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Non-relativistic electric charge in electromagnetic field

M. Apostol
Department of Theoretical Physics, Institute of Atomic Physics,
Magurele-Bucharest MG-6, POBox MG-35, Romania
email: apoma@theory.nipne.ro

Abstract

The motion of a non-relativistic point electric charge in an electromagnetic radiation field is analyzed in a few particular cases of physical interest. It is shown that in usual cases the effect of the interaction with the radiation field may amount to what is known as the adiabatic hypothesis for a quasi-free particle. A point free charge q with mass m in moderately high radiation fields with the amplitude of the vector potential A_0 and frequency ω is "accelerated" into a (mean) quasi-uniform motion along the direction of propagation of the radiation and oscillates, mainly with the double frequency 2ω, such that it radiates quasi-classically an electromagnetic field; this radiation looks as if it is produced by an effective charge renormalized by the factor η^2, where $\eta = qA_0/2mc^2$; it is a lateral radiation, in the sense that its maximum is at the right angle with the direction of propagation of the accelerating field. Also, the approximation employed allows an estimation of the amplitude of ionization of a bound state under the action of a field of electromagnetic radiation. It is also shown that within this quasi-classical approximation the quantum transitions among the bound states of the charge subject to a classical field of electromagnetic radiation are absent.

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Key words: electromagnetic interaction; charge acceleration and radiation; ionization

Introduction. The motion of point electric charges subject to high-intensity electromagnetic fields is currently receiving a great deal of attention, especially in connection with the development of high-power optic lasers.[1]-[4] Apart from charge acceleration, many other novel phenomena are envisaged, like vacuum polarization, vacuum breakdown, photon-photon scattering, electron-positron pair creation or non-linear quantum electrodynamics effects, as well as multiple Compton scattering, generation of higher harmonics, atomic and even nuclear effects. A quantum relativistic charge in a plane wave of classical electromagnetic radiation in vacuum is described by the well-known Volkov wavefunction derived as early as 1930.[5, 6] The classical relativistic Hamilton-Jacobi equation helps illuminating many interesting points of this problem.[7] The non-relativistic quantum charge in a moderately intense electromagnetic field exhibits acceleration along the direction of propagation of the radiation and oscillations, as well as its own radiation and, for bound states, ionization.[8] Two particular features are related to these problems. One refers to the possible non-linearities brought about by an intense field of classical radiation, as a consequence of the photon high density and undetermined number of photons; this would be an important departure point from the usual treatment of electron-photon interaction in quantum electrodynamics. Another feature refers to the presence of many electrons in ionized gaseous plasmas (a usual experimental situation in laser physics), where the internal polarization field and plasma oscillations are present.

Let a non-relativistic particle with electric charge q and mass m be subject to a field of electromagnetic radiation with the vector potential $\mathbf{A}$, $div\mathbf{A}=0$. The Schrodinger equation reads

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{1}{2m}\left(\mathbf{P}-\frac{q}{c}\mathbf{A}\right)^2\psi+V\psi\ , \tag{1}$$

where $\mathbf{P}=-i\hbar\partial/\partial\mathbf{r}$ is the canonical (generalized) momentum and V is an external, static potential; c is the speed of light in vacuum and $\hbar$ is Planck's constant. If the particle is an electron and spin is considered, then Pauli's equation can be used instead of equation (1). The problem discussed here is to see whether, and in what conditions, a solution of the form

$$\psi=F\phi \tag{2}$$

is possible (exists), where ϕ satisfies the Schrodinger equation without electromagnetic field,

$$i\hbar\frac{\partial\phi}{\partial t}=\frac{p^2}{2m}\phi+V\phi\ ,\ \mathbf{p}=-i\hbar\partial/\partial\mathbf{r}\ , \tag{3}$$

and the function F becomes unity ($F=1$) for $\mathbf{A}=0$.

We note immediately that if the field would derive from a gauge transformation, *i.e.* if $\mathbf{A}=grad\chi$, (*i.e.*, it would be a gauge field), there would exist a scalar potential $\varphi=-\frac{1}{c}\frac{\partial\chi}{\partial t}$, such that $\frac{1}{c}\frac{\partial\varphi}{\partial t}+div\mathbf{A}=0$; then $\mathbf{A}$ changes the momentum, $\mathbf{P}\rightarrow\mathbf{P}+\frac{q}{c}\partial\chi/\partial\mathbf{r}$ ($\partial/\partial\mathbf{r}\rightarrow\partial/\partial\mathbf{r}+\frac{iq}{c\hbar}\partial\chi/\partial\mathbf{r}$), an additional term $q\varphi=-\frac{q}{c}\frac{\partial\chi}{\partial t}$ appears in the hamiltonian and $\psi=e^{\frac{iq}{c\hbar}\chi}\phi$; the function F would be $F=e^{\frac{iq}{c\hbar}\chi}$ in this case. However, a radiation field, for instance of the form $A_x=A_y=0$, $A_z=A=A_0\cos\frac{\omega}{c}(ct-x)$, $\varphi=0$, where ω is the frequency of the radiation, is not a gauge field.

We note also that the problem formulated above is the non-relativistic counterpart of the relativistic charge in an electromagnetic wave, whose solution is the Volkov wavefunction;[5] therefore, it may appear that a route to the solution of our problem here would be the non-relativistic limit of the Volkov wavefunction. This is not so, because in the non-relativistic limit the contribution of the anti-particles, which is included in the Volkov wavefunction, are lost. The situation is similar with the zero mass limit of the ultrarelativistic case, which cannot be taken directly on the Volkov wavefunction, because this wavefunction is not analytic in the particle mass.[1]

Inserting equation (2) into equation (1) and making use of equation (3) we get

$$i\hbar\frac{\partial F}{\partial t}\phi=\frac{1}{2m}(p^2F)\phi-\frac{q}{mc}\mathbf{A}(\mathbf{p}F)\phi+\frac{q^2}{2mc^2}A^2F\phi+\frac{1}{m}(\mathbf{p}\phi)(\mathbf{p}F)-\frac{q}{mc}\mathbf{A}(\mathbf{p}\phi)F\ , \tag{4}$$

where $\mathbf{p}=-i\hbar\partial/\partial\mathbf{r}$ and we use $div\mathbf{A}=0$. We take $A_x=A_y=0$, $A_z=A=A_0\cos\frac{\omega}{c}(ct-x)$ and equation (4) becomes

$$i\hbar cF^{'}\phi=-\frac{\hbar^2}{2m}F^{''}\phi+\frac{q^2}{2mc^2}A^2F\phi-\frac{i\hbar}{m}(p_x\phi)F^{'}-\frac{q}{mc}A(p_z\phi)F\ , \tag{5}$$

where F is a function of $\xi=ct-x$ only. Introducing the (reduced) Compton wavelength $\lambda_c=\hbar/mc$ of the charge with mass m and the parameter $\eta=qA_0/2mc^2$, equation (5) can also be written as

$$iF^{'}\phi=-\frac{1}{2}\lambda_cF^{''}\phi+\frac{2\eta^2}{\lambda_c}F\phi\cos^2\frac{\omega}{c}\xi-i\lambda_c\left(\frac{p_x}{\hbar}\phi\right)F^{'}-2\eta\left(\frac{p_z}{\hbar}\phi\right)F\cos\frac{\omega}{c}\xi\ . \tag{6}$$

[1]An ultrarelativistic massless charge does not couple with the electromagnetic field; the phase of the Volkov function oscillates indefinitely in this case, and the interaction part of the Volkov wavefunction may be viewed as being reduced to zero.

The term $\lambda_c F^{''}$ compared with $F^{'}$ shows the variation of $F^{'}$over distances of the order λ_c; we may neglect such variations, as they imply relativistic corrections; in addition, the uncertainty principle implies a limitation to distances much larger than λ_c. For instance, for an electron $\lambda_c \simeq 3\times 10^{-11}cm$, which is such a short distance that the non-relativistic approximation becomes meaninglesss. Therefore we neglect the term $\lambda_c F^{''}$ in equation (6). The term $\lambda_c(p_x\phi/\hbar)F^{'}$ should be compared with $F^{'}\phi$; it is of the order λ_c/λ_q, where λ_q is the characteristic wavelenghth in the potential V; for instance, for a bound state it is of the order of the characteristic wavelength of the bound state; for scattering, it is of the order of the scattering wavelength ($\hbar^2/m\lambda_q^2 \simeq |V|$; for a Coulomb potential it is of the order of the Bohr radius, $\hbar^2/m\lambda_q^2 \simeq q^2/\lambda_q$, $\lambda_q \simeq \hbar^2/mq^2$); for a free motion $\lambda_c/\lambda_q = v_x/c$, where v_x is the velocity along the x-direction. We can see that this term also can be neglected in equation (6). Therefore, we can write approximately

$$iF^{'}\phi \simeq \frac{2\eta^2}{\lambda_c}F\phi\cos^2\frac{\omega}{c}\xi - 2\eta\left(\frac{p_z}{\hbar}\phi\right)F\cos\frac{\omega}{c}\xi \ . \tag{7}$$

The term $\eta(p_z\phi/\hbar)F$ may imply variations over large distances, such that it may bring relevant contributions. In the weak coupling limit $\eta \ll 1$ it brings the main contribution. However, for usual fields of interest (*i.e.* for fields which are not extremely weak) the last term in equation (7) may be neglected. Under these circumstances, the approximation which turns out to be relevant for determining the function F is the approximation of a free charge, where F depends only on the interaction and does not depend on the wavefunction ϕ.

In this context, we also note that the ansatz given by equation (2) may look too restrictive, since, in general, we expect the function F to be a matrix. If we label the states ϕ by a suffix n, we would expect, in general, that the wavefunction ϕ_n be taken by interaction into a superposition of wavefunctions $F_{n^{'}n}\phi_{n^{'}}$. The assumption embedded into equation (2) amounts to what is known as the quasi-adiabatic hypothesis. It implies that the interaction varies slowly in comparison with the periods of transitions among the bound states of the charge (this may be not true for the ionization process).

Free particle. For a free particle ($V=0$) $p_z\phi$ is a constant momentum multiplied by ϕ ($\phi \sim e^{\frac{i}{\hbar}p_z z}$); we get from equation (7)

$$F = e^{-i\frac{\eta^2}{\lambda_c}\xi - i\frac{\eta^2\lambda}{2\lambda_c}\sin\frac{2\omega}{c}\xi + i\frac{2\eta\lambda}{\lambda_q}\sin\frac{\omega}{c}\xi} \ , \tag{8}$$

where $\lambda = \omega/c$ is the wavelength of the radiation field.

For a free charge an interesting situation appears for moderately high fields; we may consider that the non-relativistic limit is preserved up to $\eta \simeq 1$; for an electron $\eta = 1$ corresponds to $A_0 = 10^3 statvolt$ and an electric field $E_0 \simeq 10^8 stavolt/cm$ for an optical frequency $\omega = 10^{15}s^{-1}$ (for comparison the atomic fields are of the order $10^6 statvolt/cm$); the intensity of the radiation in this case is $I = cE_0^2/4\pi \simeq 10^{18}w/cm^2$. The Compton wavelength for the electron is $\lambda_c \simeq 3\times 10^{-11}cm$; we take the optical wavelength $\lambda = 1\mu m = 10^{-4}cm$ and the electron wavelength $\lambda_q = 1\text{Å} = 10^{-8}cm$ (corresponding to an initial energy $\simeq 3eV$ and an initial momentum $\simeq 10^{-19}g\cdot cm/s$). Under these circumstances we may neglect the last term in the phase of the function F in equation (8), and write

$$F \simeq e^{-i\frac{\eta^2}{\lambda_c}\xi - i\frac{\eta^2\lambda}{2\lambda_c}\sin\frac{2\omega}{c}\xi} \ . \tag{9}$$

Making use of

$$e^{iz\cos\varphi} = \sum_n i^n e^{in\varphi}J_n(z) \ , \tag{10}$$

where J_n are the Bessel functions, we can write

$$F = \sum_n e^{i\left(\frac{2n\omega}{c} - \frac{\eta^2}{\lambda_c}\right)\xi} J_{-n}(\eta^2\lambda/2\lambda_c) \ . \tag{11}$$

We can see that a free charge acquires a superposition of plane waves in the presence of the radiation field, with energies

$$\mathcal{E}_n = -2n\hbar\omega + \eta^2 mc^2 + \mathcal{E}_0 \ , \tag{12}$$

momenta (along the direction of propagation of the radiation)

$$p_n = -2n\hbar k + \eta^2 mc + p_{x0} \tag{13}$$

and very small weights $J_n(\eta^2\lambda/2\lambda_c) \simeq J_n(10^6)$ (for our numerical estimates); $\mathcal{E}_0$ is the initial energy, p_{x0} is the initial momentum along the direction of propagation of the radiation and $k = \omega/c$ is the wavevector of the radiation.

It is worth noting that the charge has a distribution of momentum (momentum density) along the direction of propagation of the radiation; it can be computed most conveniently from the wavefunction given by equation (9); we get

$$p(\mathbf{r}, t) = \frac{1}{V}\eta^2 mc(1 + \cos\frac{2\omega}{c}\xi) \ , \tag{14}$$

where V is the volume and p_{x0} is set equal to zero; the total momentum is

$$\int p(\mathbf{r}, t)d\mathbf{r} = \sum_n p_n J_n^2(\eta^2\lambda/2\lambda_c) = \eta^2 mc \ . \tag{15}$$

We can see that the charge is "accelerated" along the direction of propagation of the radiation, in the sense that it acquires a constant momentum $\eta^2 mc$ and a variable momentum corresponding to the oscillating term in equation (14) ; the momentum and energy of the particle depend on the time. Similarly, there exists a distribution of velocity

$$v(\mathbf{r}, t) = \frac{1}{V}\eta^2 c(1 + \cos\frac{2\omega}{c}\xi) \ . \tag{16}$$

We note that both the energy and the momentum scales of the charge in the radiation field are the relativistic quantities mc and, respectively, mc^2, according to equations (12) and (14), although we are in the non-relativistic limit. This is due to the fact that the electromagnetic radiation (even in its classical limit) is essentially a relativistic entity; the non-relativistic limit is expressed by the inequality $\eta^2 \ll 1$. It is also worth noting that the higher harmonics appearing in equations (13) are not present in equation (14).

A charge which oscillates according to equation (14) radiates; however, for a charge distribution extended to the whole space this radiation is not observable (the radiating charge distribution absorbs its own radiation). In accordance with experimental situation we assume that the charge in the radiation field is confined in fact to a small spatial region, of linear dimension d, $d \gg \lambda$ (and λ_q). Inside this region we can view the charge as a wavepacket, localized at $\mathbf{r}_0$ and moving with velocity v given by equaton (16); we neglect the motion along the other two directions. The wavepacket assumption is not necessary, though even for moderate values of η the wavelength of the wavefunction in equation (9) is of the order λ_c/η^2, which is much smaller than our distances of interest. This amounts to a quasi-classical approximation. Therefore, we assume a current density of the form

$$j_x(\mathbf{r},t)=\eta^2qc\left[1+\cos\frac{2\omega}{c}(ct-x_0)\right]\delta(\mathbf{r}-\mathbf{r}_0)\ , \tag{17}$$

as given by equation (16). We get immediately the vector potential

$$A_x=\eta^2q\left[1+\cos\frac{2\omega}{c}(ct-|\mathbf{r}-\mathbf{r}_0|-x_0)\right]\frac{1}{r}\simeq\eta^2q\left[1+\cos\frac{2\omega}{c}(ct-r)\right]\frac{1}{r} \tag{18}$$

for $r\gg r_0$. From the gauge condition $\partial\Phi/c\partial t+div\mathbf{A}=0$ we get the leading contribution $\Phi=(\eta^2qx/r^2)\cos\frac{2\omega}{c}(ct-r)$. We compute now the fields according to $\mathbf{E}=-\partial\mathbf{A}/c\partial t-grad\Phi$, $\mathbf{H}=curl\mathbf{A}$. We get

$$\begin{gathered}E_x=\tfrac{2\eta^2q\omega}{c}\sin\tfrac{2\omega}{c}(ct-r)\tfrac{1}{r}(1-x^2/r^2)\ ,\\ E_y=-\tfrac{2\eta^2q\omega}{c}\sin\tfrac{2\omega}{c}(ct-r)\tfrac{xy}{r^3}\ ,\ E_z=-\tfrac{2\eta^2q\omega}{c}\sin\tfrac{2\omega}{c}(ct-r)\tfrac{xz}{r^3}\end{gathered} \tag{19}$$

and

$$H_x=0\ ,\ H_y=\tfrac{2\eta^2q\omega}{c}\sin\tfrac{2\omega}{c}(ct-r)\tfrac{z}{r^2}\ ,\ H_z=-\tfrac{2\eta^2q\omega}{c}\sin\tfrac{2\omega}{c}(ct-r)\tfrac{y}{r^2}\ . \tag{20}$$

The Poyinting vector $\mathbf{S}=(c/4\pi)\mathbf{E}\times\mathbf{H}$ leads to the radiated differential intensity

$$dI=Sdf=H_y=\frac{\eta^4q^2\omega^2}{\pi c}\sin^2\frac{2\omega}{c}(ct-r)(1-x^2/r^2)do\ , \tag{21}$$

where do is the element of the solid angle and $df=r^2do$ is the element of area. We can see that the maximum radiated field is at the angle $\theta=\pi/2$, where $\cos\theta=x/r$; it is a "lateral" radiation. It is worth noting that this is a classical radiation with frequency 2ω, and an effective charge reduced by the factor η^2. If we consider the rather unphysical case of very weak fields, such that $\eta^2\ll 1$, the function F given by equation (8) reduces to

$$F=e^{i\frac{2\eta\lambda}{\lambda_q}\sin\frac{\omega}{c}\xi}\ ; \tag{22}$$

the charge radiates frequency ω, and it is renormalized by a factor $\sim(\lambda_c/\lambda_q)\eta$. However, the last term $i\frac{2\eta\lambda}{\lambda_q}\sin\frac{\omega}{c}\xi$ in the phase of the function F in equation (8) becomes important in comparison with the second term for very low fields ($\eta^2\lambda/\lambda_c\ll\eta\lambda/\lambda_q$, $E_0\simeq 10^4 statvolt/cm$); we may neglect it in our subsequent discussion.

Ionization. The theory described here offers the opportunity of formulating a simple model of ionization under the action of an electromagnetic wave. We assume a bound state $e^{-\frac{i}{\hbar}\mathcal{E}_bt}\phi_b$ of a charge, where the $\mathcal{E}_b$-energy factor is shown explicitly, as, for instance, a periferic electron in a heavy atom. Under the action of the electromagnetic radiation this state becomes $Fe^{-\frac{i}{\hbar}\mathcal{E}_bt}\phi_b$; it contains many states, among which a free particle state, which may be viewed as the ionization state; it may be taken as being of the form $\phi_i=\frac{1}{\sqrt{V}}e^{-\frac{i}{\hbar}\mathcal{E}_it+i\mathbf{Q}\mathbf{r}}$, where $\mathcal{E}_i$ is the energy and $\hbar\mathbf{Q}$ is the momentum of the free charge. Then, the amplitude of ionization is given by

$$a=\int d\mathbf{r}\phi_i^*Fe^{-\frac{i}{\hbar}\mathcal{E}_bt}\phi_b=\frac{e^{\frac{i}{\hbar}(\mathcal{E}_i-\mathcal{E}_b)t}}{\sqrt{V}}\sum_n\int d\mathbf{r}e^{-i\mathbf{Q}\mathbf{r}}\phi_be^{i\left(\frac{2n\omega}{c}-\frac{\eta^2}{\lambda_c}\right)\xi}J_{-n}(\eta^2\lambda/2\lambda_c)\ , \tag{23}$$

where we use the function F given by equation (11) for moderately strong fields. We can insert in equation (23) the Fourier expansion of the function ϕ_b

$$\phi_b=\frac{1}{\sqrt{V}}\sum_{\mathbf{q}}c(\mathbf{q})e^{i\mathbf{q}\mathbf{r}}\ ,\ c(\mathbf{q})=\frac{1}{\sqrt{V}}\int d\mathbf{r}\phi_be^{-i\mathbf{q}\mathbf{r}}\ , \tag{24}$$

which leads to

$$a = e^{\frac{i}{\hbar}(\mathcal{E}_i - \mathcal{E}_b)t} \sum_n c(Q_x + 2n\omega/c - \eta^2/\lambda_c, \mathbf{Q}_t) e^{i\left(\frac{2n\omega}{c} - \frac{\eta^2}{\lambda_c}\right)ct} J_{-n}(\eta^2\lambda/2\lambda_c) \ , \tag{25}$$

where $\mathbf{Q}_t$ is the transverse momentum of the free particle in the (y, z)-plane.

First, we can check that for a very localized wavefunction ϕ_b, for which all the coefficients $c(\mathbf{q})$ may be taken equal to unity, the ionization amplitude $|a|^2 = 1$, since

$$1 = \left|e^{iz\cos\varphi}\right|^2 = \sum_{nn'} i^{n-n'} e^{i(n-n')\varphi} J_n(z) J_{n'(z)} \ ; \tag{26}$$

this indicates that such a very localized wavefunction (corresponding to a point particle) is not a bound state. For a reasonably bound state the coefficients $c(\mathbf{q})$ decrease with increasing q, such that we may assume that the main contribution to the summation in equation (25) comes from $\mathbf{Q}_t = 0$ and $Q_x + 2n_0\omega/c - \eta^2/\lambda_c \simeq 0$, where n_0 is the integral part of $(c/2\omega)(\eta^2/\lambda_c - Q_x)$; under these circumstances we may take $c(0) \simeq 1$ and the amplitude a becomes

$$a \simeq e^{\frac{i}{\hbar}(\mathcal{E}_i - \mathcal{E}_b)t} e^{-iQ_x ct} J_{-n_0}(\eta^2\lambda/2\lambda_c) \simeq \sqrt{\frac{4\lambda_c}{\eta^2\lambda}} e^{\frac{i}{\hbar}(\mathcal{E}_i - \mathcal{E}_b)t} e^{-iQ_x ct} \cos\left[\eta^2\lambda/2\lambda_c - (n_0 + 1/2)\frac{\pi}{2}\right] \tag{27}$$

for $\eta^2\lambda/2\lambda_c \gg 1$. In this context, we can note that the asymptotic formula

$$e^{iz\cos\varphi} \simeq \sqrt{\frac{2\pi}{z}} \sum_n \left[e^{iz}\delta(\varphi - \pi/4 + 2\pi n) + e^{-iz}\delta(\varphi + 3\pi/4 + 2\pi n)\right] \ , \ z \gg 1 \tag{28}$$

(Poisson formula) is not much more helpful in estimating the ionization amplitude.

It is worthwhile noting that the ionization process analyzed here in a classical electromagnetic wave is different from the ionization which proceeds by absorbing one, or more, radiation quanta (photoelectric effect), and, as well as, it is different from the ionization in static electric or magnetic fields, in quasi-static electric fields, or in the quasi-classical tunneling approximation.[9]-[12] In addition, we note that the result presented here refers especially to a periferic electron in a heavy atom.

Absence of quantum transitions. Let $e^{-\frac{i}{\hbar}\mathcal{E}_n t}\phi_n$ be a bound state of the charge; it is taken by interaction into $Fe^{-\frac{i}{\hbar}\mathcal{E}_n t}\phi_n$; a state $e^{-\frac{i}{\hbar}\mathcal{E}_{n'} t}\phi_{n'}$ is found in the state with interaction with a probabiliy amplitude

$$a_{nn'} = \int d\mathbf{r}\phi^*_{n'} F\phi_n e^{-i\omega_{nn'}t} = \sum_s J_s(\eta^2\lambda/2\lambda_c) \int d\mathbf{r}\phi^*_{n'}\phi_n e^{i\left(\frac{2s\omega}{c} + \frac{\eta^2}{\lambda_c}\right)x} e^{i\left(\omega_{nn'} - 2s\omega - \eta^2 mc^2/\hbar\right)t} \ , \tag{29}$$

where $\omega_{nn'} = (E_{n'} - E_n)/\hbar$. With the notation

$$C_{nn'}(s) = \int d\mathbf{r}\phi^*_{n'}\phi_n e^{i\left(\frac{2s\omega}{c} + \frac{\eta^2}{\lambda_c}\right)x} \tag{30}$$

we can write

$$a_{nn'} = \sum_s C_{nn'}(s) J_s(\eta^2\lambda/2\lambda_c) e^{i\left(\omega_{nn'} - 2s\omega - \eta^2 mc^2/\hbar\right)t} \ . \tag{31}$$

Since η^2/λ_c is very large in equation (30) we may neglect the s-term in the phase in this equation, such that $C_{nn'}$ does not depend practically on s; in this case, the summation over s in equation (31) gives zero. Therefore, we conclude that there is no quantum transition among the bound states of a charge in a classical field of electromagnetic radiation, as expected. The ansatz given by equation (2) implies that the interaction is much slower than any transition among the bound states, but not necessarily so for the ionization process, especially with high energy of the ejected charge.

Also, it is worth noting that the presence of photons with energy $n\hbar\omega$ in the dressed wavefunction of the charge indicates merely the presence of virtual states, while the multiple Compton effect with absorption of n photons is cancelled out by the multiple Compton effect with emission of n photons. This process is distinct from the external Compton effect in the presence of the electromagnetic radiation.[13]

Concluding remarks. It is worth noting another form of the solution of equation (7), written as

$$\dot{F} = \frac{i}{\hbar} V F \ \ , \tag{32}$$

where we use the time derivative; V stands for the interaction generated by A. The solution can also be written as

$$F = 1 + \frac{1}{\hbar}\int^t dt^{'} iV + \frac{1}{\hbar^2}\int^t dt^{'} iV \int^{t^{'}} dt^{''} iV + ... \tag{33}$$

with the initial condition $F = 1$ for $t \rightarrow -\infty$. If we extend the integration with respect to the intermediate times to t and introduce the factor $n!$ we recover immediately the exponential form. The form of the solution given by equation (33) is useful for the case when F and V are matrices (operators); then V includes also factors of the form $e^{i\omega_{nn'} t}$, and we can recognize easily in equation (33) the perturbation theory with intermediate states. As long as we require $\phi \rightarrow F\phi$ as in equation (2) we lose all transitions which are slower than the rate of switching on the interaction; indeed, such slow $e^{i\omega_{nn'} t}$-terms may be neglected in equation (33), which amounts to a quasi-classical character of the interaction V; we recover in this case the exponential form of solution which we used in the main text and which led to the absence of quantum transitions; this may not be necessarily the case for the ionization process, especially when the energy of the ejected particle is high. This circumstane may be termed a quasi-classical approximation; it may be seen formally by integrating by parts in equation (33) where terms like

$$\frac{1}{\hbar}\int dt V e^{i\omega_{nn'} t} = \frac{V}{\hbar\omega_{nn'}} e^{i\omega_{nn'} t} - \int dt \frac{1}{i\hbar\omega_{nn'}} \frac{\partial V}{\partial t} e^{i\omega_{nn'} t} \tag{34}$$

appear; they allow a direct comparison between $\hbar\omega_{nn'}$ and the rate $\partial V/\partial t$ (the first term in equation is the change in the wavefunction due to a constant interaction).

A special attention deserves the electrons in ionized gaseous plasmas subject to an electromagnetic radiation field. In normal conditions a gaseous plasma has a density of electrons $n \simeq 10^{19} cm^{-3}$; the electrons oscillate under the action of an external electric field E_{ex} and their own internal electric field E_{in}, according to the classical law

$$m\ddot{\mathbf{u}} + m\omega_0^2\mathbf{u} = q\mathbf{E}_{ex} + q\mathbf{E}_{in} \ \ , \tag{35}$$

where ω_0 is a characteristic frequency for bound electrons and $\mathbf{u}$ is their displacement from equilibrium positions; for free electrons $\omega_0 = 0$. The charge imbalance is $\delta n = -n div\mathbf{u}$; since the potential φ is given by $\Delta\varphi = -4\pi q\delta n = 4\pi n^2 q div\mathbf{u}$, we have the internal electric field $\mathbf{E}_{in} = -grad\varphi = -4\pi n^2 q\mathbf{u}$ and the equation of motion

$$\ddot{\mathbf{u}} + \omega_p^2\mathbf{u} = q\mathbf{E}_{ex} \ \ , \tag{36}$$

where $\omega_p = \sqrt{4\pi n q^2/m}$ is the plasma frequency. The displacement u is much smaller than the mean separation distance between the electrons, and, since the plasma moves as a macroscopic body (as a whole), its dynamics is entirely classical. The external radiation field superposes its oscillations with wavelength $\lambda \simeq 10^{-4}cm$ over this small, uniform displacement, according to equation (36). Consequently, they behave as free electrons subject to electromagnetic radiation. In a plane wave they are "accelerated", oscillate and radiate; in a focused beam they are distributed over the surface of a spherical polariton.[14]

In conclusions, it is shown in this paper that the interaction with a plane-wave field of classical electromagnetic radiation can be introduced in the dynamics of a point electric charge through an exponential factor within a quasi-classical and adiabatic approximation. This procedure allows the description of the "acceleration" and radiation of a free charge, as well as the ionization process for bound states of the charge. Since the swithching on of the interaction is slower than the quantum-mechanical transitions, the latter cannot be described within this approximation.

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