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Motion of an electric charge under the action of laser fields

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Abstract

The motion of an electric charge (electron) under the action of an external electromagnetic radiation is analyzed both in the classical limit and in the quantum-mechanical theory. The classical solution is provided by the Hamilton-Jacobi equation for the mechanical action, while the quantum-mechanical solution is the well-known Volkov wavefunction. It is shown that in high-intensity radiation fields, as those generated by laser pulses, the charge can be accelerated up to ultrarelativistic velocities in a drift motion along the direction of propagation of the radiation. As a consequence of high, ultrarelativistic velocities the charge oscillations become slow and the charge does not emit radiation anymore; in addition, the ultrarelativistic charge does not "feel" anymore the radiation field. The negative-energy states get lower and lower energy, a negative momentum in the radiation field, and move in the opposite direction (as if they would possess a negative mass), such that the gap between the positive-energy and negative-energy states is increased by radiation.

Also, the motion of an electric charge (electron) is analyzed in a standing electromagnetic wave, both for a classical relativistic charge and a quantum-mechanical charge. A classical relativistic charge suffers multiple Compton collisions in a standing radiation wave. Since the usual electron flux is much lower than usual photon density (flux), the Compton collisions do not destroy the standing wave; the charge propagates almost in a straight line in the wave, with a very short mean free path; since the mean fee path is much shorter than the radiation wavelength, the charge does not "feel", practically, the radiation. For low energies, a quantum charge suffers diffraction by a standing electromagnetic wave, either by transmission, or by reflection (Kapitza-Dirac effect). In usual cases, the diffraction of the electrons by a standing radiation wave proceeds similarly as the diffraction by a classical grating.

Introduction. The Breit-Wheeler process[1]

$$\gamma + \gamma \rightarrow e^- + e^+ \ , \tag{1}$$

where an electron (e^-) - positron (e^+) pair is created by collision of two gamma photons, has not yet been observed. The related process

$$\gamma + n\omega \rightarrow e^- + e^+ \ , \tag{2}$$

where a gamma photon collides with n optical photons with frequency ω, has been observed indirectly[2, 3] in laser fields by the cross-symmetrical multi-photon Compton effect

$$e^- + n\omega \rightarrow e^- + \gamma \ ; \tag{3}$$

positron emission has been observed, intrepreted as the process given by equation (2), subsequent to the process given by equation (3). The multi-photon processess are allowed by electrons moving in an electromagnetic radiation field, where the electrons get "dressed" by an indefinite number of photons, as if photons would be carried on by the electron. The colliding electron in the multi-photon Compton effect is stopped and reversed in its motion in the radiation field, a process which generates gamma photons. The dressed electron in the radiation field is described by the Volkov wavefunctions.[4] The oscillating factors in the phase of the Volkov wavefunction, coming from the radiation field, generates a Fourier decomposition in the radiation phase, which indicates a superposition of multi-photon states.[5]

The Breit-Wheeler process brings into discussion the vacuum polarization. In a constant and uniform electric field the vacuum may break down into electron-positron pairs[6]-[9] (an effect related to the so-called Klein paradox,[10] where the reflection coefficient increases indefinitely for an electron colliding against an infinite potential wall). The critical values of the field can be estimated by $eE \cdot \hbar/mc = mc^2$, where E is the electric field, e is the electron charge and m is the electron mass; we get the so-called Schwinger limit $E = m^2c^3/e\hbar \simeq 4.4 \times 10^{13} statvolt/cm$ ($1statvolt/cm = 3\times10^4 V/m$); a similar estimation holds for a magnetic field H ($(e\hbar/mc)H = mc^2$, where we may recognize a magnetic moment), though the pair creation in a magnetic field is unlikely. The vacuum energy in constant, uniform electric or magnetic fields, properly regularized, gives an effective, non-linear hamiltonian (lagrangian), which contains the invariants $E^2 - H^2$and EH. The non-linear character leads to vacuum birefringence and photon splitting; in a plane waves (leaving aside the spatial and temporal oscillations) the two invariants vanish. The static limit ($\omega \to 0$) of the cross-section of the process given by equation (2) recovers the transition (break-up) probability of the vacuum in a static electric field.

Classical charge in an electromagnetic plane wave.[11] The relativistic equation

$$(\frac{1}{c}\mathcal{E} - \frac{e}{c}\Phi)^2 = m^2c^2 + (\mathbf{p} - \frac{e}{c}\mathbf{A})^2 \ , \tag{4}$$

where $\mathcal{E}$ is the energy and $\mathbf{p}$ is the momentum of a charge e subjected to the electromagnetic potentials Φ and $\mathbf{A}$, leads to the Hamilton-Jacobi equation

$$g^{ij}\left(\frac{\partial S}{\partial x^i} + \frac{e}{c}A_i\right)\left(\frac{\partial S}{\partial x^j} + \frac{e}{c}A_j\right) = m^2c^2 \ , \tag{5}$$

where the metrics is $g^{ij} = (1, -1, -1, -1)$, through $\mathcal{E} \to -\partial S/\partial t$, $\mathbf{p} = \partial S/\partial \mathbf{r}$, S being the mechanical action. We consider a plane wave; since the potentials A_i are functions of the phase $\xi = k_i x^i$ only, we seek a solution of the form $S = -f_i x^i + F(\xi)$, where $f_i f^i = m^2c^2$; f^i is the momentum of a free particle; since $k_i k^i = 0$ and $k^i A_i = 0$ (transversality condition), with zero scalar potential, we get

$$F' = -\frac{e}{c\gamma}f_i A^i + \frac{e^2}{2\gamma c^2}A_i A^i \ , \tag{6}$$

where $\gamma = k_i f^i$.

We assume that the plane wave propagates along the x-direction, so that $\xi = ct - x$ and $k^i = (1, 1, 0, 0)$; since $\gamma = f^0 - f^1$ and $f^i = (f^0, f^1, \overrightarrow{\kappa})$, $f_i f^i = (f^0)^2 - (f^1)^2 - \kappa^2 = m^2c^2$, we get

$$f^0 = \frac{1}{2}\gamma + \frac{m^2c^2 + \kappa^2}{2\gamma} \ , \ f^1 = -\frac{1}{2}\gamma + \frac{m^2c^2 + \kappa^2}{2\gamma} \tag{7}$$

and

$$S = -\frac{1}{2}\gamma(ct + x) - \frac{m^2c^2 + \kappa^2}{2\gamma}\xi + \overrightarrow{\kappa}\overrightarrow{r} + \frac{e}{c\gamma}\int^{\xi} d\xi' \overrightarrow{\kappa}\overrightarrow{A} - \frac{e^2}{2c^2\gamma}\int^{\xi} d\xi' A^2 \ , \tag{8}$$

where $\overrightarrow{\kappa}$ is the transverse momentum and $\mathbf{r} = (y, z)$ is the transverse position. Now the coordinates are given by the derivatives of S with respect to the momentum $\overrightarrow{\kappa}$ and the parameter γ, and the momenta $\mathbf{p} + \frac{e}{c}\mathbf{A}$ are given by the derivatives of S with respect to the cordinates; the energy is the derivative of S with respect to the time. We get

$$y = \tfrac{\kappa_y}{\gamma}\xi - \tfrac{e}{c\gamma}\int^{\xi} d\xi' A_y \ ,$$
$$z = \tfrac{\kappa_z}{\gamma}\xi - \tfrac{e}{c\gamma}\int^{\xi} d\xi' A_z \ , \tag{9}$$
$$x = \tfrac{1}{2}\left(\tfrac{m^2c^2+\kappa^2}{\gamma^2} - 1\right)\xi - \tfrac{e}{c\gamma^2}\int^{\xi} d\xi' \overrightarrow{\kappa}\overrightarrow{A} + \tfrac{e^2}{2c^2\gamma^2}\int^{\xi} d\xi' A^2$$

and

$$p_y = \kappa_y - \tfrac{e}{c}A_y \ ,$$
$$p_z = \kappa_z - \tfrac{e}{c}A_z \ , \tag{10}$$
$$p_x = -\tfrac{1}{2}\gamma + \tfrac{m^2c^2+\kappa^2}{2\gamma} - \tfrac{e}{c\gamma}\overrightarrow{\kappa}\overrightarrow{A} + \tfrac{e^2}{2c^2\gamma}A^2 \ ;$$

the energy is given by

$$\mathcal{E} = c(\gamma + p_x) \ . \tag{11}$$

It is worth noting the linear dependence of the energy on the momentum.

For a charge at rest at $x = y = z = 0$ at the initial moment of time $t = 0$ ($\overrightarrow{\kappa} = 0$, $f^1 = 0$), and a vector potential $A = A_z = A_0\cos(\omega t - kx) = A_0\cos\frac{\omega}{c}(ct - x) = A_0\cos\frac{\omega}{c}\xi$ (linear polarization) we get $\gamma = mc$ ($\gamma^2 = m^2c^2$) and

$$z = -\tfrac{eA_0}{mc^2}\lambda\sin(\omega t - kx) \ , \ y = 0 \ ,$$
$$x = \tfrac{e^2A_0^2/4m^2c^4}{1+e^2A_0^2/4m^2c^4}\left[ct + \tfrac{\lambda}{2}\sin 2(\omega t - kx)\right] \ , \tag{12}$$
$$p_x = \tfrac{e^2A_0^2}{2mc^3}\cos^2(\omega t - kx) \ , \ p_z = -\tfrac{eA_0}{c}\cos(\omega t - kx) \ , \ p_y = 0$$

and

$$\mathcal{E} = mc^2 + \frac{e^2A_0^2}{2mc^2}\cos^2(\omega t - kx) \ , \tag{13}$$

where $\lambda = c/\omega$ is the radiation wavelength. For time-averaged quantities an effective mass can be defined by

$$\mathcal{E}^2/c^2 - p_x^2 = m^{*2}c^2 \ , \ m^{*2} = m^2\left(1 + \frac{e^2A_0^2}{2m^2c^4}\right) \ , \tag{14}$$

by analogy with the free particle.[12]

We can see that, apart from oscillations (with frequency 2ω for the coordinate x!), the charge exhibits a drift motion, governed by the ratio of the field energy eA_0 to the rest energy mc^2. There exist also solutions with negative energy (and negative momentum p_x, $\mathcal{E} = c(-\gamma - p_x)$), corresponding to $\gamma = -mc$, which move in opposite direction.

Application to laser fields, 1. We introduce the parameter

$$\eta = \frac{eA_0}{2mc^2} \tag{15}$$

(or $\eta = eE_0/2m\omega c$, where $E_0 = \omega A_0/c$ is the electric field); from equation (12) the drift velocity of the charge is given approximately by[1]

$$v \simeq \frac{\eta^2}{1+\eta^2}c \ , \tag{16}$$

and the coordinates x and z can be written as

$$\begin{gathered} x \simeq vt + \tfrac{1}{2}\tfrac{v\lambda}{c}\sin 2(\omega - kv)t = vt + \tfrac{1}{2}\tfrac{v\lambda}{c}\sin 2\omega(1-\tfrac{v}{c})t \ , \\ z = -2\eta\lambda \sin\omega(1-\tfrac{v}{c})t \ ; \end{gathered} \tag{17}$$

a current $J = ev$ occurs, along the direction of propagation of the radiation.

We assume an electron beam moving with small velocity along the y-direction, perpendicular to the x-axis along which the high-intensity laser radiation is focalized; initially, the electrons are delocalized, but they get rapidly localized, classical and relativistic, as a consequence of the accleeration in the wave.

Usually, the parameter η is small ($\eta \ll 1$). However, a laser intensity $I = 10^{24} - 10^{25}w/cm^2$, focalized in a pulse of dimension d, generates an electric field $E_0 \simeq \sqrt{I/c} = 10^{10}statvolt/cm$ ($E^2d^3 = Id^2\tau = Id^3/c$, where $\tau = d/c$ is the duration of the pulse). This is a very high electric field;[2] the vector potential is $A_0 = cE_0/\omega = 10^{-5}E_0 = 10^5 statvolt$ for the optical frequency $\omega = 10^{15}s^{-1}$ (or $\nu = \omega/2\pi = 10^{15}s^{-1}$); the corresponding energy for an electron is $eA_0 = 10^{-5}erg = 10MeV$. This energy is much higher than the rest energy of the electron $mc^2 = 0.5MeV$, so that the ratio $\eta = eA_0/2mc^2 = 10$ is much larger than unity. It follows that the electron can be accelerated, during the short duration τ of the pulse, up to velocities close to the speed of light, along the direction of propagation of the radiation field.[3]

If the radiation is propagated in a gaseous plasma,[13] then a radiation pulse is a wavepacket; when focalized, it is a three-dimensional wavepacket which distributes the electrons over its surface, such as to compensate the radiation field. Under such circumstances, there is no field available in the pulse to accelerate charges; the charges are accelerated by the the transport motion of the wavepacket (pulse; pulsed polariton).[14]

Quantum charge in an electromagnetic plane wave. By applying $\gamma(p - \frac{e}{c}A) + mc$ to the Dirac equation $[\gamma(p - \frac{e}{c}A) - mc]\psi = 0$ we get the second-order equation

$$[(p - \frac{e}{c}A)^2 - m^2c^2 - \frac{i}{2}\frac{e\hbar}{c}F_{\mu\nu}\sigma^{\mu\nu}]\psi = 0 \ , \tag{18}$$

where $p_\mu = i\hbar\partial_\mu = (i\hbar\frac{\partial}{c\partial t}, i\hbar\frac{\partial}{\partial \mathbf{r}})$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the strength of the electromagnetic field and $\sigma^{\mu\nu} = (\overrightarrow{\alpha}, i\overrightarrow{\Sigma})$,

$$\overrightarrow{\alpha} = \begin{pmatrix} 0 & \overrightarrow{\sigma} \\ \overrightarrow{\sigma} & 0 \end{pmatrix} \ , \ \overrightarrow{\Sigma} = \begin{pmatrix} \overrightarrow{\sigma} & 0 \\ 0 & \overrightarrow{\sigma} \end{pmatrix} \ , \tag{19}$$

[1]For $x = f(t, x)$ the velocity is given by $v = \frac{dx}{dt} = \frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x}$, or $v = \frac{\partial f/\partial t}{1-\partial f/\partial x}$.

[2]The linear dimension of the pulse is cca $10\mu m$; $10^{24}w/cm^2$ corresponds to cca $10pw$.

[3]The duration of the accelerating regime is governed by a factor $1 - e^{-t/\Delta t}$, where Δt is the time needed to introduce the charge in the beam; it is of the order ω^{-1}, corresponding to the transient region localized at the surface of the laser focus; compared with the pulse duration $\tau \simeq s\omega^{-1}$, where s is approximately 10 or more, we can see that the electron is accelerated very quickly. For high intensities, when the electron moves almost with the pulse velocity c (therefore it is subjected to the field over the pulse length, and duration), the oscillation phase is $\simeq \omega\xi/c = \omega t/(1+\eta^2)$ (according to equations (10) and (12)); for $\omega\tau/(1+\eta^2) = s/(1+\eta^2) \ll 1$ the oscillations may be neglected (their frequency is very low, the length ξ is much shorter than the radiation wavelength λ); the coordinate z oscillates very slowly, such that it does go a period in time τ; we have $z \simeq \lambda\eta s/(1+\eta^2) = d\eta/(1+\eta^2)$, and we can see that it is smaller than the dimension d of the pulse. If the electron is injected along the y-axis, then $y = \kappa_y\xi/\gamma \ll \kappa_y\lambda/\gamma$, and the momentum κ_y should be sufficiently small for the electron to stay inside of the pulse.

$\vec{\sigma}$ being the Pauli matrices. In deriving equation (18) we used $\gamma^\mu\gamma^\nu = g^{\mu\nu} + \sigma^{\mu\nu}$, $\sigma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$ for the Dirac matrices γ and

$$(\mathbf{a}, \mathbf{b}) = \begin{pmatrix} 0 & a_x & a_y & a_z \\ -a_x & 0 & -b_z & b_y \\ -a_y & b_z & 0 & -b_x \\ -a_z & -b_y & b_x & 0 \end{pmatrix} \tag{20}$$

($\frac{1}{2}(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu) = g^{\mu\nu}$). Equation (18) can also be written as

$$\left[\left(\frac{i\hbar}{c}\frac{\partial}{\partial t} - \frac{e}{c}\Phi\right)^2 - \left(i\hbar\frac{\partial}{\partial \mathbf{r}} + \frac{e}{c}\mathbf{A}\right)^2 - m^2c^2 - \frac{i}{2}\frac{e\hbar}{c}F_{\mu\nu}\sigma^{\mu\nu}\right]\psi = 0 \ ,. \tag{21}$$

where $A_\mu = (\Phi, -\mathbf{A})$.

In equation (18) we use $p_\mu = i\hbar\partial_\mu$ and $\sigma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$; the electromagnetic potentials are functions of the phase $\xi = kx$ only, and they are transverse, $\partial_\mu A^\mu = k_\mu A^{'\mu} = 0$ ($k^2 = 0$); we get

$$\left[-\hbar^2\partial_\mu\partial^\mu - \frac{2ie\hbar}{c}A^\mu\partial_\mu + \frac{e^2}{c^2}A^2 - \frac{ie\hbar}{c}(\gamma k)(\gamma A^{'}) - m^2c^2\right]\psi = 0 \ ; \tag{22}$$

the solution is of the form[4]

$$\psi = e^{-\frac{i}{\hbar}px}F \ , \tag{23}$$

where $p^2 = m^2c^2$; we get

$$2i\hbar(pk)F^{'} + \left[-\frac{2e}{c}(pA) + \frac{e^2}{c^2}A^2 - \frac{ie\hbar}{c}(\gamma k)(\gamma A^{'})\right]F = 0 \tag{24}$$

and

$$F = e^{-\frac{i}{\hbar}\int^\xi d\xi^{'}\left[\frac{e}{c(pk)}(pA) - \frac{e^2}{2(pk)c^2}A^2\right] + \frac{e}{2c}\frac{(\gamma k)(\gamma A)}{(pk)}}u \ , \tag{25}$$

where u is a constant bispinor; since[4]

$$(\gamma k)(\gamma A)(\gamma k)(\gamma A) = -(\gamma k)(\gamma k)(\gamma A)(\gamma A) + 2(kA)(\gamma k)(\gamma A) = -k^2A^2 = 0 \ , \tag{26}$$

we get

$$F = \left[1 + \frac{e}{2c}\frac{(\gamma k)(\gamma A)}{(pk)}\right]e^{-\frac{i}{\hbar}\int^\xi d\xi^{'}\left[\frac{e}{c(pk)}(pA) - \frac{e^2}{2(pk)c^2}A^2\right]}u \tag{27}$$

and

$$\psi = \left[1 + \frac{e}{2c}\frac{(\gamma k)(\gamma A)}{(pk)}\right]e^{\frac{i}{\hbar}S}u \ , \tag{28}$$

where

$$S = -px - \int^\xi d\xi^{'}\left[\frac{e}{c(pk)}(pA) - \frac{e^2}{2(pk)c^2}A^2\right] \ . \tag{29}$$

We assume that the interaction is introduced adiabatically; then u is the solution of the free Dirac equation $(\gamma p - mc)u = 0$, *i.e.* it is the plane wave constant bispinor. Therefore, the wavefunctions are

$$\psi_{p\sigma} = \frac{1}{\sqrt{2\varepsilon V}}\left[1 + \frac{e}{2c}\frac{(\gamma k)(\gamma A)}{(pk)}\right]e^{\frac{i}{\hbar}S}u_{p\sigma} \ , \tag{30}$$

[4]We use $(\gamma a)(\gamma b) = ab - ia^\mu b^\nu\sigma_{\mu\nu}$, $(\gamma a)(\gamma a) = ab$.

where $\sigma=\pm 1$ is the spin label, V is the volume ($\mathbf{p}$ is a discrete variable) and $u_{p\sigma}$ is normalized such as $\overline{u}_{p\sigma}u_{p\sigma'}=2mc^2\delta_{\sigma\sigma'}$, $\overline{u}_{-p\sigma}u_{-p\sigma'}=-2mc^2\delta_{\sigma\sigma'}$.[15] We give here these constant bispinors

$$u_{p\sigma}=\left(\begin{array}{c}(\varepsilon+mc^2)^{1/2}w_\sigma\\(\varepsilon-mc^2)^{1/2}(\overrightarrow{n}\overrightarrow{\sigma})w_\sigma\end{array}\right)\ ,\ u_{-p-\sigma}=\left(\begin{array}{c}(\varepsilon-mc^2)^{1/2}(\overrightarrow{n}\overrightarrow{\sigma})w'_\sigma\\(\varepsilon+mc^2)^{1/2}w'_\sigma\end{array}\right)\ , \tag{31}$$

where $\overrightarrow{n}=\mathbf{p}/p$, $w'_\sigma=-\sigma_y w_{-\sigma}$ and w_σ can be taken as the eigenvectors of σ_z. We note that $u^*_{p\sigma}u_{p\sigma'}=\overline{u}_{p\sigma}\gamma^0u_{p\sigma'}=2\varepsilon\delta_{\sigma\sigma'}$; the Dirac matrices are

$$\gamma^0=\left(\begin{array}{cc}1&0\\0&-1\end{array}\right)\ ,\ \overrightarrow{\gamma}=\left(\begin{array}{cc}0&\overrightarrow{\sigma}\\-\overrightarrow{\sigma}&0\end{array}\right)\ . \tag{32}$$

It is easy to see that $\psi_{p\sigma}$ are orthonormal, by using the adiabatic cutoff $e^{-\varepsilon|\xi|}$; similarly, by steepest descent, the "completeness" can be proved (see, for instance, Refs. [16]-[18]). The phase S given by equation (29) is the classical mechanical action; it contains the drift motion of the electron along the propagation of the wave; while the pre-exponential factor in the wavefunction given by equation (30) includes the oscillations of the electron in the radiation field.

The current $j^\mu=c\overline{\psi}\gamma^\mu\psi$ (with the probability density $\rho=\overline{\psi}\gamma^0\psi=\psi^*\psi=j^0/c$) is computed by using

$$\overline{\psi}_{p\sigma}=\frac{1}{\sqrt{2\varepsilon V}}\overline{u}_{p\sigma}e^{-\frac{i}{\hbar}S}\left[1+\frac{e}{2c}\frac{(\gamma A)(\gamma k)}{(pk)}\right]e^{\frac{i}{\hbar}S}\ , \tag{33}$$

the commutation relations of the matrices γ ($(\gamma k)^2=k^2=0$, $(\gamma A)^2=A^2$) and $\overline{u}\gamma^\mu u=2cp^\mu$ (from the Dirac equation).[5] We get

$$j^\mu=\frac{c}{\varepsilon V}\left\{p^\mu-\frac{e}{c}A^\mu+k^\mu\left[\frac{e}{c(pk)}(pA)-\frac{e^2}{2(pk)c^2}A^2\right]\right\}\ . \tag{34}$$

The momentum can also be computed in a similar manner; we get

$$\begin{gathered}q^\mu=\psi^*_{p\sigma}(p^\mu-\tfrac{e}{c}A^\mu)\psi_{p\sigma}=p^\mu-\tfrac{e}{c}A^\mu+k^\mu\left[\tfrac{e}{c(pk)}(pA)-\tfrac{e^2}{2(pk)c^2}A^2\right]+\\+k^\mu\tfrac{ie}{8\hbar(pk)\varepsilon}F_{\lambda\nu}(u^*\sigma^{\lambda\nu}u)\ .\end{gathered} \tag{35}$$

The time average of this quantity

$$q^\mu=p^\mu-k^\mu\frac{e^2\overline{A^2}}{2(pk)c^2} \tag{36}$$

has the property

$$q^2=p^2-\frac{e^2\overline{A^2}}{c^2}=m^2c^2(1-e^2\overline{A^2}/m^2c^4)\ ; \tag{37}$$

it defines an effective mass m^*, which increases with increasing interaction ($A^2=-\mathbf{A}^2$).

Application to laser fields, 2. Consider an electromagnetic wave propagating along the x-direction, $k^\mu=(1,1,0,0)$, $\xi=k_\mu x^\mu=ct-x$, with the electromagnetic potentials $A^\mu=(0,0,0,A)$, $A=A_0\cos\frac{\omega}{c}\xi$ (linear polarization) and an electron moving initially along the y-direction, $p^\mu=(mc,0,p_y,0)$ (with small p_y); then, the pre-exponential factor of the wavefunction in equation (30) is

$$1+\frac{e}{2c}\frac{(\gamma k)(\gamma A)}{(pk)}=1-\frac{eA_0\cos\frac{\omega}{c}\xi}{2mc^2}\left(\begin{array}{cc}-i\sigma_2&\sigma_3\\\sigma_3&-i\sigma_2\end{array}\right)\ . \tag{38}$$

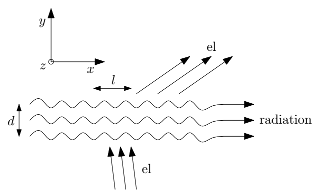

Figure 1: Electrons injected in a laser beam and accelerated by the radiation field.

The mechanical action given by equation (29) becomes

$$S = -\left(mc^2 + \frac{e^2A_0^2}{4mc^2}\right)t + p_y y + \frac{e^2A_0^2}{4mc^3}x - \frac{e^2A_0^2}{8mc^2\omega}\sin\frac{2\omega}{c}\xi \ ; \tag{39}$$

for a high-intensity interaction the electron acquires a drift momentum

$$P_x \simeq \frac{e^2A_0^2}{4mc^3} \ , \tag{40}$$

an energy

$$\mathcal{E} \simeq mc^2 + \frac{e^2A_0^2}{4mc^2} = mc^2 + cP_x \tag{41}$$

and a phase velocity

$$v_x \simeq \frac{\mathcal{E}}{P_x} = \frac{1 + e^2A_0^2/4m^2c^4}{e^2A_0^2/4m^2c^4}c \ , \tag{42}$$

which is higher than the speed of ligh in vacuum c; the group velocity is approximately c. Noteworthy, there is a momentum along the z-coordinate, given by $P_z = -\frac{eA_0}{c}\cos\frac{\omega}{c}\xi$ (from $P_z + eA/c = 0$). Further, we assume that the electromagnetic field is very high, such as $eA_0/2mc^2 \gg 1$; it may correspond to a laser beam focalized in vacuum;[19] Under such circumstances, the charge becomes ultrarelativistic; making use of the pre-exponential factor given by equation (38), the wavefunction becomes

$$\psi_{p\sigma} \simeq \frac{1}{\sqrt{2V}}\begin{pmatrix} w_\sigma \\ \sigma_1 w_\sigma \end{pmatrix} e^{\frac{i}{\hbar}S} \ , \ \psi_{-p-\sigma} \simeq \frac{1}{\sqrt{2V}}\begin{pmatrix} i\sigma_3 w_{-\sigma} \\ -\sigma_2 w_{-\sigma} \end{pmatrix} e^{-\frac{i}{\hbar}S} \ , \tag{43}$$

where

$$S \simeq -\frac{e^2A_0^2}{4mc^2}t + \frac{e^2A_0^2}{4mc^3}x = -\frac{e^2A_0^2}{4mc^3}(ct - x) \tag{44}$$

(the fast-oscillating term in the exponent can be neglected). We can check that the current is $\overline{\psi}_{p\sigma}\gamma^\mu\psi_{p\sigma} = \frac{1}{V}(1,1,0,0)$, corresponding to a plane wave which describes an ultrarelativistic particle. We can also see that the pre-exponential factor in the wavefunction reduces to that of a free particle in this case.

We can see that $\psi_{-p-\sigma}$ corresponds to negative energy (and momentum). The negative-energy electrons in the Dirac Fermi sea get lower and lower (negative) energy (as if they would have a negative mass); such that the gap between the negative-energy states and positive-energy states is increased by radiation.

The injection of the electrons in the laser beam can be done as shown in Fig.1. The role of the adiabatic parameter is played by the transverse momentum p_y; it generates an uncertainty $\Delta t = dm/p_y$ in the time, where d is the transverse dimension of the laser beam (compare with the radiation wavelength) and m is the mass of the electron, an uncertainty $\Delta x = c\Delta t = cdm/p_y$ in the longitudinal coordinate, an uncertainty $\Delta P_x = \hbar p_y/cdm$ in longitudinal momentum and an uncertainty $\Delta\mathcal{E} = \hbar p_y/dm$ in energy; and an uncertainty $\Delta v = \hbar p_y/dmE$ in velocity. We can see that the electron is transported by the radiation field with ultrarelativistic velocities along the direction of propagation of the radiation. The oscillations of the charge become slow in this case, since the phase $\xi = ct - x$ is vanishing for the phase velocity close to c; the motion is, practically, uniform and the charge does not radiate.

It is worth noting that the accelerated electron "feels" not anymore the radiation; indeed, in its rest frame we have the fields

$$E_z^{'} = \frac{\omega A_0}{c} \sin \frac{\omega}{c}(ct - x) \cdot \sqrt{\frac{1 - v/c}{1 + v/c}} \ , \ H_y^{'} = -\frac{\omega A_0}{c} \sin \frac{\omega}{c}(ct - x) \cdot \sqrt{\frac{1 - v/c}{1 + v/c}} \tag{45}$$

and the frequency

$$\omega^{'} = \omega \sqrt{\frac{1 - v/c}{1 + v/c}} \tag{46}$$

(Doppler effect); for $v \to c$ these quantities vanish.

Standing electromagnetic wave. Consider a standing electromagnetic wave with the vector potential

$$A = A_z = \frac{1}{2} A_0 [\cos(\omega t - kx) + \cos(\omega t + kx)] = A_0 \cos \omega t \cos kx \tag{47}$$

(linear polarization); the frequency of the wave is in the optical range, $\nu = \omega / 2\pi \simeq 10^{15} s^{-1}$, and the wavelength is $\lambda = 2\pi / k = c/\nu \simeq 3 \times 10^{-5} cm = 0.3 \mu m$. Consider the motion of a relativistic electron in this standing wave, with energy $\mathcal{E} \gg mc^2 = 0.5 MeV$ and momentum $p \gg mc$; the electron wavelength a is much shorter than its Compton wavelength, $a \ll h/mc = 2 \times 10^{-10} cm$. Since $a \ll \lambda$ we may consider the motion of the electron as being classical. The electrons are injected into the electromagnetic wave; in a radiation beam of thickness, say, $1mm$, they spent a time $10^{-1}/c = 3 \times 10^{-12}$ (or much longer), which is much longer than the wave period $1/\nu = 10^{-15} s$; consequently, we may average the Hamilton-Jacobi equation

$$\frac{1}{c^2} (\partial S / \partial t)^2 = (gradS - \frac{e}{c} \mathbf{A})^2 + m^2 c^2 \tag{48}$$

with respect to the time; equation (48) becomes

$$\frac{1}{c^2} (\partial S / \partial t)^2 = (\partial S / \partial x)^2 + (\partial S / \partial y)^2 + (\partial S / \partial z)^2 + \frac{e^2 A_0^2}{2c^2} \cos^2 kx + m^2 c^2 \ ; \tag{49}$$

the solution of this equation is the mechanical action

$$S = \pm \frac{eA_0}{\sqrt{2}\omega} \cos kx + p_y y + p_z z - Et \ , \tag{50}$$

where $p_{y,z}$ are constant momenta and $\mathcal{E}$ is the energy, given by

$$\mathcal{E}^2 = m^2 c^4 + \frac{1}{2} e^2 A_0^2 + (p_y^2 + p_z^2) c^2 \ . \tag{51}$$

In the standing electromagnetic wave the electron acquires a longitudinal momentum (along the x-direction) given by

$$P_x = \frac{\partial S}{\partial x} = \pm \frac{eA_0}{\sqrt{2} c} \sin kx \ . \tag{52}$$

The conservation of energy and momentum, corresponding to the Compton effect, leads to

$$\mathcal{E}_0 + \hbar\omega = \sqrt{m^2c^4 + c^2p_t^2 + e^2A_0^2/2} + \hbar\omega' \ ,$$

$$p_{0x} + \hbar k = \pm\tfrac{eA_0}{\sqrt{2}c}\sin kx + \hbar k_x' \ , \tag{53}$$

$$p_{0t} = p_t + \hbar k_t' \ ;$$

in equations (53) $\mathcal{E}_0 = \sqrt{m^2c^4 + c^2p_{0x}^2 + c^2p_{0t}^2}$ is the energy of the incident electron, with longitudinal momentum p_{0x} and transverse momentum p_{0t} ($p_{0t} = \sqrt{p_{0y}^2 + p_{0z}^2}$), ω and ω' are the frequencies of the photon before and, respectively, after collision, k and k_x' are the momenta of the photon along the x-axis before and, respectively, after collision and p_t and k_t' are the transverse momenta of the electron and, respectively, the photon after collision. The spatial average of the electron momentum $P_x = (\pm eA_0/\sqrt{2}c)\sin kx$ inside the wave is zero; we may suggest that, for stability, the spatial average of the photon momentum after collision must be zero, $\overline{k_x'} = 0$; since two photons with opposite momenta $\pm\hbar k$ are present in the standing wave in equal proportions, it follows that the original momentum of the electron along the x-direction must also be vanishing, $p_{0x} = 0$. Similarly, in order the wave be stable, the spatial average of the transverse momentum of the photon after collision must be zero, $\overline{k_t'} = 0$; it follows $\overline{p}_t = p_{0t}$. However, we must allow for fluctuations, so that we have

$$\overline{p_t^2} = p_{0t}^2 + \hbar^2\overline{k_t'^2} \ ; \tag{54}$$

similarly, we have, from the second equation (53),

$$\frac{e^2A_0^2}{4c^2} = \hbar^2\overline{k_x'^2} \ . \tag{55}$$

Inserting equations (54) and (55) in the first equation (53) we get the frequency shift

$$\hbar\Delta\omega = \hbar(\omega' - \omega) = -\frac{e^2A_0^2/4 + \hbar^2\omega^2}{2(\mathcal{E}_0 + \hbar\omega)} \ ; \tag{56}$$

since $\overline{k_t'^2} > 0$ we must have $eA_0/2 < \hbar\omega'$ ($eA_0 > 0$), which leads, approximately, to $eA_0/2 < \hbar\omega$. This indicates that 1) either the electrons do not penetrate the standing wave for high values of the electromagnetic field ($eA/2 > \hbar\omega$), or 2) the standing wave is destroyed by the electron beam, or 3) the electrons simply suffer Compton collisions without destroying the wave, and the fluctuations hypothesis is not valid. Indeed, the usual electron beams have such a weak electron flow, that the Compton effect they produce in a standing electromagnetic wave do not cause any damage to the wave.[5]

It is instructive at this moment to have an estimation of the photon density in a laser pulse with moderate intensity $I = 10^{18}w/cm^2$; this intensity corresponds to an electric field of the order $E_0 \simeq \sqrt{I/c} = 10^7 statvolt/cm$ (and a similar magnetic field). The energy density is of the order $w \simeq I/c = 10^{14}erg/cm^3$, with a density of photons $n \simeq 10^{25}cm^{-3}$ with energy $\hbar\omega = 1eV$; the photon flow (flux) is $cn \simeq 10^{35}/cm^2 \cdot s$. Relativistic electrons can be accelerated to give an electric current $\simeq 100mA$, which corrresponds to $\simeq 10^{17}$ electrons per second (electron charge $1.6\times10^{-19}C$) (such an electric current can be produced over a cross-sectional area $1cm^2$); it follows that we may have an electron flow $\simeq 10^{17}/cm^2 \cdot s$. We may see that electron flows are much weaker than

[5]For comparison, the frequency of a Compton-scattered photon is $\omega' = \omega[1 + (\hbar\omega/mc^2)(1 - \cos\varphi 0]^{-1}$, where φ is the scattering angle; or $\Delta\omega = -(\hbar\omega\omega'/mc^2)(1 - \cos\varphi)$.

photon flows. Therefore, we may conclude that the disruption of a standing electromagnetic wave by electron beams is highly unlikely. In fact, electrons in a standing electromagnetic wave simply suffer Compton collisions.

Moreover, because the photon density is very high, an electron suffers many collisions, such that its mean free path is very short; consequently, it moves practically in a straight line, and its mean free path is much shorter than the wavelength of the wave. It follows that the electron does not "feel" the structure of the standing wave, and it behaves, practically, as a free electron, suffering many collisions; therefore, its intrusion in the standing wave has, practically, no effect. The injection of electrons in a standing electromagnetic wave is practically a multiple Compton effect in vacuum. The mean free path of the electron is of the order of the mean separation distance between the photons ($\simeq 10^{-8}cm$), the Compton cross-section σ is of the order of the square of the classical electromagnetic radius of the electron ($r_e = e^2/mc^2 \simeq 2.8 \times 10^{-13}cm$), and the radiation wavelength is $\simeq 3 \times 10^{-5}cm$ ($\hbar\omega = 1eV$).[6]

The creation of electron-positron pairs from vacuum is currently considered in a standing electromagnetic wave of high-power lasers.[20] Since the electron or positron Compton wavelength is much shorter than the radiation wavelength, it is suggested that the spatial variation of the standing wave may be disregarded; then, in high-intensity laser fields, we may get a high electric field, variable in time, which may attain the Schwinger limit. However, both the current and the envisaged lasers are far from the Schwinger limit, and the magnetic field, arising from the spatial variation of the radiation, seems to diminish considerably the rate of pair production. It is also suggested that a gamma radiation would help much to enhance the effect of the electric field, thus leading to pair creation by multi-photon collisions (catalytic effect). However, the time needed for creating a pair is much longer than the wave period, so that the time variation of the wave is in fact averaged. Apart from such dificulties, electrons in a high-intensity standing electromagnetic wave acquire a high energy ($\sqrt{m^2c^4 + e^2A_0^2/2}$) for positive-energy levels and a low energy ($-\sqrt{m^2c^4 + e^2A_0^2/2}$) for negative-energy levels, such that the gap between these states is enlarged by the wave. Similar considerations are valid for electron-positron pairs created in laser fields in the presence of a Coulomb potential (Bethe-Heitler process[21, 22]).

Oscillations of a charge in a standing electromagnetic wave. An electron at rest can be "caught" very quickly by a standing electromagnetic wave; if the wave intensity is high, the electron gets a high momentum P_x along the x-axis, according to equation (52); leaving aside the numerous Compton scatterings, the average electron motion proceeds according to momentum $P_x = mv_x/(1 - v_x^2/c^2)^{1/2}$, where v_x is the electron velocity; the electron acquires an oscillating acceleration given by

$$\frac{dv_x}{dt} = v_x\frac{dv_x}{dx} = -\frac{1}{2}m^2c^4\frac{d}{dx}\frac{1}{m^2c^2 + P^2} \ , \tag{57}$$

or

$$\frac{dv_x}{dt} = kc^2\frac{\frac{e^2A_0^2}{2m^2c^4}\sin kx\cos kx}{\left(1 + \frac{e^2A_0^2}{2m^2c^4}\sin^2 kx\right)^2} \ ; \tag{58}$$

the coordinate $x(t)$ is obtained from

$$\frac{dx}{dt} = v_x = c\frac{\frac{eA_0}{\sqrt{2}mc^2}\sin kx}{\sqrt{1 + \frac{e^2A_0^2}{2m^2c^4}\sin{}^2 kx}} \tag{59}$$

[6]The (invariant) number of collisions in volume dV in time dt is $d\nu = \sigma[(\mathbf{v}_1 - \mathbf{v}_2)^2 - (\mathbf{v}_1 \times \mathbf{v}_2)^2/c^2]^{1/2}n_1n_2dVdt$, where $\mathbf{v}_{1,2}$ are the velocities of the two particles with densities $n_{1,2}$; the velocity factor is the relative velocity.

(these equations should be read in absolute value). An approximate solution is given by

$$x = (2n+1)\frac{\lambda}{4} + \frac{c}{\omega}\sin\frac{eA_0}{\sqrt{2}mc^2}\omega t \ , \tag{60}$$

where n is any integer; indeed, using equation () we can verify, approximately, $P_x^2 = (e^2A_0^2/2c^2)\sin^2 kx$, for $eA_0/\sqrt{2}mc^2 \ll 1$. We can see that the solution x oscillates around the values x_n given by $kx_n = (2n+1)\pi/2$. The mean square of the momentum is $\overline{P_x^2} = e^2A_0^2/4c^2$, the mean square of the velocity is

$$\overline{v_x^2} = c^2\frac{\frac{e^2A_0^2}{4m^2c^4}}{1+\frac{e^2A_0^2}{4m^2c^4}} \ ; \tag{61}$$

with $x = x_n + a\sin\Omega t$ and $\overline{(x-x_n)^2} = \lambda^2/4$ we get

$$\Omega = \frac{1}{\pi}\omega\frac{\frac{eA_0}{2mc^2}}{\sqrt{1+\frac{e^2A_0^2}{4m^2c^4}}} = \frac{1}{\pi}\omega\frac{\eta}{\sqrt{1+\eta^2}} \ ; \tag{62}$$

we can see that the electron oscillates with a mean frequency ranging from 0 to ω/π; it emits, as a dipole, radiation with the same mean frequency.

Electron diffraction by a standing electromagnetic wave. We assume now that the electrons have a wavelengh a of the order of the radiation wavelength $\lambda \simeq 10^{-5}cm$; the electron momentum is of the order $p \simeq 6\times 10^{-22}g\cdot cm/s$, and the energy cp is of the order $\simeq 10^{-11}erg$ ($\simeq 10eV$); it follows that the electrons can be treated as non-relativistic (quantum) particles. The electron velocity is of the order $v = p/m \simeq 10^6cm/s$; the motion time of the electron is much longer than the radiation period, so we may average over the time. The electron energy reads

$$\mathcal{E} = \frac{1}{2m}(\mathbf{p}-\frac{e}{c}\mathbf{A})^2 = \frac{p_x^2}{2m}+\frac{p_y^2}{2m}+\frac{p_z^2}{2m}-\frac{e}{mc}p_zA+\frac{e^2}{2mc^2}A^2 \ ; \tag{63}$$

or, taking the temporal average,

$$\mathcal{E} = \frac{p_x^2}{2m}+\frac{p_y^2}{2m}+\frac{p_z^2}{2m}+\frac{e^2A_0^2}{4mc^2}\cos^2 kx \ ; \tag{64}$$

we can see that the electron moves in a periodic potential

$$U(x) = \frac{e^2A_0^2}{4mc^2}\cos^2 kx \tag{65}$$

along the x-axis, with momentum $p_x = (eA_0/\sqrt{2}c)\sin kx$.

Electrons moving with free momentum $\mathbf{p} = \hbar\mathbf{q}$ are diffracted by the standing electromagnetic wave; this is the Kapitza-Dirac effect.[23] The incident electron wave $\psi^{(0)} = e^{i\mathbf{qr}}$ satisfies the Schrodinger equation

$$-\frac{\hbar^2}{2m}\Delta\psi^{(0)} = \mathcal{E}\psi^{(0)} \ ; \tag{66}$$

the interaction modifies it by $\psi^{(1)}$; in the Born approximation this wavefunction satisfies the Schrodinger equation

$$-\frac{\hbar^2}{2m}\Delta\psi^{(1)} + U\psi^{(0)} = \mathcal{E}\psi^{(1)} \ ; \tag{67}$$

or

$$\Delta\psi^{(1)} + q^2\psi^{(1)} = \frac{2m}{\hbar^2}U\psi^{(0)} \ , \tag{68}$$

where $q=\sqrt{2mE/\hbar^2}$ is the incident wavevector. Equation (68) has the solution

$$\psi^{(1)}(\mathbf{r})=-\frac{m}{2\pi\hbar^2}\int d\mathbf{r}^{'}U(\mathbf{r}^{'})\psi^{(0)}(r^{'})\frac{e^{iq\left|\mathbf{r}-\mathbf{r}^{'}\right|}}{\left|\mathbf{r}-\mathbf{r}^{'}\right|}\ ; \tag{69}$$

for large distances r we get

$$\psi^{(1)}(\mathbf{r})=-\frac{m}{2\pi\hbar^2}\int d\mathbf{r}^{'}U(\mathbf{r}^{'})e^{i\mathbf{q}-\mathbf{q}^{'})\mathbf{r}^{'}}\cdot\frac{e^{iqr}}{r}\ , \tag{70}$$

where $\mathbf{q}^{'}=q\mathbf{r}/r$ is the scattering wavevector. The scattering amplitude is

$$f=-\frac{m}{2\pi\hbar^2}\int d\mathbf{r}^{'}U(\mathbf{r}^{'})e^{i\mathbf{q}-\mathbf{q}^{'})\mathbf{r}^{'}} \tag{71}$$

and the cross-section is

$$d\sigma=\left|f\right|^2 do=\left|\frac{m}{2\pi\hbar^2}\int d\mathbf{r}^{'}U(\mathbf{r}^{'})e^{i\mathbf{q}-\mathbf{q}^{'})\mathbf{r}^{'}}\right|^2 do\ , \tag{72}$$

where do is the solid angle. The Born aproximation is valid for

$$\frac{mU_0}{\hbar^2}\frac{V}{r}\ll 1\ , \tag{73}$$

where V is the interaction volume and $U=U_0\cos^2 kx$. We can see that the diffraction occurs for $\mathbf{q}-\mathbf{q}^{'}=\pm 2\mathbf{k}$ (with $q=q^{'}$) which is the Bragg condition $q^2(1-\cos\theta)=2k^2$ for momentum transfer $2\hbar\mathbf{k}$, θ being the scattering angle.

The Born aproximation encounters difficulties for high intensity of radiation (great U_0), or a large interaction volume; for instance, for a large interaction volume and low electron energy we may have multiple scatterings. For higher electron energy and a thin standing wave we may have transmission diffraction according to the classical diffraction law $\frac{\lambda}{2}\sin\theta=na$, where n is any integer. For high radiation intensity we may have reflection diffraction according to the same classical law; in both cases the electrons "feel" a truncated potential $U(x)$.

Concluding remarks. The motion of an electric charge (electron) has been analyzed here in an electromagnetic radiation and in a standing electromagnetic wave. The wavelength of a relativistic electron is much shorter than the radiation wavelength, so that the electron can be treated classically for many purposes, although the quantum-mechanical motion has also been analyzed. In addition, the time spent by an electron in a standing electromagnetic wave is much longer than the wave period, so that a time-averaged approach is appropriate. Under such conditions, a classical relativistic electron in high-intensity electromagnetic radiation (laser pulses) in vacuum is accelerated by a drift, uniform motion up to ultrarelativistic velocities, where the electron do not "feel" anymore the electromagnetic field. Similar conclusions are valid for a quantum-mechanical electron in electromagnetic radiation (Volkov wavefunction), where the ultrarelativistic regime brings appreciable technical simplifications. In a standing electromagnetic wave a relativistic electron suffers multiple Compton collisions, with a very short mean free path, moving, practically, in a straight line. The standing wave is not destroyed by the electron beam, since the usual electron fluxes are much weaker than the photon fluxes (density); moreover, since the electron mean free path is much shorter than the radiation wavelength, the electron does not "feel", practically, the wave; it suffers Compton scatterings as if it would be in vacuum. For low energy the electrons are diffracted by the standing wave (Kapitza-Dirac effect), either by transmission, or by reflection; for high-intensity radiation the wave behaves as a classical diffraction grating.

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