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**Damping (reaction) force for a charge in high-intensity radiation field**

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**Abstract**

The equations of motion of an electric charge in a plane wave of radiation field is solved. It is shown that in high radiation field the charge becomes ultrarelativistic and the Landau-Lifshitz reaction (damping) force is very small and has no measurable effects.

Let us consider, with usual notation, the motion of a charge $q$ with mass $m$ in an electromagnetic field generated by the vector potential $A_z = A_0 \cos\frac{\omega}{c}(ct - x) = A$; the equation of motions are

$$mc\frac{du^i}{ds} = \frac{q}{c}F^{ik}u_k \quad , \tag{1}$$

where $u^i$ is the four-velocity. The field intensity ($\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$, $\mathbf{H} = curl\mathbf{A}$) is given by

$$F^{ik} = \begin{pmatrix} 0 & 0 & 0 & -E \\ 0 & 0 & 0 & -E \\ 0 & 0 & 0 & 0 \\ E & E & 0 & 0 \end{pmatrix} \ , \ F_{ik} = \begin{pmatrix} 0 & 0 & 0 & E \\ 0 & 0 & 0 & -E \\ 0 & 0 & 0 & 0 \\ -E & E & 0 & 0 \end{pmatrix} \ , \tag{2}$$

where $E = E_z = \frac{\omega}{c}A_0 \sin\frac{\omega}{c}(ct - x)$ (and $H_y = -E$). The world-line element is usually written as $ds = cdt/\gamma$, $\gamma = 1/\sqrt{1 - v^2/c^2}$; however, this is not convenient for the integration of the equations of motion, since the field depends on $s = ct - x$. Consequently, the equations of motion are re-written as

$$\begin{gathered} mc\tfrac{du^0}{ds} = \tfrac{qE}{c}u^3 \ , \ mc\tfrac{du^1}{ds} = \tfrac{qE}{c}u^3 \ , \\ mc\tfrac{du^3}{ds} = \tfrac{qE}{c}(u^0 - u^1) \end{gathered} \tag{3}$$

($u^2 = 0$); we get immediately $u^0 - u^1 = const = 1$, for a charge initially at rest in the origin (we recall $u^0 = \gamma$, $\mathbf{u} = \gamma\mathbf{v}/c$); it follows

$$\begin{gathered} u^1 = \tfrac{q^2A^2}{2m^2c^4} \ , \ u^0 = \gamma = 1 + \tfrac{q^2A^2}{2m^2c^4} \ , \\ u^3 = -\tfrac{q}{mc^2}A \end{gathered} \tag{4}$$

(we can check $u_iu^i = 1$). The velocities are given by

$$v_x = c\frac{q^2A^2/2m^2c^4}{1 + q^2A^2/2m^2c^4} \ , \ v_z = -c\frac{qA/mc^2}{1 + q^2A^2/2m^2c^4} \tag{5}$$

and the energy is

$$\mathcal{E} = mc^2\gamma = mc^2 + \frac{q^2A^2}{2mc^2} \ . \tag{6}$$

We can see that for high fields the charge reaches the ultrarelativistic limit along the direction of propagation of the radiation field ($v_x \to c$, $v_z \to 0$). From $u^i = dx^i/ds$ we get the coordinates

$$\begin{aligned} x &= \tfrac{q^2A_0^2/4m^2c^3}{1+q^2A_0^2/4m^2c^4}\left[t + \tfrac{1}{2\omega}\sin 2\tfrac{\omega}{c}(ct-x)\right] \ , \\ z &= -\tfrac{qA_0}{mc\omega}\sin\tfrac{\omega}{c}(ct-x) \ . \end{aligned} \tag{7}$$

The reaction force[1] is given by

$$g^i = \frac{2q^3}{3mc^3}\frac{\partial F^{ik}}{\partial x^l}u_k u^l - \frac{2q^4}{3m^2c^5}\left[F^{il}F_{kl}u^k - (F_{kl}u^l)(F^{km}u_m)u^i\right] \ ;$$

it must be much smaller than $q^2/c\gamma a^2$, where $a = q^2/mc^2$ is the classical electromagnetic radius of the charge; this condition, which indicates the limits of applicability of the electromagnetism, does not imply always that the damping (reaction) force is smaller than the electromagnetic (Lorentz) force; in the limit $v \to c$ the damping may be higher than the driven force.

Making use of the field given by equations (2) we get the reaction force

$$\begin{aligned} g^0 &= \tfrac{2q^3}{3mc^3}(u^0-u^1)\left\{\tfrac{\omega^2}{c^2}Au^3 + \tfrac{qE^2}{mc^2}\left[1-(u^0-u^1)u^0\right]\right\} \ , \\ g^1 &= \tfrac{2q^3}{3mc^3}(u^0-u^1)\left\{\tfrac{\omega^2}{c^2}Au^3 + \tfrac{qE^2}{mc^2}\left[1-(u^0-u^1)u^1\right]\right\} \ , \\ g^3 &= \tfrac{2q^3}{3mc^3}(u^0-u^1)^2\left(\tfrac{\omega^2}{c^2}A - \tfrac{qE^2}{mc^2}u^3\right) \end{aligned}$$

(we can check $u_i g^i = 0$). Making use of the four-velocity given in equation (4) we get

$$\begin{aligned} g^0 &= -\tfrac{2a^2\omega^2A^2}{3c^3}\left(1 + a\tfrac{E^2}{2m\omega^2}\right) \ , \\ g^1 &= -\tfrac{2a^2\omega^2}{3c^3}\left[A^2 - \tfrac{c^2E^2}{\omega^2}\left(1 - \tfrac{q^2A^2}{2m^2c^4}\right)\right] \ , \\ g^3 &= \tfrac{2aqA\omega^2}{3c^3}\left(1 - a\tfrac{E^2}{m\omega^2}\right) \ , \end{aligned}$$

or, in the limit of high-intensity fields,

$$g^0 = g^1 \simeq -\frac{a^3A^2E^2}{3mc^3} \ , \ g^3 \simeq -\frac{2a^2qAE^2}{3mc^3} \ .$$

We consider an electron in a high-intensity field $E = 10^{10} statvolt/cm$, corresponding to a laser radiation with intensity $I = 10^{22} w/cm^2$, focalized in a pulse of dimension $d = 10\mu m$. The vector potential is $A_0 = cE_0/\omega = 10^{-5}E_0 = 10^5 statvolt$ for the optical frequency $\omega \simeq 3\times10^{15}s^{-1}$ ($\lambda = 0.5\mu m$); the corresponding energy for an electron is $qA_0 = 4.8\times10^{-5}erg \simeq 30MeV$. This energy is much higher than the rest energy of the electron $mc^2 = 0.5MeV$; the electron is accelerated along the direction of propagation of the radiation up to energies of the order $1GeV$. Under these conditions the damping force ($g^0 = g^1 \simeq 10^{-12}g/s$, $g^3 \simeq 10^{-13}g/s$) is much smaller than the Lorentz force ($qEu^3/c \simeq 10^{-8}g/s$, $qE/c \simeq 10^{-10}g/s$; $\gamma \simeq 10^4$) and the validity condition $g^i \ll q^2/c\gamma a^2$ is satisfied ($q^2/c\gamma a^2 \simeq 10^{-8}g/s$).

We may conclude that in high-intesnity radiation field the motion of the charge becomes ultrarelativistic and the damping force has no measurable effect on the motion.

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# References

[1] L. Landau and E. Lifshitz, *Course of Theoretical Physics*, vol. 2, *The Classical Theory of Fields*, Elsevier (2004).