

Coupling of (ultra-) relativistic atomic nuclei with photons

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Abstract

The coupling of photons with (ultra-) relativistic atomic nuclei is presented in two particular circumstances: very high electromagnetic fields and very short photon pulses. We consider a typical situation where the (bare) nuclei (fully stripped of electrons) are accelerated to energies $\simeq 1\text{TeV}$ per nucleon (according to the state of the art at LHC, for instance) and photon sources like petawatt lasers $\simeq 1\text{eV}$ -radiation (envisaged by ELI-NP project, for instance), or free-electron laser $\simeq 10\text{keV}$ -radiation, or synchrotron sources, etc. In these circumstances the nuclear scale energy can be attained, with very high field intensities. In particular, we analyze the nuclear transitions induced by the radiation, including both one- and two-photon processes, as well as the polarization-driven transitions which may lead to giant dipole resonances. The nuclear (electrical) polarization concept is introduced. It is shown that the perturbation theory for photo-nuclear reactions is applicable, although the field intensity is high, since the corresponding interaction energy is low and the interaction time (pulse duration) is short. It is also shown that the description of the giant nuclear dipole resonance requires the dynamics of the nuclear electrical polarization degrees of freedom.

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1 Introduction. Accelerated ions

It is well known that the nuclear photoreactions occur in the $\text{keV} - \text{MeV}$ -energy range. In particular, the characteristic energy of the giant dipole resonance (which implies oscillations of protons with respect to neutrons) is $10 - 20\text{MeV}$.¹⁻⁴ In order to get this energy scale typical mechanisms are used, like Compton backscattering (for instance a laser-electron system), or electron bremsstrahlung (usually with the same nucleus acting both as converter and target), etc.⁵⁻¹⁸ High intensity laser pulses can be used for accelerating electrons in compact laser-plasma configurations.^{3,17} High-power and short-pulsed lasers are pursued nowadays for increasing the intensity of the electromagnetic field.¹⁹ Photon-ion or photon-photon mediated ion-ion interactions

are also well known in the so-called peripheral reactions.^{20,21} Vacuum polarization effects have also been discussed recently in high-energy photon-proton collisions,²² or light-by-light scattering in multi-photon Compton effect.^{23–25} We describe here a high-energy and high-field intensity coupling of the atomic nucleus to photons from various sources (*e.g.*, optical laser, free electron laser, synchrotron radiation) by using (ultra-) relativistic atomic nuclei.

We consider (ultra-) relativistically accelerated ions moving with velocity v along the x -axis. We envisage acceleration energies of the order $\varepsilon = 1\text{TeV}$ per nucleon (according to the state of the art at LHC, for instance).²⁶ At these energies the ion is fully stripped of its electrons, so we have a bare atomic nucleus. We assume that a beam of photons of frequency ω_0 is propagating counterwise (from a laser source, or a free electron laser, or a synchrotron source, etc), such that the photons suffer a head-on collision with the nucleus. The moving nucleus will "see" a photon frequency

$$\omega = \omega_0 \sqrt{\frac{1+\beta}{1-\beta}}, \quad \beta = v/c \quad (1)$$

in its rest frame, according to the Doppler effect. For (ultra-) relativistic nuclei ($\beta \simeq 1$) this frequency may acquire high values. For instance, we have

$$\beta \simeq 1 - \frac{\varepsilon_0^2}{2\varepsilon^2}, \quad \omega \simeq 2\omega_0 \frac{\varepsilon}{\varepsilon_0}, \quad (2)$$

where $\varepsilon_0 \simeq 1\text{GeV}$ is the nucleon rest energy; for $\varepsilon = 1\text{TeV}$ we get a photon frequency $\omega \simeq 2 \times 10^3 \omega_0$ ($\gamma = (1 - \beta^2)^{-1/2} \simeq \varepsilon/\varepsilon_0 = 10^3$). We can see that for a 1eV -laser we get 2keV -photons in the rest frame of the accelerated nucleus; for a 10keV -free electron laser we get 20MeV -photons, etc. The effect is tunable by varying the energy of the accelerated ions. This idea has been discussed in relation to hydrogen-like accelerated heavy ions, which may scatter resonantly X - or gamma-rays photons.²⁷ Similarly, a frequency up-shift was discussed for photons reflected by a relativistically flying plasma mirror generated by the laser-driven plasma wakefield,²⁸ or photons in the rest frame of an ultra-relativistic electron beam.^{24,29}

For a typical laser radiation (see, for instance, ELI-NP project,³⁰) we take a photon energy $\hbar\omega_0 = 1\text{eV}$ (wavelength $\lambda \simeq 1\mu\text{m}$), an energy $\mathcal{E} = 50\text{J}$ and a pulse duration $\tau = 50\text{fs}$. The pulse length is $l = 15\mu\text{m}$ (cca 15 wavelengths), the power is $P = 10^{15}\text{W}$ (1 pettawatt). For a $d^2 = (15\mu\text{m})^2$ -pulse cross-sectional area the intensity is $I = P/d^2 = 4 \times 10^{20}\text{W/cm}^2$. The electric field is $E \simeq 10^9\text{statvolt/cm}$ ($1\text{statvolt/cm} = 3 \times 10^4\text{V/m}$) and the magnetic field is $H = 10^9\text{Gs}$ ($1\text{T} = 10^4\text{Gs}$). These are very high fields (higher than atomic fields). The (ultra-) relativistic ion will see a shortened pulse of length $l' = \sqrt{1 - \beta^2}l$, with a shortened duration $\tau' = \sqrt{1 - \beta^2}\tau$ and an energy $\mathcal{E}' = \mathcal{E} \sqrt{(1 + \beta)/(1 - \beta)}$ (the number of photons $N_{ph} \simeq 10^{20}$ is invariant). It follows that the power and intensity are increased by the factor $(1 - \beta)^{-1}$ ($\simeq 2\gamma^2$) and the fields are increased by the factor $(1 - \beta)^{-1/2}$; for instance, $E' = E/\sqrt{1 - \beta} = \sqrt{2}(\varepsilon/\varepsilon_0)E \simeq 10^{12}\text{statvolt/cm}$; this figure is two orders of magnitude below Schwinger limit.

A higher enhancement can be obtained by taking into account the aberration of light, even from a collimated laser.^{31–33} Indeed, for a cross-sectional beam area $D^2 = (0.5\text{mm})^2$ we get an intensity $I = P/D^2 = 4 \times 10^{17}\text{W/cm}^2$ and an electric field $E \simeq 5 \times 10^7\text{statvolt/cm}$ (all the other parameters being the same). In the rest frame of the ion the power increases by a factor $(1 - \beta)^{-1}$, as before, but the cross-sectional area D'^2 of the beam, decreases by a factor $(1 - \beta)/(1 + \beta)$ ($\simeq 1/4\gamma^2$), as a consequence of the "forward beaming" (aberration of light);²⁸ we have $D'^2 = D^2(1 - \beta)/(1 + \beta)$, which leads to an enhancement factor $(1 + \beta)/(1 - \beta)^2$ for intensity and a factor $(1 + \beta)^{1/2}/(1 - \beta)$ ($\simeq 2\sqrt{2}\gamma^2$) for field. We get, for instance, $I' \simeq 3 \times 10^{24}\text{W/cm}^2$ and an electric field $E' \simeq 2\sqrt{2}\gamma^2 E \simeq 10^{14}\text{statvolt/cm}$.

Similarly, we can take as typical parameters for a free electron laser the photon energy $\hbar\omega_0 = 10\text{keV}$, the pulse duration $\tau = 50\text{fs}$ and a much lower energy $\mathcal{E} = 5 \times 10^{-5}J$ (power $P = 10\text{Gw}$); the fields may decrease by 3 orders of magnitude, but still they are very high ($10^9 - 10^{11}\text{statvolt/cm}$) in the rest frame of the accelerated ion.

Under these circumstances, the photons can attain energies sufficiently high for photonuclear reactions, or giant dipole resonances, with additional features arising from the electron-positron pair creation, vacuum polarization, etc; indeed, above $\simeq 1\text{MeV}$ the pair creation in the Coulomb field of the atomic nucleus becomes important. Vacuum polarization effects at very high intensity fields and high field frequency are still insufficiently explored. Beside, all these happen in two particular circumstances: very short times and very high electromagnetic fields. We discuss here the effect of these particular circumstances on typical phenomena related to photon-nucleus interaction.

2 Nuclear transitions

Let us consider an ensemble of interacting particles, some of them with electric charge, like protons and neutrons in the atomic nucleus, subjected to an external radiation field. We envisage quantum processes driven by field energy quantum of the order $\hbar\Omega = 10\text{MeV}$, as discussed above. First, we note that the motion of the particles at this energy is non-relativistic, since the particle rest energy $\simeq 1\text{GeV}$ is much higher than the energy quantum (we can check that the acceleration qE/m is much smaller than the "relativistic acceleration" $c\Omega$, where q and m is the particle charge and, respectively, mass and E denotes the electric field). Consequently, we start with the classical lagrangian $L = mv^2/2 - V + q\mathbf{v}\mathbf{A}/c - q\Phi$ of a particle with mass m and charge q , moving in the potential V and subjected to the action of an electromagnetic field with potentials Φ and \mathbf{A} ; \mathbf{v} is the particle velocity. We get immediately the momentum $\mathbf{p} = m\mathbf{v} + q\mathbf{A}/c$ and the hamiltonian

$$H = \frac{1}{2m}p^2 + V - \frac{q}{mc}\mathbf{p}\mathbf{A} + \frac{q^2}{2mc^2}A^2 + q\Phi . \quad (3)$$

Usually, the particle hamiltonian $p^2/2m + V$ is separated and quantized (V may be viewed as the mean-field potential of the nucleus), and the remaining terms are treated as a perturbation. In the first order of the perturbation theory we limit ourselves to the external radiation field, which is considered sufficiently weak. Consequently, we put $\mathbf{A} = \mathbf{A}_0$ and $\Phi = 0$ in equation (3) and take approximately $\mathbf{p} \simeq m\mathbf{v}$. We get the well known interaction hamiltonian

$$H_1 = -\frac{q}{c}\mathbf{v}\mathbf{A}_0 = -\frac{1}{c}\mathbf{J}\mathbf{A}_0 , \quad (4)$$

where $\mathbf{J} = q\mathbf{v}$ is the current; in the non-relativistic limit we include also the spin currents in \mathbf{J} . If we leave aside the spin currents, the interaction hamiltonian given by equation (4) can also be written as $q\mathbf{r}(d\mathbf{A}_0/dt)/c$. Usually, the field does not depend on position over the spatial extension of the ensemble of particles. Indeed, in the present case the wavelength of the quantum $\hbar\Omega = 10\text{MeV}$ is $\lambda \simeq 10^{-12}\text{cm}$, which is larger than the nucleus dimension $\simeq 10^{-13}\text{cm}$; therefore we may neglect the spatial variation of the field and write the interaction hamiltonian as

$$H_1 = \frac{q}{c}\mathbf{r}\frac{d\mathbf{A}_0}{dt} = \frac{q}{c}\mathbf{r}\frac{\partial\mathbf{A}_0}{\partial t} = -q\mathbf{r}\mathbf{E}_0 = -\mathbf{d}\mathbf{E}_0 , \quad (5)$$

where $\mathbf{d} = q\mathbf{r}$ is the dipole moment. This is the well-known dipole approximation. For an ensemble of N particles we write the interaction hamiltonian given by equation (4) as

$$H_1 = -\frac{1}{c}\sum_i \mathbf{J}_i\mathbf{A}_0 \quad (6)$$

(within the dipole approximation) and its matrix elements between two states a and b are given by

$$H_1(a, b) = -\frac{1}{c} \mathbf{J}(a, b) \mathbf{A}_0 =$$

$$= -\frac{1}{c} [\sum_i \int d\mathbf{r}_1 \dots d\mathbf{r}_i \dots d\mathbf{r}_N \psi_a^*(\mathbf{r}_1 \dots \mathbf{r}_i \dots \mathbf{r}_N) \mathbf{J}_i \psi_b(\mathbf{r}_1 \dots \mathbf{r}_i \dots \mathbf{r}_N)] \mathbf{A}_0 , \quad (7)$$

where $\psi_{a,b}$ are the wavefunctions of the two states a and b ; the notation \mathbf{r}_i in equation (7) includes also the spin variable. As it is well known, the transition amplitude is given by

$$c_{ab} = -\frac{i}{\hbar} \int dt H_1(a, b) e^{i\omega_{ab}t} , \quad (8)$$

where $\omega_{ab} = (E_a - E_b)/\hbar$ is the frequency associated to the transition between the two states a and b with energies E_a and, respectively, E_b . We take

$$\mathbf{A}_0(t) = \mathbf{A}_0 e^{-i\Omega t} + \mathbf{A}_0^* e^{i\Omega t} \quad (9)$$

(with $\Omega > 0$) and note that the pulse duration $\tau' = \sqrt{1 - \beta^2} \tau \simeq 5 \times 10^{-17} s$ is much longer than the transition time $1/\Omega \simeq 10^{-22} s$; we can extend the integration in equation (8) to infinity and get

$$c_{ab} = \frac{2\pi i}{\hbar c} \mathbf{J}(a, b) \mathbf{A}_0 \delta(\omega_{ab} - \Omega) ; \quad (10)$$

making use of $\delta(\omega = 0) = t/2\pi$, we get the number of transitions per unit time

$$P_{ab} = |c_{ab}|^2 / t = 2\pi \left| \frac{\mathbf{J}(a, b) \mathbf{A}_0}{\hbar c} \right|^2 \delta(\omega_{ab} - \Omega) . \quad (11)$$

This is a standard calculation. Usually, the field and the wavefunctions of the atomic nuclei are decomposed in electric and magnetic multiplets, and the selection rules of conservation of the parity and the angular momentum are made explicit (see, for instance,³⁴). It relates to the absorption (emission) of one photon.

It is worth estimating the number of transitions per unit time as given by equation (11). First, we may approximate $J(a, b)$ by qv . For an energy $\hbar\Omega = 10 MeV$ and a rest energy $1 GeV$ we have $v/c = 10^{-1}$. Next, from $\mathbf{E}_0 = (-1/c) \partial \mathbf{A}_0 / \partial t$ we deduce $A_0 \simeq 10^{-3} statvolt$ (for $E_0 = 10^9 statvolt/cm$ and $\Omega = 10^{22} s^{-1}$); it follows that the particle energy in this field is $qA_0 \simeq 1 eV$ (which is a very small energy). We get from equation (11) $P_{ab} \simeq (10^{28} / \Delta\Omega) s^{-1}$, where $\Delta\Omega \simeq 1/\tau' \simeq 10^{16} s^{-1}$ is the uncertainty in the pulse frequency, such that the number of transitions per unit time is $P_{ab} \simeq 10^{12} s^{-1}$ (much smaller than $\Omega = 10^{22} s^{-1}$). We can see that, under these circumstances, the first-order calculations of the perturbation theory are justified.

For higher fields we should include the second-order terms in the interaction hamiltonian given by equation (3); this second-order interaction hamiltonian reads

$$H_2 = -\frac{q^2}{2mc^2} \mathbf{A}_0^2 . \quad (12)$$

We can see that within the dipole approximation this interaction does not contribute to the transition amplitude, since the field does not depend on position and the wavefunctions are orthogonal. For field wavelengths shorter than the dimension of the ensemble of particles (*i.e.*, beyond the dipole approximation) we write

$$\mathbf{A}_0(\mathbf{r}, t) = \mathbf{A}_0 e^{-i\Omega t + i\mathbf{k}\mathbf{r}} + \mathbf{A}_0^* e^{i\Omega t - i\mathbf{k}\mathbf{r}} , \quad (13)$$

where $\mathbf{k} = \Omega/c$ is the wavevector, and get

$$H_2(a, b) = -\frac{q^2}{2mc^2} [A_0^2(a, b)e^{-2i\Omega t} + A_0^{*2}(b, a)e^{2i\Omega t}] , \quad (14)$$

where

$$A_0^2(a, b) = [\sum_i \int d\mathbf{r}_1 \dots d\mathbf{r}_i \dots d\mathbf{r}_N \psi_a^*(\mathbf{r}_1 \dots \mathbf{r}_i \dots \mathbf{r}_N) e^{2i\mathbf{k}\mathbf{r}_i} \psi_b(\mathbf{r}_1 \dots \mathbf{r}_i \dots \mathbf{r}_N)] A_0^2 . \quad (15)$$

This interaction gives rise to two-photon pocesses, with the transition amplitude

$$c_{ab} = \frac{2\pi i}{\hbar} \frac{q^2}{2mc^2} A_0^2(a, b) \delta(\omega_{ab} - 2\Omega) . \quad (16)$$

Comparing the transition amplitudes produced by the interaction hamiltonians H_1 (equation (10)) and H_2 (equation (16)) we may get an approximate criterion: qA_0/mc^2 (two-photons) compared with v/c (one photon). Since $v/c \simeq 10^{-1}$ (as estimated above), we should have $qA_0 > 10^{-1} \times 1GeV = 100MeV$ in order to get a relevant contribution from two-photon processes. As estimated above, $qA_0 \simeq 1eV$, so we can see that the second-order interaction hamiltonian and the two-photon processes bring a very small contribution to the transition amplitudes.

3 Giant dipole resonance

There is another process of excitation of the ensemble of particles described by the hamiltonian given by equation (3). Indeed, let us write the interaction hamiltonian

$$H_{int} = -\frac{q}{mc} \mathbf{p} \mathbf{A} + \frac{q^2}{2mc^2} A^2 + q\Phi , \quad (17)$$

or

$$H_{int} = -\frac{q}{c} \mathbf{v} \mathbf{A} - \frac{q^2}{2mc^2} A^2 + q\Phi . \quad (18)$$

Under the action of the electromagnetic field the mobile charges (*e.g.*, protons in atomic nucleus) acquire a displacement \mathbf{u} , which, in general, is a function $\mathbf{u}(\mathbf{r}, t)$ of position and time. This is a collective motion associated with the particle-density degrees of freedom; in the limit of long wavelengths (*i.e.* for \mathbf{u} independent of position) it is the motion of the center of mass of the charges. Therefore, an additional velocity $\dot{\mathbf{u}}$ should be included in equation (18). It is easy to see that this \mathbf{u} -motion implies a variation $\rho_p = -nq \text{div} \mathbf{u}$ of the (volume) charge density and a current density $\mathbf{j}_p = nq\dot{\mathbf{u}}$, where n is the concentration of mobile charges. Obviously, these are polarization charge and current densities (the suffix p comes from "polarization"). The charge and current densities ρ_p and \mathbf{j}_p give rise to an internal, polarization electromagnetic field, with the potentials \mathbf{A}_p and Φ_p (related through the Lorenz gauge $\text{div} \mathbf{A}_p + (1/c) \partial \Phi_p / \partial t = 0$), which should be added to the potential of the external field in equation (18). Indeed, the retardation time $t_r = a/c \simeq 10^{-23} s$, where $a \simeq 10^{-13} cm$ is the dimension of the atomic nucleus, is shorter than the excitation time $\Omega^{-1} = 10^{-22} s$, so the atomic nucleus gets polarized. In particular the scalar potential Φ in equation (18) is the polarization scalar potential Φ_p . We get

$$H_{int} = H_1 - \frac{1}{c} \mathbf{J} \mathbf{A}_p - \frac{q}{c} \dot{\mathbf{u}} (\mathbf{A}_0 + \mathbf{A}_p) - \frac{q^2}{2mc^2} (\mathbf{A}_0 + \mathbf{A}_p)^2 + q\Phi_p , \quad (19)$$

where H_1 is given by equation (4). Within the dipole approximation we may take \mathbf{u} independent of position, except for the surface of the particle ensemble, where the density falls abruptly to zero.

A similar behaviour extends to the vector and scalar polarization potentials (inside the ensemble); in addition, through the Lorenz gauge, the scalar potential Φ_p can be taken independent of time within this approximation. The surface effects can be neglected as regards the scalar product of two orthogonal wavefunctions. All these simplifications amount to neglecting all the terms in equation (19) except the first two; therefore, we are left with

$$H_{int} \simeq H_1 + H_{1p} , \quad H_{1p} = -\frac{1}{c} \mathbf{J} \mathbf{A}_p ; \quad (20)$$

in order to get \mathbf{A}_p we need a dynamics for the displacement field \mathbf{u} .

We can construct a dynamics for the displacement field \mathbf{u} by assuming that it is subjected to internal forces of elastic type, characterized by frequency ω_c ; the (non-relativistic) equation of motion is given by

$$m\ddot{\mathbf{u}} = q(\mathbf{E}_0 + \mathbf{E}_p) - m\omega_c^2 \mathbf{u} , \quad (21)$$

where $\mathbf{E}_0 = -(1/c)\partial\mathbf{A}_0/\partial t$ is the external electric field and \mathbf{E}_p is the polarization electric field. Within the dipole approximation, Gauss's equation $\text{div}\mathbf{E}_p = 4\pi\rho_p = -4\pi nq\text{div}\mathbf{u}$ gives $\mathbf{E}_p = -4\pi nq\mathbf{u}$ for matter of infinite extension (polarization $\mathbf{P} = nq\mathbf{u}$). For polarizable bodies of finite size there appears a (de-) polarizing factor f within the same dipole approximation, as a consequence of surface charges (for instance, $f = 1/3$ for a sphere). Therefore, we can write equation (21) as

$$\ddot{\mathbf{u}} + (\omega_c^2 + f\omega_p^2)\mathbf{u} = \frac{q}{m}\mathbf{E}_0 , \quad (22)$$

where $\omega_p = \sqrt{4\pi nq^2/m}$ is the plasma frequency. For nucleons we can estimate $\hbar\omega_p \simeq Z^{1/2} MeV$, where Z is the atomic number. An estimation for the characteristic frequency ω_c can be obtained from $m\omega_c^2 d^2/2 = \mathcal{E}_c(d/a)$, where d is the displacement amplitude, a is the dimension of the nucleus and $\mathcal{E}_c (\simeq 7 - 8 MeV)$ is the mean cohesion energy per nucleon; the maximum value of d is the mean inter-particle separation distance $d = a/A^{1/3}$, where A is the mass number. We get $\hbar\omega_c \simeq 10A^{1/6} MeV$. It is convenient to introduce the frequency $\Omega_0 = (\omega_c^2 + f\omega_p^2)^{1/2}$, which, as we can see from the preceding estimations, is of the order of $10 MeV$, and write the equation of motion (22) as

$$\ddot{\mathbf{u}} + \Omega_0^2 \mathbf{u} = \frac{q}{m} \mathbf{E}_0 . \quad (23)$$

This is the equation of motion of a linear harmonic oscillator under the action of an external force $q\mathbf{E}_0$. Making use of equation (9), we get the external field

$$\mathbf{E}_0 = \frac{i\Omega}{c} \mathbf{A}_0 e^{-i\Omega t} - \frac{i\Omega}{c} \mathbf{A}_0^* e^{i\Omega t} ; \quad (24)$$

for frequency Ω approaching the oscillator frequency Ω_0 the motion described by equation (23) is a classical motion, and we get

$$\mathbf{u} = -\frac{iq\Omega}{mc} \cdot \frac{1}{\Omega^2 - \Omega_0^2} (\mathbf{A}_0 e^{-i\Omega t} - \mathbf{A}_0^* e^{i\Omega t}) . \quad (25)$$

According to the discussion made above, the polarization field is

$$\mathbf{E}_p = -4\pi f n q \mathbf{u} = \frac{if\omega_p^2\Omega}{c} \cdot \frac{1}{\Omega^2 - \Omega_0^2} (\mathbf{A}_0 e^{-i\Omega t} - \mathbf{A}_0^* e^{i\Omega t}) \quad (26)$$

and the corresponding vector potential is

$$\mathbf{A}_p = \frac{f\omega_p^2}{\Omega^2 - \Omega_0^2} (\mathbf{A}_0 e^{-i\Omega t} + \mathbf{A}_0^* e^{i\Omega t}) . \quad (27)$$

A damping factor Γ can be included in equation (23),

$$\ddot{\mathbf{u}} + \Omega_0^2 \mathbf{u} + \Gamma \dot{\mathbf{u}} = \frac{q}{m} \mathbf{E}_0, \quad (28)$$

and we can write the solution as

$$\mathbf{u} = -\frac{q}{m} \mathbf{E}_0 \frac{1}{\Omega^2 - \Omega_0^2 + i\Omega\Gamma} e^{-i\Omega t} + c.c.; \quad (29)$$

the polarization reads

$$\mathbf{P} = nqf\mathbf{u} = -\frac{f\omega_p^2}{4\pi} \frac{1}{\Omega^2 - \Omega_0^2 + i\Omega\Gamma} \mathbf{E}_0 e^{-i\Omega t} + c.c., \quad (30)$$

so that we can define the polarizability

$$\alpha = -\frac{f\omega_p^2}{4\pi} \frac{1}{\Omega^2 - \Omega_0^2 + i\Omega\Gamma}. \quad (31)$$

Therefore, the vector potential \mathbf{A}_p given by equation (27) can be written as

$$\mathbf{A}_p = -4\pi (\alpha \mathbf{A}_0 e^{-i\Omega t} + \alpha^* \mathbf{A}_0^* e^{i\Omega t}). \quad (32)$$

Now, we can estimate the transition amplitude between two states a and b , making use of the interaction hamiltonian H_{1p} given by equation (20). We get the amplitude

$$c_{ab} = -\frac{8\pi^2 i}{\hbar c} \alpha \mathbf{J}(a, b) \mathbf{A}_0 \delta(\omega_{ab} - \Omega) \quad (33)$$

and the number of transitions per unit time

$$P_{ab} = 32\pi^3 \left| \frac{\mathbf{J}(a, b) \mathbf{A}_0}{\hbar c} \right|^2 |\alpha|^2 \delta(\omega_{ab} - \Omega). \quad (34)$$

Comparing this result with equation (11) we can see that, apart from a numerical factor, the rate of polarization-driven transitions are modified by the factor

$$|\alpha|^2 = \left(\frac{f\omega_p^2}{4\pi} \right)^2 \frac{1}{(\Omega^2 - \Omega_0^2)^2 + \Omega^2 \Gamma^2}. \quad (35)$$

This is a typical resonance factor, which indicates that the polarization of the particle ensemble is important for $\Omega \simeq \Omega_0$ (at resonance), where the ensemble can be disrupted. Obviously, this is a giant dipole resonance.^{35,36} For Ω far away from the resonance frequency Ω_p the polarization is practically irrelevant, and it may be neglected in comparison with the transitions brought about by the interaction hamiltonian H_1 (equation (11)). It is worth noting that we can define an electric susceptibility χ and a dielectric function ε for the polarizable ensemble of particles, by combining equations (4), (20) and (32). We get

$$H_1 + H_{1p} = -\frac{1}{c} \mathbf{J} [(1 - 4\pi\alpha) \mathbf{A}_0 e^{-i\Omega t} + c.c.] = -\frac{1}{c} \mathbf{J} \left[\frac{1}{\varepsilon} \mathbf{A}_0 e^{-i\Omega t} + c.c. \right], \quad (36)$$

since $1 - 4\pi\alpha = (1 + 4\pi\chi)^{-1} = 1/\varepsilon$, as expected (according to their definitions, we have $\mathbf{P} = \alpha \mathbf{E}_0 = \chi(\mathbf{E}_0 - 4\pi\mathbf{P})$, where \mathbf{P} is the polarization, *i.e.* the dipole moment per unit volume). Therefore,

the total interaction hamiltonian is proportional to $1/\varepsilon = (\Omega^2 - \omega_c^2)/(\Omega^2 - \Omega_0^2)$, and we note that, beside the Ω_0 -pole, it has a zero for $\Omega = \omega_c$, where the transitions are absent.

A similar description holds for ions (or neutral atoms) in an external electromagnetic field. Perhaps the most interesting case is a neutral, heavy atom, for which we can estimate the plasma energy $\hbar\omega_p \simeq 10Z^{1/2}eV$. For the cohesion energy per electron we can use the Thomas-Fermi estimation $16Z^{7/3}/ZeV = 16Z^{4/4}eV$, which leads to $\hbar\omega_c \simeq 13Z^{5/6}eV$. We can see that the typical scale energy where we may expect to occur a giant dipole resonance is $\hbar\Omega_0 \simeq 1keV$. However, the motion of the electrons under the action of a high-intensity electromagnetic field is relativistic (see, for instance,³⁷).

4 Discussion and conclusions

The direct photon-nucleus coupling processes described here are hampered by electron-positron pairs creation in the Coulomb field of the nucleus. For photons of energy $\hbar\Omega = 10MeV$ we may consider the (ultra-) relativistic limit of the pair creation cross-section. As it is well known,^{38,39} in this case the cross-section is derived within the Born approximation, the pair partners are generated mainly in the forward direction, they have not very different energies from one another and the recoil momentum (energy) transmitted to the nucleus is small. For bare nuclei (absence of screening) the total cross-section of pair production is given by

$$\sigma_{pair} = \frac{Z^2 r_0^2}{137} \left(\frac{28}{9} \ln \frac{2\hbar\Omega}{mc^2} - \frac{218}{27} \right) \simeq 10^{-28} Z^2 cm^2, \quad (37)$$

where $r_0 = e^2/mc^2$ is the classical electron radius, $-e$ is the electron charge and m is the electron mass. We can get an order of magnitude estimation of the efficiency of the processes described here by comparing this cross-section with the nuclear cross-section $a^2 \simeq 10^{-26} cm^2$. We can see that $\sigma_{pair}/a^2 \simeq 10^{-2} Z^2$, which may go as high as 10^2 for heavy nuclei.

In conclusion, we may say that in the rest frame of (ultra-) relativistically accelerated heavy ions (atomic nuclei) the electromagnetic radiation field produced by high-power optical or free electron lasers may acquire high intensity and high energy, suitable for photonuclear reactions. In particular, the excitation of dipole giant resonance may be achieved. Nuclear transitions are analyzed here under such particular circumstances, including both one- and two-photon processes. It is shown that the perturbation theory is applicable, although the field intensity is high, since the interaction energy is low (as a consequence of the high frequency) and the interaction time (pulse duration is short). It is also shown that the giant nuclear dipole resonance is driven by the nuclear (electrical) polarization degrees of freedom, whose dynamics may lead to disruption of the atomic nucleus when resonance conditions are met. The concept of nuclear (electrical) polarization is introduced, as well as the concept of nuclear electrical polarizability and dielectric function.

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