) (1-4).

[10] C. Rechatin, J. Faure, A. Ben-Ismail, J. Lim, R. Fitour, A. Specka, H. Videau, A. Tafzi, F. Burgy and V. Malka, "Controlling the phase-space volume of injected electrons in a laser-plasma accelerator," Phys. Rev. Lett. **102** 164801 (2009) (1-4).

[11] S. F. Martins, R. A. Fonseca, W. Lu, W. B. Mori and L. O. Silva, "Exploring laser-wakefield-accelerator regimes for near-term lasers using particle-in-cell simulation in Lorentz boosted frames," Nature Phys. **6** 311-316 (2010).

[12] A. Giulietti, N. Bourgeois, T. Ceccotti, X. Davoine, S. Dobosz, P. D'Oliveira, M. Galimberti, J. Galy, A. Gamucci, D. Giulietti, L. A. Gizzi, D. J. Hamilton, E. Lefebvre, L. Labate, J. R. Marques, P. Monat, H. Popescu, F. Reau, G. Sarri, P. Tomassini and P. Martin, "Intense γ-ray source in the giant-dipole-resonance range driven by $10-Tw$ laser pulses," Phys. Rev. Lett. **101** 105002 (2008) (1-4).

[13] A. G. Mordovanakis, J. Easter, N. Naumova, K. Popov, P.-E. Masson-Laborde, B. Hou, I. Sokolov, G. Mourou, I. V. Glazyrin, W. Rozmus, V. Bychenkov, J. Nees and K. Krushelnick, "Quasimonoenergetic electron beams with relativistic energies and ultrashort duration from laser-solid interactions at $0.5\,kHz$," Phys. Rev. Lett. **103** 235001 (2009) (1-4).

[14] S. Kalmykov, S. A. Yi, V. Khudik and G. Shvets, "Electron self-injection and trapping into an evolving plasma bubble," Phys. Rev. Lett. **103** 135004 (2009) (1-4).

[15] I. Kostyukov, E. Nerush, A. Pukhov and V. Seredov, "Electron self-injection in multidimensional relativistic-plasma wake fields," Phys. Rev. Lett. **103** 175003 (2009) (1-4).

[16] P. Dong, S. A. Reed, S. A. Yi, S. Kalmykov, G. Shvets, M. C. Downer, N. H. Mattis, W. P. Leemans, C. McGuffey, S. S. Bulanov, V. Chvykov, G. Kalintchenko, K. Krushelnick, A. Maksimchuk, T. Matsuoka, A. G. R. Thomas and V. Yanovsky, "Formation of optical bullets in laser-driven plasma bubble accelerators," Phys. Rev. Lett. **104** 134801 (2010) (1-4).

[17] J. Xu, B. Shen, X. Zhang, M. Wen, L. Ji, W. Wang, Y. Yu and K. Nakajima, New J. Phys. **12** 023037 (2010), doi: 10.1088/1367-2630/12/2/023037 (1-9).

[18] C. McGuffey, A. G. R. Thomas, W. Shumaker, T. Matsuoka, V. Chvykov, F. J. Dollar, G. Kalintchenko, V. Yanovsky, A. Maksimchuk, K. Krushelnick, V. Yu. Bychenkov, I. V. Glazyrin and A. V. Karpeev, "Ionization induced trapping in a laser wakefield accelerator," Phys. Rev. Lett. **104** 025004 (2010) (1-4).

[19] T. Tajima and S. Ushioda, "Surface polaritons in LO-phonon-plasmon coupled systems in semiconductors," Phys. Rev. **B18** 1892-1897 (1978).

[20] L. Landau and E. M. Lifshitz, *Course of Theoretical Phyiscs*, vol. 4, (*Quantum Electrodynamics*, V. B. Berestetskii, E. M. Lifshitz and L. P. Pitaevskii), Butterworth-Heinemann, Oxford (2002).

[21] J. Weber, "Method for observation for neutrinos and antineutrinos," Phys. Rev. **C31** 1468-1475 (1985).

[22] J. Weber, "Apparent observation of abnormally large coherent scattering cross sections using keV and MeV range antineutrinos, and solar neutrions," Phys. Rev. **D38** 32-39 (1988).

[23] G. Preparata, *QED Coherence in Matter*, World Scientific (1995).

[24] M. Apostol, "Weber's coherent scattering and neutrino detection," Roum. Reps. Phys. **60** 315-325 (2008).

[25] M. Apostol and M. Ganciu, "Coherent polarization driven by external electromagnetic fields," Phys. Lett. A, doi: 10.1016/j.physleta. 2010. 10.017.

[26] S. Miyamoto, Y. Asano, S. Amano, D. Li, K. Imasaki, H. Kinugasa, Y. Shoji, T. Takagi, T. Mochizuki, "Laser Compton back-scattering gamma-ray beam line on NewSUBARU," et al, Rad. Meas. **41**, Suppl. 2, S179-S185 (2006).

[27] W. Guo, W. Xu, J. G. Chen, Y. G. Ma, X. Z. Cai, H. W. Wang, Y. Xu, C. B. Wang, G. C. Lu, W. D. Tian, R. Y. Yuan, J. Q. Xu, Z. Y. Wei, Z. Yan, W. Q. Shen, "A high intensity beam line of γ-rays up to 22 MeV energy based on Compton backscattering," Nucl. Instr.&Methods **A578** 457-462 (2007).

[28] D. Habs, M. Hegelich, J. Schreiber, M. Gross, A. Henig, D. Kiefer and D. Jung, "Dense laser-driven electron sheets as relativistic mirrors for coherent production of brilliant X-ray and γ-ray beams," Appl. Phys. **B93** 349-354 (2008).

[29] K. Kawase, Y. Arimoto, M. Fujiwara, S. Okajima, M. Shoji, S. Suzuki, K. Tamura, T. Yorita and H. Ohkuma, "MeV γ-generation from backward Compton scattering at SPring-8," Nucl. Instr.&Methods, **A592** 154-161 (2008).

[30] J. Meyer-ter-Vehn and H. C. Wu, "Coherent Thomson backscattering from laser-driven relativistic ultra-thin electron layers," Eur. Phys. J. **D55** 433-441 (2009).

[31] B. Qiao, M. Zepf, M. Borghesi, B. Dromey and M. Geissler, "Coherent X-ray production via pulse reflection from laser-driven dense electron sheets" New. J. Phys. **11** 103042 (2009), doi: 10.1088/1367-2630/11/10/10.3042 (1-11).

[32] H. C. Wu, J. Meyer-ter-Vehn, J. Fernandez and B. M. Hegelich, "Uniform laser-driven relativistic electron layer for coherent Thomson scattering," Phys. Rev. Lett. **104** 234801 (2010) (1-4).