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Magnetostatic modes excited by spin-transfer torques. The case of a spin valve

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Abstract

The coupled equations of motion are established for the dynamics of the spin-polarization of an electric flow and the magnetization of a ferromagnetic sample. The analysis is performed for isotropic samples, in the long-wavelength limit and with disregard of finite-size effects. Both quantal and classical motion are investigated. It is shown that magnetostatic modes are excited in a ferromagnetic sample by a spin-polarized electric flow, through the spin-transfer torque. An optical mode is identified, beside spin-waves modes. Similar excitations of a spin valve are analyzed, and shown to occur in a resonance-like regime.

Spintronics. A spin valve consists of two ferromagnetic, conducting samples separated by an insulating, non-magnetic thin layer. It is easy to see that the spin-polarized electric flow in such a valve exhibits a conductivity which depends on $\cos\theta$, where θ is the angle between the two magnetizations, as a consequence of the spin-magnetization coupling. This has been pointed out by Julliere[1] and calculated more explicitly by Slonczewski.[2] An additional magnetic exchange between the two samples does also contribute similarly. The electric resistance of the junction may vary appreciably with the angle θ, this being the giant magnetoresistance effect[3] (distinct from the colossal magnetoresistance, which is a change of resistance by orders of magnitude in the presence of a magnetic field). The discovery of the giant magnetoresisance is considered to be the birth of the spintronics.

We deal herein with the excitation of magnetostatic modes in ferromagnetic samples and spin valves by spin-transfer torques.

Motion of the spin polarization. The motion of a spin $\mathbf{s}$ in a magnetic field $\mathbf{H}$ is governed by Schrodinger's equation

$$i\hbar\partial\psi/\partial t = \mu\mathbf{H}\mathbf{s}\psi \ , \tag{1}$$

where ψ is the spinor wavefunction and $s\mu$ is the magnetic moment. For electrons, $\mu = e\hbar/mc$ is twice the Bohr magneton and $\mathbf{s} = \sigma/2$, where σ are Pauli's matrices.

The spin polarization of an electric flow is given by

$$\mathbf{S} = (\psi, \mathbf{s}\psi) \ , \tag{2}$$

where the scalar product is taken over the spin coordinates only. Therefore, $\mathbf{S}$ is a density of spin $\mathbf{S}(\mathbf{r})$, with a possible spatial dependence on position $\mathbf{r}$. Making use of $[s_i, s_j] = i\varepsilon_{ijk}s_k$, where ε_{ijk} is the totally antisymmetric unit tensor, it is easy to see that Schrodinger's equation (1) leads to the motion of the spin polarization as being described by

$$\partial\mathbf{S}/\partial t = \gamma\mathbf{H}\times\mathbf{S} \ , \tag{3}$$

where $\gamma = \mu/\hbar = e/mc$ is the gyromagnetic ratio. This is the basic Larmor equation of motion of a spin under the action of its torque with respect to the magnetic field.

We note that $\mathbf{m} = -\mu\mathbf{S}$ is the magnetization of the electric flow (density of the magnetic moment), and $c \cdot curl\mathbf{m}$ is the spin contribution to the electric flow (charge per unit area per unit time).

With the commutation relations

$$[S_i(\mathbf{r}), S_j(\mathbf{r}')] = i\varepsilon_{ijk}S_k(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') \ , \tag{4}$$

the equation of motion (3) is obtained by the effective hamiltonian

$$\mathcal{H} = \gamma\hbar\int d\mathbf{r}\cdot\mathbf{HS} = \mu\int d\mathbf{r}\cdot\mathbf{HS} = -\int d\mathbf{r}\cdot\mathbf{mH} \tag{5}$$

through $\partial\mathbf{S}/\partial t = (i/\hbar)\,[\mathcal{H},\mathbf{S}]$.

Ferromagnet. In a ferromagnet the spin experiences a magnetic field $\mathbf{H} = 4\pi\mathbf{M}$,[1] where $\mathbf{M}$ is the magnetization. It follows that the equation of motion (3) reads

$$\partial\mathbf{S}/\partial t = 4\pi\gamma\mathbf{M}\times\mathbf{S} \tag{6}$$

and the corresponding hamiltonian (5) becomes

$$\mathcal{H} = 4\pi\gamma\hbar\int d\mathbf{r}\cdot\mathbf{MS} \ . \tag{7}$$

In a magnetic field $\mathbf{H}$ the magnetization obeys an equation of motion

$$\partial\mathbf{M}/\partial t = \gamma_f\mathbf{H}\times\mathbf{M} \tag{8}$$

similar to (3), where γ_f is the gyromagnetic ratio of the ferromagnetic moments. The magnetic field $\mathbf{H}$ in (8) comes from the spin polarization of the electric flow, through $\mathbf{H} = 4\pi\mathbf{m} = -4\pi\mu\mathbf{S}$, so equation (8) becomes

$$\partial\mathbf{M}/\partial t = -4\pi\mu\gamma_f\mathbf{S}\times\mathbf{M} \ . \tag{9}$$

Equations (6) and (9) describe the coupled motion of the ferromagnetic magnetization of the sample and the spin polarization of the electric flow. Equation (9) is given by the same hamiltonian (7) with the commutation relations

$$[M_i(\mathbf{r}), M_j(\mathbf{r}')] = -\mu_f\cdot i\varepsilon_{ijk}M_k(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') \ , \tag{10}$$

where μ_f is the magnetic moment of the ferromagnet ($\gamma_f = \mu_f/\hbar$).

We note that the hamiltonian density given by (7) can also be written as $-\mathbf{MH}$, where $\mathbf{H} = 4\pi\mathbf{m} = -4\pi\mu\mathbf{S}$, and, in this form, it is the well-known term in the thermodynamic potential of the ferromagnet. The full contribution to this thermodynamic potential must also include $-\mathbf{H}^2/8\pi = -2\pi\mu^2\mathbf{S}^2$, which does not affect the motion of the spin polarization $\mathbf{S}$.[2] Additional contribution to the feromagnetic energy can also be included, which may only affect the motion of the magnetization.

[1] By the general relation $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$, where $\mathbf{B}$ is the magnetic induction and $\mathbf{H} = 0$. We leave aside the finite-size effects.

[2] Making use of the commutation relations (4) we get easily that the net result is proportional to a vanishing product of the antisymmetric tensor ε by a symmetric one.

Magnetization (non-uniformity) energy. The exchange energy brings the main contribution to the magnetization (non-uniformity) energy.[3] It comes from terms of the form $\mathbf{M}(\mathbf{r})\mathbf{M}(\mathbf{r}\pm\varepsilon)$ in the limit $\varepsilon\rightarrow 0$.[4] It is easy to see that, for an isotropic material, the density of the magnetization energy looks like $(\alpha_f/2)(\partial\mathbf{M}/\partial x_i)(\partial\mathbf{M}/\partial x_i)$ (where α_f must be positive for ferromagnetism). We introduce similar interactions between the spins in the electric flow and between the spins in the electric flow and the ferromagnetic sample. This leads to the magnetization hamiltonian

$$\begin{gathered}\mathcal{H}_m=\int d\mathbf{r}\cdot[\tfrac{1}{2}\alpha_f(\partial\mathbf{M}/\partial x_i)(\partial\mathbf{M}/\partial x_i)+\tfrac{1}{2}\alpha\mu^2(\partial\mathbf{S}/\partial x_i)(\partial\mathbf{S}/\partial x_i)-\\ -\beta\mu(\partial\mathbf{M}/\partial x_i)(\partial\mathbf{S}/\partial x_i)]\ .\end{gathered}\tag{11}$$

Making use of the commutation relations (4) and (10) it is easy to see that the first two terms in (11) bring no contribution to the equations of motion.[5] The only contribution brought by the magnetization energy to the equations of motion arises from the mixed term in (11). The equations of motion (6) and (9) become

$$\begin{gathered}\partial\mathbf{S}/\partial t=4\pi\gamma\mathbf{M}\times\mathbf{S}+\beta\gamma\Delta\mathbf{M}\times\mathbf{S}\ ,\\ \partial\mathbf{M}/\partial t=-4\pi\mu\gamma_f\mathbf{S}\times\mathbf{M}-\beta\mu\gamma_f\Delta\mathbf{S}\times\mathbf{M}\ .\end{gathered}\tag{12}$$

Linearization. We write $\mathbf{M}=\mathbf{M}_0+\mathbf{m}$ and $\mathbf{S}=\mathbf{S}_0+\mathbf{s}$, where $\mathbf{M}_0$ and $\mathbf{S}_0$ are classical variables and $\mathbf{m}$ and $\mathbf{s}$ are quantal ones. That means that we take $\hbar\rightarrow 0$ and $\mu\mathbf{S}_0=finite$, and similarly for the ferromagnetic spins, such as $\mathbf{M}_0=finite$. On the other hand, the (spin) quantization of $\mathbf{m}$ and $\mathbf{s}$ are not relevant anymore for solving the equations of motion.[6] Their elementary excitations as resulting from the equations of motion will obey their own quantization. We assume in addition that $\mathbf{M}_0$ and $\mathbf{S}_0$ are colinear and uniform and constant. Moreover, we introduce a susceptibility χ of the spin-polarized electric flow through $-\mu S_0=4\pi\chi M_0$. We neglect other magnetic interactions, like the anisotropy energy or the interaction with an external magnetic field, as being irrelevant for the subsequent discussion. We assume that the ferromagnetic sample looks like a single ferromagnetic domain, an assumption justified by the weakness of the anisotropy energy and the correspondingly large width of the domain wall. This assumpption is also valid in the long-wavelength limit.

The linearized equations of motion read now

$$\begin{gathered}\partial\mathbf{s}/\partial t=4\pi\gamma\mathbf{M}_0\times\mathbf{s}+4\pi\gamma(4\pi\chi/\mu)\mathbf{M}_0\times\mathbf{m}+\beta\gamma(4\pi\chi/\mu)\mathbf{M}_0\times\Delta\mathbf{m}\ ,\\ \partial\mathbf{m}/\partial t=4\pi\gamma_f(4\pi\chi)\mathbf{M}_0\times\mathbf{m}+4\pi\mu\gamma_f\mathbf{M}_0\times\mathbf{s}+\beta\mu\gamma_f\mathbf{M}_0\times\Delta\mathbf{s}\ .\end{gathered}\tag{13}$$

They can be written in a more symmetrical form

$$\begin{gathered}\partial\mathbf{m}_e/\partial t=A\mathbf{M}_0\times\mathbf{m}_e-AC\mathbf{M}_0\times\mathbf{m}-BC\mathbf{M}_0\times\Delta\mathbf{m}\ ,\\ \partial\mathbf{m}/\partial t=A_fC\mathbf{M}_0\times\mathbf{m}-A_f\mathbf{M}_0\times\mathbf{m}_e-B_f\mathbf{M}_0\times\Delta\mathbf{m_e}\ ,\end{gathered}\tag{14}$$

[3]Other contributions are relativistic effects, typically smaller by factors $10^{-2}-10^{-5}$, with notable exceptions however (rare-earths).

[4]Far from the Curie point this energy implies in fact $\mathbf{M}/M$.

[5]This is in contrast with some claims made occasionally, suggested probably by the deficient Holstein-Primakoff technique (where the spin waves are rigurously valid only in the long-wavelength limit, or at zero temperature, see, for instance, F. Dyson, Phys. Rev. **102** 1217, 1230 (1956)), or by a completely different situation where the spins (magnetization) move classically (see the original paper by L. Landau and E. Lifshitz, Phys. Z. Sowjet. **8** 153 (1935)).

[6]It is worth stressing upon the circumstance that the spin fields do not obey a classical dynamics, as there is no classical counterpart of them; the correspondence principle allows such a clasical limit for magnetization.

by introducing the magnetization $\mathbf{m}_e = -\mu\mathbf{s}$ of the electric flow and the parameters

$$A = 4\pi\gamma \ , \ A_f = 4\pi\gamma_f \ ,$$
$$B = \beta\gamma \ , \ B_f = \beta\gamma_f \ , \ C = 4\pi\chi \ . \tag{15}$$

Eigenmodes. Equations (14) can easily be solved. We assume $\mathbf{M}_0$ oriented along the z-axis and $\mathbf{m}_e\, , \mathbf{m}$ in the xy-plane, depending only on the x-coordinate. It is convenient to use the combinations $m^+ = m_x + im_y$, etc. Equations (14) become

$$\partial m_e^+/\partial t = iAM_0 m_e^+ - iACM_0 m^+ - iBCM_0 \Delta m^+ \ ,$$
$$\partial m^+/\partial t = iA_f CM_0 m^+ - iA_f M_0 m_e^+ - iB_f M_0 \Delta m_e^+ \ . \tag{16}$$

In the limit of long-wavelengths we get the eigenmodes

$$\omega_1 = M_0(A + CA_f) - CM_0 k^2 (AB_f + A_f B)/(A + CA_f) \ ,$$
$$\omega_2 = CM_0 k^2 (AB_f + A_f B)/(A + CA_f) \ . \tag{17}$$

The optical mode ω_1 corresponds approximately to anti-phase oscillations of the two magnetizations $\mathbf{m}$ and $\mathbf{m}_e$; hence we may say that magnetostatic modes in the ferromagnetic sample are excited by the spin-transfer torque in the spin-polarized electric flow. Both modes have a strong dispersion, since typical value for parameter β are $\beta \sim 10^{-12} cm^2$. A numerical estimation for the optical mode is given by $\omega_1 \sim M_0\mu/\hbar$, which leads to $\omega_1 \sim 10GHz$ for $M_0 = 0.1Ts$ and $\mu \sim \mu_B \simeq 10^{-23} J/Ts$, where $\mu_B = e\hbar/2mc$ is the Bohr magneton.[7]

Spin valve. In the long-wavelength limit we may neglect the terms involving derivatives in equations (16). Then, we have magnetostatic modes excited in the *rhs* ferromagnetic sample of the spin valve by the spin-polarized electric current flowing from the *lhs* of the valve into the *rhs* of the valve, of the form $m_e^{+0} \sim e^{i\omega_1 t}$, where $\omega_1 = M_0(A + CA_f)$ is the optical mode given by (17) for the *lhs* sample.

If we redefine $m^+ \rightarrow Cm^+$ it is easy to see that equations (16) reduce to

$$\partial(m_e - m)/\partial t = i\omega_2(m_e - m) \ , \tag{18}$$

where we have omitted the upper suffix + and $\omega_2 = M_0(A + CA_f)$ corresponds to the *rhs* sample. We introduce $m_e \rightarrow m_e + m_e^0 e^{i\omega_1 t}$ in the *lhs* of this equation, which accounts for an external force, and get

$$\partial d/\partial t = i\omega_2 d - i\omega_1 m_e^0 e^{i\omega_1 t} \ , \tag{19}$$

where $d = m_e - m$. The solution of this equation is a typical resonance one,

$$d = d_0 e^{i\omega_2 t} - \frac{\omega_1}{\omega_1 - \omega_2} e^{i\omega_1 t} \ . \tag{20}$$

We may say that magnetostatic modes are excited in the spin valve by the spin-polarized electric flow, through the spin-transfer torque, in a resonance regime. A similar conclusion was probably pointed out by Berger.[4]

[7] For $\chi \sim 1$ and $M_0 = 0.1Ts$ we get the spin-density $S_0 \sim M_0/\mu \sim 10^{16} cm^{-3}$.

Classical motion. The above treatment pertains to the quantal motion of the spins and magnetization. In macroscpic ferromagetic samples and for macroscopic electric currents the magnetization and spin polarization are classical variables. The commutation relations (4) and (10) vanish. The magnetization energy $\mathcal{H}_m$ given by (11) rests the same, but the interacting energy given by (5) or (7) should now be completed to the full interaction energy, which is given by

$$\mathcal{H} = -(1/8\pi)\int d\mathbf{r}\cdot(4\pi)^2(\mathbf{M}+\mathbf{M}_e)^2 \ , \tag{21}$$

where $\mathbf{M}_e = -\mu\mathbf{S}$ denotes now the magnetization of the electric flow (associated to the spin polarization). The mixed term in (21) coincides with the previous hamiltonian given by (7). Similarly, we write the hamiltonian (11) as

$$\begin{gathered}\mathcal{H}_m = \int d\mathbf{r}\cdot[\tfrac{1}{2}\alpha_f(\partial\mathbf{M}/\partial x_i)(\partial\mathbf{M}/\partial x_i) + \tfrac{1}{2}\alpha(\partial\mathbf{M}_e/\partial x_i)(\partial\mathbf{M}_e/\partial x_i) + \\ +\beta(\partial\mathbf{M}/\partial x_i)(\partial\mathbf{M}_e/\partial x_i)] \ .\end{gathered} \tag{22}$$

We write the variation of the energy as $\delta(\mathcal{H}+\mathcal{H}_m) = -\int d\mathbf{r}\cdot\mathbf{H}_m\delta\mathbf{M} = -\int d\mathbf{r}\cdot\mathbf{H}_e\delta\mathbf{M}_e$, where $\mathbf{H}_{m,e}$ are the corresponding magetic fields experienced by the two magnetizations. In the classical limit these magnetizations can be represented as being produced by torques of currents, so they obey the Larmor equation of motion $\partial\mathbf{M}_{m,e}/\partial t = \gamma_{m,e}\mathbf{H}_{m,e}\times\mathbf{M}_{m,e}$. Alternately, the equation of motion (3) holds also in the classical limit. This way, we get the Landau-Lifshitz equations of motion (without relaxation)

$$\begin{gathered}\partial\mathbf{M}_e/\partial t = \gamma(\alpha\Delta\mathbf{M}_e + 4\pi\mathbf{M} + \beta\Delta\mathbf{M})\times\mathbf{M}_e \\ \partial\mathbf{M}/\partial t = \gamma_f(\alpha_f\Delta\mathbf{M} + 4\pi\mathbf{M}_e + \beta\Delta\mathbf{M_e})\times\mathbf{M} \ .\end{gathered} \tag{23}$$

We solve these equations as before, by linearizing them with respect to small deviations $\mathbf{m}\,,\mathbf{m}_e$ in the (x,y)-plane, which depend only on the x-coordinate. We get finally the equations

$$\begin{gathered}\partial m_e^+/\partial t = iAM_0m_e^+ - iACM_0m^+ - iDCM_0\Delta m_e^+ - iBCM_0\Delta m^+ \ , \\ \partial m^+/\partial t = iA_fCM_0m^+ - iA_fM_0m_e^+ - iD_fM_0\Delta m^+ - iB_fM_0\Delta m_e^+ \ ,\end{gathered} \tag{24}$$

where $D = \alpha\gamma$ and $D_f = \alpha_f\gamma_f$. They differ from equations (16) only by the D-terms . In the long wavelength limit the eigenfrequencies are given by

$$\begin{gathered}\omega_1 = M_0(A+CA_f) - M_0k^2[A(CB_f - D_f) + CA_f(B - CD)]/(A+CA_f) \ , \\ \omega_2 = CM_0k^2[A(B_f+D) + A_f(B+D_f)]/(A+CA_f) \ .\end{gathered} \tag{25}$$

All the above discussion made for quantal spins remains qualitatively valid. There is only slight quantitative changes in the spectrum. It is worth noting that in the absence of the current the magnetization of the sample moves by spin-density waves given by $\omega_1 = M_oD_fk^2$.

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