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On the phase diagram of the Quantum Chromodynamics

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Abstract

It is suggested that the hadronization of the quark-gluon plasma is a first-order phase transition described by a critical curve in the temperature-(quark) density plane which terminates in a critical point. Such a critical curve is derived from the van der Waals equation with zero pressure and its parameters are estimated by using the theoretical approach given in M. Apostol, Roum. Reps. Phys. 59 249 (2007). The main assumption is that quark-gluon plasma created by high-energy nucleus-nucleus collisions is a gas of (massless) ultrarelativistic quarks at equilibrium with gluons (vanishing chemical potential, indefinite number of quarks). This plasma expands, gets cooler and dilute and hadronizes at a certain transition temperature and transition density. The transition density is very close to the saturation density of the nuclear matter and, it is suggested that both these points are very close to the critical point. This point is given by $n \simeq 1fm^{-3}$ and $T \simeq 200MeV$ and it can have a universal character.

As it is well known, the Quantum Chromodynamics (QCD) developed in the past 50 years describes the quark-gluon strong interaction. The QCD lagragian is

$$L = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \overline{\psi}_{f\alpha}\left[i\gamma^\mu(\partial_\mu\delta^{\alpha\beta} - igt^{\alpha\beta}_a A^a_\mu) - m_f\delta^{\alpha\beta}\right]\psi_{f\beta} \ , \tag{1}$$

where

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^a_{bc}A^b_\mu A^c_\nu \tag{2}$$

and

$$[t_a, t_b] = if^c_{ab}t_c \ . \tag{3}$$

A^a_μ are gluon potentials, with μ the Lorentz label and $a = 1...8$ the gluon label; $G^a_{\mu\nu}$ are the gluon fields, g is the coupling constant and f^a_{bc} is the structure factor of the SU(3) group. The eight 3×3 matrices t_a ($\alpha, \beta = 1, 2, 3$) are the SU(3) generators. $\psi_{f\alpha}$ are the quark fields (bispinors), labelled by flavour $f = 1...6$ and color $\alpha = 1, 2, 3$, with mass m_f; γ^μ are the Dirac matrices. For a review of QCD the redear can consult Refs. [1]-[3]

The lagrangian is constructed by close analogy with the Quantum Electrodynamics (QED), with two major differences: the Yang-Mills fields (quadratic term in the gluon fields in equation (2)) and the underlying SU(3) symmetry (color group). Perturbation-theory calculations indicate that effective (renormalized) coupling strength becomes weaker and weaker for higher- and higher-energy processes, a phenomenon known as the quark (de-) confinement; the quarks and gluons are free at high energy and get confined at low energy; for instance, they are bound in hadrons (mesons and baryons), which are color-singlet states (zero color charge).[4, 5] The confinement phenomenon, which is opposite to QED, is due to the non-linear Yang-Mills contribution. Both coupling strength and masses get renormalized, but they remain undefined as long as free, independent quarks are not observables. The theory is renormalizable indeed, but the renormalization is useless. The usual type of calculations in QCD is the lattice-gauge theory calculations:[6] the path integral is used on a discretized lattice and computed numerically. The results are more of a qualitative nature. A certain consensus claims that QCD describe the strong interactions, in particular the hadronic jet production in high-energy collisions and parton distributions. Various simplifications are customary in calculations, in particular the limitation to only u and d quarks (the lightest), whose mass is put equal to zero. In this case, the chiral symmetry (handedness) of the theory should be broken at low energies (the hadrons are not chiraly symmetric). Broken symmetries, associated phase transitions and, in general, methods borrowed from condensed matter physics, are employed with the hope of getting more quantitative results in QCD.

In nucleus-nucleus collisions a high energy can be transferred to the internal structure of the nucleons (*e.g.*, $1TeV$ per nucleon as compared with the nucleon binding energy $1GeV$), such that we may expect the liberation of quarks and gluons for a short while, followed by a quick hadronization.[7]-[10] An ultrarelativistic gas of quarks can be formed in such collisions, reaching quickly the thermal equilibrium at a temperature produced by the collision energy; we may speak of a quark-gluon plasma, with a threshold (ignition) temperature of cca $125-180MeV$, and a hadron-quark-gluon plasma transition.[11] In a high-energy collision the plasma expands, its volume and number of quarks and gluons (at local equilibrium) increase, the quark density and temperature decrease, and the quarks in the outer shell hadronize.[12] It is tempting to assign a second order to such a transition, but it is difficult to see what symmetry is broken (except for zero-mass quarks, where it is the chiral symmetry which may be broken; but the real situation involves non-vanishing masses). We may only assume that the hadron-quark-gluon plasma is first order, involving a hadron binding energy, very similar with the van der Waals liquid (solid)-gas transition. As it is well known, the van der Waals isotherms are given by

$$(p+an^2)(1-bn)=nT \ , \tag{4}$$

where p is the presssure, n is the density, T is the temperature and a, b are constants. It is difficult to vary pressure in hadron assemblies, as it is for the density. Very high densities are encountered in neutron stars. We may assume that the variation of the pressure is simulated by the variation of the density, and, since the van der Waals characteristic pressure is $\sim an^2$, we may also assume $p=cn^2$, where c is a constant. The van de Walls isotherms become

$$(a+c)n(1-bn)=T \ , \tag{5}$$

or

$$T=-Bn^2+An \ , \tag{6}$$

where A and $B>0$ are constants . We can see that equation (6) is equivalent with the van der Waals equation (4) for zero pressure. Since $\partial T/\partial n<0$ for a physical transition, we can see that we should consider the above equation from $n=A/2B$ up to $n=A/B$ ($T>0$), so we should have $A>0$. Under these conditions the above equation gives the curve corresponding to the hadron-quark-gluon plasma transition in the (T, n) plane, the point $n_c=A/2B$, $T_c=A^2/4B$ being the critical point. Equation (6) can also be written as $T=An[1-(B/A)n]$, where we can see that the ratio B/A is a limiting volume, which may be viewed as corresponding to a nominal "volume" v_n of the quarks, $B/A=v_n$; the quark density n can be written as $n=N/V=1/v_q$, where v_q is the mean volume assigned to a quark in the volume V occupied by N quarks. Since we may expect a mixture of various quark species, the density n can be generalized to the mean density involving partial densities.

Now we describe briefly the theoretical approach given in Ref. [12], because it gives us access to the parameters A and B in equation (6). We consider a nucleus with N_n nucleons in a volume $V_0 = R_0^3$, where R_0 is, approximately, the radius of the nucleus; the nucleus is subjected to a high-energy collision, with an energy $E/N_n = 1TeV$, for instance.We limit ourselves to the lightest quarks, for which we may neglect their mass at these energy values ($m_u \simeq 2MeV$, $m_d \simeq 5MeV$). The energy is dominated by gluons, except for assuming an ultrarelativistic gas of an indefinite number of quarks (vanishing chemical potential) in equilibrium with gluons, *i.e.* a quark-gluon plasma. In this case, the plasma energy (quarks plus gluons, almost equal energy) is given by

$$E = VT^4/(\hbar c)^3 \tag{7}$$

and the mean number of quarks is

$$N = VT^3/(\hbar c)^3 \tag{8}$$

(up to some immaterial numerical factors). (The pressure is $p = E/3$, the entropy is $S \simeq 4E/3T \simeq N$ and the density is given by $T = \hbar c n^{1/3}$). At the initial moment we have $E_0 = V_0 T_0^4/(\hbar c)^3$ and $N_0 = V_0 T_0^3/(\hbar c)^3$; for an energy $E/N_n = 1TeV$ we get $T_0 = 1GeV$ and $N_0 = 10^3 N_n$ ($N_n \simeq 100$) (we assume the nucleon radius $a = 2fm$ and $R_0 = aN_n^{1/3}$; $\hbar c = 200MeV \cdot fm$). This plasma expands in time according to the laws[1]

$$\begin{gathered} R = R_0(1 + ct/R_0) \ , \ V = V_0(1 + ct/R_0)^3 \ , \\ T = T_0(1 + ct/R_0)^{-3/4} \ , \ N = N_0(1 + ct/R_0)^{3/4} \ ; \end{gathered} \tag{9}$$

its density goes like

$$n = N/V = n_0(1 + ct/R_0)^{-9/4} = n_0(T/T_0)^3 \tag{10}$$

or, using $N_0 = V_0 T_0^3/(\hbar c)^3$ (*i.e.* $n_0 = (T_0/\hbar c)^3$),

$$T = \hbar c n^{1/3} \ ; \tag{11}$$

if we put here the quark density in the cold nucleus ($n \simeq N_n/V_0$ or $n \simeq 3N_n/V_0$) we get the threshold (ignition) temperature $100 - 150MeV$ (for $a = 2fm$; the values $125 - 180MeV$ given above are obtained for $a = 1.5fm$). We can see that, during expansion, plasma gets cooler and the quark density decreases according to equations (9); at the same time, the energy is conserved and the entropy increases. Equation (11) defines also the chemical potential of a degenerate ultrarelativistic gas of quarks ($\mu = \hbar c n^{1/3}$).

Further on, a mechanism of condensation (hadronization) has been put forward in Refs. [12, 16] (a first-order phase transition). The transition temperature is given by

$$T_t \simeq T_q(T_q/T_m)^{1/2} \ , \tag{12}$$

where T_q is a characteristic quark temperature and $T_m \simeq m_0 c^2$ is a characteristic temperature given by the average mass m_0 of the condensed quarks (up to some immaterial numerical factors). It is shown in Refs. [12] that only a fraction f of the quark number is affected by hadronization (and dominates the hadronization process, with a classical Boltzmann statistics), so that we have in fact

$$T_t \simeq f^{1/2} T_q(T_q/T_m)^{1/2} \ . \tag{13}$$

[1] Compare with the hydrodynamical model of particle production, Refs. [13]-[15].

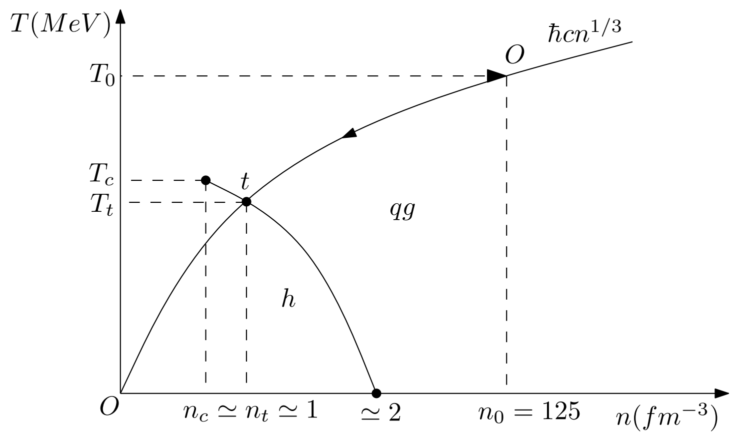

Figure 1: Hadronization of the quark-gluon plasma. Phase diagram temperature (T) *vs* quark density (n). Note the hadronization curve $T = \hbar c n^{1/3}$.

At transition $T_t = T_q$ and

$$T_t = f^{-1} T_m \ , \ \hbar c n_t^{1/3} \simeq f^{-1} m_0 c^2 \ , \tag{14}$$

an expected and plausible result. We can see that the transition density

$$n_t = f^{-3} \left(\frac{m_0 c}{\hbar} \right)^3 \tag{15}$$

is related to the Compton wavelength $\hbar / m_0 c$ of the "average" condensed quark. We can take $m_0 = 4MeV$ (mean mass of the u and d quarks), and get $n_t = f^{-3}(50fm)^{-3}$. In Ref. [12] it is suggested that fraction f is given approximately by $f = 1/N_0^{1/3} \simeq 2 \times 10^{-2}$ (an argument derived from the saturation of the nuclear forces), so we get the transition density $n_t \simeq 1fm^{-3}$. It corresponds to a transition temperature $T_t = f^{-1} T_m \simeq 200MeV$ and a transition radius $R_t = R_0 (n_t/n_0)^{-4/9} = R_0 (T_0/T_t)^{4/3} \simeq 8R_0$. This hadronization happens after $t = 5 \times 10^{-23} s$ from the collision and involves $fN_t = fN_0 (R_t/R_0)^{3/4} \simeq 100 N_n$ hadronized quarks. (The transition implies a latent heat, discontinuities of the thermodynamic potentials, etc, as for a first-order (van der Waals) transition). We can see that the transition density $n_t = 1fm^{-3}$ is very close to the saturation density of the nuclear matter. It is worth noting that during condensation the pressure is practically vanishing.

The transition temperature and density must obey the critical curve give by equation (6). With temperature measured in MeV and density measured in fm^{-3} we get

$$200 = A - B \ , \tag{16}$$

which gives a relation between the two parameters A and B of the critical curve. Since the transition density is very close to the saturation density of the nuclear matter, we may take tentatively the critical density $n_c = A/2B = n_t = 1fm^{-3}$; we get $A = 400$ and $B = 200$ (and the critical temperature $T_c = A^2/4B = T_t = 200MeV$); the density where the critical curve crosses the n-axis is $A/B = 2fm^{-3}$. It is worth noting that the critical point $n_c = 1fm^{-3}$, $T_c = 200MeV$ derived here can have a universal character; indeed it lies on the curve $T = \hbar c n^{1/3}$ for n equal to the saturation density of the nuclear matter (*e.g.*, $n = 1fm^{-3}$).

The hadronization of the quark-gluon plasma is shown in Fig. 1. According to the description given above the hadronization process starts with the creation of a quark-gluon plasma at the initial quark density n_0 (e.g., $\simeq 10^3 N_n / V_0 = 125 fm^{-3}$) and temperature T_0 (*e.g.*, $1GeV$), followed by a cooling of the plasma along the curve $T = \hbar c n^{1/3}$ untill it encounters the critical curve at the transition point n_t (*e.g.*, $1fm^{-3}$) and T_t (*e.g.*, $200MeV$) where the hadronization occurs. With our units (MeV and fm) the curve $T = \hbar c n^{1/3}$ reads $T = 200n^{1/3}$. The transition temperature is very close to the saturation density of the nuclear matter and, very likely, it is very close to the critical density.

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