Journal of Theoretical Physics

Founded and Edited by M. Apostol **255** (2016)

ISSN 1453-4428

Ten naive commentaries of the non-expert layman M. Apostol on the Seismological Problem

recorded by B. F. Apostol
Department of Engineering Seismology, Institute of Earth's Physics,
Magurele-Bucharest MG-6, POBox MG-35, Romania
email: afelix@theory.nipne.ro

1. It is another strange particularity of the human nature that such impressive disasters as the earthquakes received so little scientific attention in the history of the Natural Sciences. It was only in 1966, after more than half a century since the invention of the seismographs, when Aki estimated for the first time the energy released in an earthquake.[1] For some fancy reason this energy was called seismic moment and was denoted by M. After two decades, starting 1935, Richter and Gutenberg provided a magnitude scale of measuring this energy, and the "intensity" of the earhquakes;[2]-[6] it reads $\lg M = \frac{3}{2}M_w + 16.1$, where M_w denotes the magnitude (the suffix w - another fancy notation) and M is measured in erg; the constant 16.1 was chosen arbitrarily. It is a curious coincidence that $M^{2/3}$ is a surface energy for a volume energy M (any surface S is $S = V^{2/3}$, where V is the volume it encloses).

It is strange that nobody asks what is the error of estimating M, as long as we all know that, scientifically speaking, any experimental measurement has an error. An earthquake of magnitude $M_w = 7$ releases an energy $M \simeq 10^{26} erg$; it is natural for such large figures to assume an exponent error of the order unity, which makes a magnitude error $\Delta M_w = \pm 0.33$. Having in view the big difference in energy, this is a pretty large error. As convenient as the Gutenberg-Richter law is, it does not take us too far in the understanding of the earthquakes.

2. The seismographs at the Earth's surface record a propagating disturbance of the type of the elastic waves. In most cases, the seismographs indicate that such a disturbance originates at some location $\mathbf{R}_0$ beneath the Earth's surface and lasts a finite, relatively short, lapse of time T. It follows that the focal region of such an earthquake has very small dimensions in comparison with the distances of our interest; therefore, the force which generates such an earthquake should include the Dirac delta function $\delta(\mathbf{R} - \mathbf{R}_0)$, or its derivatives, and the Dirac delta function $\delta(t)$, where $\mathbf{R}$ is the position vector and t denotes the time. Such a force defines an "elementary" earthquake. Its mathematical expression is very useful, since it generates an elastic waves equation for some sort of a Green function. For extended seismic sources, both in space and time, we can introduce a structure factor of the focal region; by deconvolution it may give information about the structure of the seismic source. It was only recently when the notions of elementary earthquake and structure factor have been introduced.[7]

3. It comes out that the most general elementary force density which ensures the zero force and zero angular momentum (as for a stationary Earth) is given by $f_i = M_{ij} T \delta(t) \partial_j \delta(\mathbf{R} - \mathbf{R}_0)$, where M_{ij} is the tensor of the seismic moment; M is a generic notation for its magnitude.[7] Therefore, it turns out that the seismic source has a tensorial character, implies anisotropy, in accordance with the seismic records, and may represent a faulting slip, in agreement with the general concepts of the tectonics. Moreover, for an isotropic tensor M_{ij} (a scalar), the force given here may represent an explosion (another source of earthquakes - artificial earthquakes). Reduced to its principal axes the tensor of the seismic moment looks like an ellipsoid, which a very useful representation of the seismic source.

It is reasonable to admit that the material in the focal region is destroyed during the activation of the seismic source; the rupture of the material appears when the density of the seismic moment M/V is of the same order of magnitude as the elastic energy density ρc^2, where V is the volume of the focal region, ρ is the density of the medium and c is the velocity of an elastic wave. From the equality $M/V = \rho c^2$, using $M = 10^{26} dyn \cdot cm$ (corresponding to an earthquake magnitude $M_w = 7$), an average Earth's density $\rho = 5g/cm^3$ and an average wave velocity $c = 5km/s$, we get a volume $V = 8 \times 10^{13} cm^3$ and a localization length $l = V^{1/3} \simeq 400m$ of the focal region. This implies an extension $l \simeq 400m$ for the function $\delta(\mathbf{R} - \mathbf{R}_0)$ and an extension $T = l/c \simeq 0.08s$ for the function $\delta(t)$ in the expression of the force. Indeed, the Fourier analysis of the seismic disturbances indicates practically the absence of wavelengths shorter than such a critical distance l (and consequently, the absence of frequencies higher than a corresponding critical frequency).

We can see that the notion of seismic moment, elementary earthquakes together with basic physical knowledge offer interesting insights and an useful estimation of the physical characteristics of the earthquakes. The textbooks and treatises of Seismology are full of obscure "theories" endowed with extremely cumbersome notations.[8]-[19] For example, "dynamic elasticity" and "elastodynamics" are just other fancy words for the Theory of Elasticity, wave equation and Gauss's theorem, notions with a long and well-founded tradition in Theoretical Physics and Equations of Mathematical Physics;[20, 21] the "kinematics and dynamics of the seismic source" is the theory of defects in solids. In the current seismological literature the Earth is represented in general as a fully anisotropic elastic medium, when there is no experimental indication of such a situation in the seismic phenomena (crystals of condensed matter are anisotropic!). In these books the "representation of the seismic sources", the "point dislocation source" and the "double-couple representation" are very imperfect approximations to the tensorial force, most likely. These books deal with "vertically heterogeneous media" and "media with depth-dependent properties", where the path of the seismic disturbance to the Earth's surface is short enough to not warrant such a general frame. Plane waves, their reflection and refraction are usual items in such books, though the seismic evidence is far from such a situation. It seems that the expert seismologists do what they know better without caring too much of the empirical seismic recordings. Vibrations of the Earth as a whole in an earthquake aftermath is a "must" in such books, as feeble as such things are. On the other hand, not much of the real, measurable seismic things in these books.

4. A seismogram has a well-defined, typical structure. It consists of two, relatively small, shocks, separated in time, followed by one main shock, with a rather sharp wavefront and a long tail; sometimes, two main shocks can be seen in a seismogram. This structure has been recognized long ago, especiallly by Love,[22, 23] but it seems that it has not yet find its way into the modern books of Seismology. It may appear as a reasonable task for a theoretical Seismology to account for this seismogram structure. To this end, it is natural to resort to the equation of the elastic waves (Navier-Cauchy equation[24, 25]) with a tensorial force. It is again strange that such a solution is absent in modern Seismology, although Stokes indicated it (for a vectorial force) since a very long time (1849!).[26] The solution was given only recently.[7] Some attempts to derive solution by using the double-couple representation are affected by very cumbersome particular manipulations, that it is impossible to make any sense from them.[8]-[10]

The explanation for the lack of solution to the afore mentioned problem may reside in a double circumstance. First, the Earth has a surface, and any practitioner of Equations of Mathematical Physics is readily inclined to think that the equation of the elastic waves must be solved, for a determinate solution, with boundary conditions on the surface. Usually, in such cases, the spherical Earth is locally approximated by an elastic half-space bounded by a plane surface and the equation of the elastic waves is considered for a homogeneous isotropic elastic half-space. In order to make use of the boundary conditions extended waves should be considered, such that a Fourier spatial expansion along the plane surface is appropriate. A particular solution of the equation may thus be obtained. A solution of the homogeneous equation should be added in order to get the full solution. Here, the second complication occurs, related to the surface Rayleigh waves. The Rayleigh waves represent a stumbling block in solving the elastic waves equation in a half-space. The Rayleigh waves are one kind of solutions of the homogeneous elastic waves equation in a half-space with a free plane surface;[27] these surface waves are damped in the depth of the half-space; they represent a set of normal modes (eigenmodes), and, as such, their amplitudes are (partially) undetermined (they belong to the same class of solutions known as Love waves[22] (for a slab) and Stoneley waves[28] (for an interface)). Added to the particular solution and imposing boundary conditions (usually free surface) these amplitudes are determined, and the full solution is thereby obtained. This strategy was championed by Oldham[29] and initiated technically by Lamb;[30, 31] it is often known as Lamb's problem. In modern times it becomes quite famous and regularly invoked as the Cagniard-de Hoop method (usually carried out with a Laplace transform).[32]-[34]

Unfortunately, the initially ill-posed problem snarls; when one attempts to recompose the Fourier (and Laplace) transforms one encounters impossible difficulties, which throw any desperate seismologist into the most dreadful, frightful, "black" approximations, which maim any reasonable solution. A lot of approximate waves are claimed to occur (some legitimate in their own context, but without any bearing here) like head (or lateral) waves, cylindrical, conical, leaking, inhomogeneous, damped, etc, waves.

5. The solution to the Lamb's problem is simple. Not any boundary requires boundary conditions. In order the boundary be active and boundary conditions appropriate the waves should act permanently on the boundary; they should be in a stationary regime, where vibrations set in. Far from this, the seismic disturbance on the Earth's surface comes and goes, propagates; the region affected by the seismic disturbance on the Earth's surface has, strictly speaking, zero thickness. This is a transient regime of propagating disturbances, characteristic of forces which are localized and last a short time. Therefore, the equations of the elastic waves in the half-space should be solved for propagating boundary conditions, *i.e.* as for an infinite space. For elementary tensorial seismic forces the solutions are spherical shells, propagating with the elastic waves velocities c_l ("longitudinal" waves, $\simeq 7km/s$) and c_t ("transverse" waves,$\simeq 3km/s$); in general, the seismic spherical shell waves are anisotropic, with mixed polarizations. They are the P- and S-waves in the seismogram; we call them primary waves.[7] These waves are responsible for the "preliminary feeble tremor" produced by any earthquake. A technical point appears in deriving these waves, related to unphysical (quasi-static) terms in elastic potentials; a calibration (regularization) procedure is devised in order to remove such terms.

6. The primary waves are shells with thickness of the order l (the dimension of the focal region). They intersect the Earth's surface along circles; it is easy to see from geometrical considerations that the intersecting seismic spot on the Earth's surface is larger than l (it depends on position) and propagates on the surface with a velocity higher than the velocity of the elastic waves. Part of the energy of the primary waves intersecting the surface is lost, corresponding to the missing spherical sector; where is gone this energy? Of course, it excites the surface, on the seismic spot, giving rise to additional wave sources, according to Huygens principle. These secondary sources generate secondary waves, which propagate in the whole half-space, surface included. Since they arise from propagating sources, the secondary waves may be called seismic radiation.

In a simplified model of propagating secondary sources the seismic radiation on the Earth's surface has a sharp wavefront, followed by a long tail; the wavefront propagates with the wave velocity, which is smaller than the velocity of the primary waves on the surface. This is the main shock recorded in seismograms (actually two main shocks, corresponding to the two primary waves); the seismic long tail enjoyed interesting discussions in the seismological literature.[35] The time lag of the main shocks in comparison with the primary waves, as well as the two primary waves, may be entangled for short distances on the Earth, as a consequence of the two different wave velocities $c_{l,t}$. The secondary waves and the seismic main shock were derived only recently in Ref. [7].

It is worth noting the effect of a discontinuity surface parallel with the Earth's surface. The primary waves ariving on the discontinuity surface generate sources of secondary waves, which propagate in the whole half-space. If the source of the primary waves is placed below the discontinuity surface, the primary waves arrived at the discontinuity surface generate secondary wave sources on this surface, which give rise to secondary waves which arrive at the free surface; they have not anymore a sharp wavefront and are reduced appreciably in magnitude. A discontinuity surface interposed between the focal region and the Earth's surface acts like a shielding for the main shock effect of the seismic waves on the Earth's surface.

7. Related to the seismic effects is the problem of static deformations of an elastic (homogeneous, isotropic) half-space with a tensorial force. For vectorial forces, applied either on the surface of the half-space or at an inner point, such problems are solved since long, being of great importance in Geotechnics; they are known as Mindlin, Boussinesq, Melan, Flamant, or Cerruti problems.[36] For a tensorial force, as that generated in a seismic source, it was solved recently.[7] A general method of solution was formulated in Ref. [7], which makes use of the extension of the equation of elastic equilibrium to including the surface by means of generalized functions (distributions), in-plane Fourier transforms (along directions parallel with the plane surface of the half-space) and Sommerfeld integrals. The solution for the tensorial force is particularly relevant for monitoring the deformation of the Earth's crust in seismogen zones, in order to get information about the seismic activity of the focal region.

8. The problem of vibrations of the spherical Earth was solved, basically, in a classical paper by Lamb.[37] Various amendmends have been done along the time (rotation, centrifugal force, inhomogeneities, external (gravitational) field, etc). The solution was achieved by means of spheroidal and toroidal functions, introduced by Lamb, which are eigenfunctions of the angular part of the equation of the elastic motion (elastic waves equation). In an earthquake aftermath the Earth vibrates, sometime for as long a time as an hour. Such slow vibrations are recorded by seismographs. What are the eigenvibrations (modes) of an elastic half-space? or an elastic slab? or an elastic cylinder? What if the seismic vibrations thermalize? Could we speak of an earthquake temperature? These rather academic problems were answered in Ref. [7]. In particular, lateral waves which move parallel with the plane surface of the half-space are another set of eigenmodes beside the damped Rayleigh surface waves.

9. There exist various other seismological problems, like what are the seismic waves produced by a meteorite which falls on the Earth's surface? What are the seismic waves produced by an explosion? What is the seismic radiation produced by a seismic source in motion? How useful is the ray approximation and how relevant are the travel times? What about boundary forces? Scattering off a small inhomogeneity? What about the waves in fluids? What is the connection between the Navier-Cauchy equation of solids and the Navier-Stokes equation of fluids? What about sound, turbulence, viscosity, anelastic matter? Explosions in seas and oceans? The effect of the surface forces on fluid waves, the effect of the seismic waves in fluids (tsunami?)? Such questions are also answered in Ref. [7].

10. One problem of immediate importance in Seismology is the protection of buildings against the damaging effects of the earthquakes (structural, earthquake engineering). The damaging effect appears at resonance, when one or more seismic frequencies equal one or more eigenfrequencies of the buildings. The seismic periods are mainly in the range $0.1 - 10s$, and, usually, the lower part of this range is shared by buidings's eigenperiods. The usual designs attempt to modify the bulidings, *e.g.* by constructing a foundation, such that the buildings's eigenfrequencies be removed from the seismic frequencies. It is very convenient to model a building as a vibrating (oscillating) bar;[38] the structure building-foundation can be modelled as two coupled harmonic oscillators. The response to a seismic excitation can be computed for such a structure. It is important to consider realistic seismic shocks, as those decaying in time and containing resonance frequencies (*e.g.*, main shocks). It was shown recently[7] that, at resonance, amplification factors occur for the oscillations of the building, which may attain large values; basically, they are given by the height of the building (elastic bar), divided by the wave velocity in the building and the attenuation factor of the seismic shock. If the seismic shock lasts for a long time, the amplification factor may be very large. In a composite structure like building-foundation with very different elastic properties the oscillator coupling lowers the low frequency (usually the building's frequency) and raises the higer frequency (foundation's frequency). As long as the resonance occurs, the amplification factors are present. Non-linearities, damping, or other structural ingredients do not appear to improve upon the situation. The results are closely related to the site amplification factors, observed experimentally in seismic effects;[39]-[41] very likely, they appear for local, buried inhomogeneities; they have a spectral character. The amplification factors are due to the excitation of the normal modes at resonance.

Acknowledgments. The author is indebted to the members of the Laboratory of Theoretical Physics at Magurele-Bucharest for many fruitful discussions. This work has been supported by the Scientific Research Agency of the Romanian Governments through Grants 04-ELI / 2016 (Program 5/5.1/ELI-RO), PN 16 42 01 01 / 2016 and PN (ELI) 16 42 01 05 / 2016.

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