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Some comments on the bound-state disintegration by electromagnetic radiation (electromagnetic effect on the alpha decay)

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Abstract

Charge emission from bound states under the action of an electromagnetic radiation is considered, with application to the ionization of heavy atoms, molecules, atomic clusters, or proton emission from heavy atomic nuclei and nuclear alpha-particle decay, or ion fragmentation. It is pointed out that in the usual experiments of this kind, conducted in laser beams, the electromagnetic radiation is suddenly applied, with respect to the relevant times of the disintegration processes. The results obtained previously in this case are briefly summarized. As a point of technical interest, the case of an adiabatically-introduced electromagnetic interaction is discussed, with particular application to proton emission and nuclear alpha-particle decay. It is shown that the adiabatic application of the electromagnetic radiation, which is valid for weak radiation fields, brings second-order corrections in the electric field to the disintegration rate corresponding to the absence of the radiation, with a slight anisotropy; these corrections give a small enhancement of the disintegration rate.

In the context of an active topical research in laser-related physics,[1]-[5] the problem of charge emission from bound-states under the action of the electromagnetic radiation is receiving a gradually increasing interest. The investigations focus especially on the effect the optical-laser radiation may have on the alpha-particle decay of the atomic nuclei,[6]-[8] or nuclear proton emission;[9, 10] but the area may be extended to atomic ionization or molecular or atomic clusters fragmentation.[11]-[14] The aim of the present paper is to offer some comments related to the theoretical approaches to such phenomena.

We adopt a simple model of many-particle bound states, consisting of single-particle states generated by a mean field. The typical example is the Thomas-Fermi model for heavy atoms. In this model the kinetic energy of a particle with mass m may be represented as $E_{kin} = \hbar^2 k^2/2m$, where $\hbar$ is Planck's constant and the wavevector is given by $k \simeq n/a$, n being a large integer and a being the dimension of the bound state; the maximum value of n is of the order $n \simeq Z^{1/3}$, where Z ($\gg 1$) is the atomic number of a heavy atom. The model may be applied to electrons in heavy atoms, molecules and atomic clusters, where Z is the number of electrons, as well as to nucleons in atomic nuclei, where the mass m is the nucleon mass and the number of nucleons is Z for protons and $A - Z$ for neutrons, A being the mass number of the nucleus. For ions in heavy molecules or atomic clusters we use a (quasi-) classical dynamics. The binding energy of a particle may be represented as $E_b = E_{kin} - U$, where $-U$ is the potential well of the mean field. For small binding energies we may approximate $E_{kin} \simeq U$ for electrons and nucleons and get an oscillation time of the order $t_0 \simeq \hbar/U$. This time is very small in comparison with any other relevant times. The relation $E_{kin} \simeq U$, where E_{kin} is the maximum kinetic energy may serve to estimate the potential well.

The ionization (fragmentation) experiments proceed usually by placing a collection of atoms, molecules, nuclei in the focal region of a laser beam, and focus radiation pulses upon that collection of particles. We consider an optical-laser radiation with a typical frequency $\omega \simeq 10^{15}s^{-1}$, corresponding to a period $T \simeq 10^{-15}s$ and a wavelength $\lambda \simeq 0.8\mu m$. We assume that the motion of the charges under the action of the electromagnetic radiation remains non-relativistic, *i.e.* $qA/mc^2 \ll 1$, where q is the particle charge and A is the amplitude of the vector potential. For electrons ($q = -4.8 \times 10^{-10} statcoulomb$, $m = 10^{-27}g$, $c = 3 \times 10^{10}cm/s$) we get a maximum electric field $E = 10^8 statavolt/cm$. For protons in atomic nuclei ($m = 2 \times 10^{-24}g$) the maximum electric field is $E = 10^{11} statvolt/cm$. It follows that the maximum intensity of the laser beam in the focal region is of the order $I = cE^2/8\pi = 10^{18}w/cm^2$ for electrons and $I = 10^{24}w/cm^2$ for nucleons. Typically, the duration of the laser pulse is of the order tens of radiation period, such that we may consider the action of the electromagnetic radiation much longer than the period of the radiation. The repetition rate of the laser pulses is usually much longer than the pulse duration. For simplification we consider linearly-polarized radiation plane waves. The laser-beam shape or multi-mode operation have little relevance upon the considerations made here.

Under these conditions the time of setting up the radiation field upon the atom, molecule, nucleus is of the order a/c; for atoms (or molecules) this time is $\Delta t \simeq 10^{-19}s$ ($a = a_H = 0.5$Å, a_H being the Bohr radius), for atomic nuclei it is $\Delta t \simeq 10^{-24}s$ ($a \simeq 10^{-13}cm$); it means that we may consider that the radiation is applied suddenly; it produced an energy uncertainty of the order $\Delta E \simeq \hbar/\Delta t \simeq 10keV$ for electrons and $\Delta E \simeq 10MeV$; such energies are much larger than the width of the one-particle energy levels, such that we cannot make use of the stationary states.

Charge emission from bound states under the action of a suddenly-applied electromagnetic radiation has been analyzed in Ref. [15]. It was shown in Ref. [15] that a succesion of Goeppert-Mayer and Kramers-Henneberger canonical transformations reduce practically to zero the bound-state mean-field potential for high electric fields (in the non-relativistic approximation the magnetic field may be left aside) and set the bound charges free in a very short time (at most of the order of the radiation period). The ionization rate is given by the reciprocal of this time. For weaker fields the charges oscillate and emit higher-order harmonics of electromagnetic radiation. The threshold field which separates the two regimes is of the order $E \simeq 10^4 statvolt/cm$ ($I = 10^{11}w/cm^2$) for electrons and $E \simeq 10^2 statvolt/cm$ ($I = 10^7 w/cm^2$) for protons. Similar considerations can be applied to the quasi-classical dynamics of ion fragmentation in large molecules or atomic clusters; the electric field in this case must lie in the range $10^7 statvolt/cm < E/A^2 < 10^{10} statavolt/cm$ ($10^{17}w/cm^2 < I/A^2 < 10^{23}w/cm^2$), where A is the mass number of the ion; these inequalities ensure fragmentation and a non-relativistic dynamics. This approach has also been applied to static electric fields in Ref. [15].

Althoug unrealistic, the adiabatic application of the electromagnetic interaction is still favoured in treating charge disintegration of bound states; noteworthy, this procedure is limited to weak electromagnetic fields. Originally, the atom ionization has been treated by using adiabatic hypothesis, either by time-dependent perturbation theory, or by imaginary-time tunneling, or other equivalent approaches.[16]-[24] Classical tunneling through the potential barrier has been applied in classical works for static fields and hydrogen-like atoms (in parabollic coordinates).[25]-[27] In all these cases it is assumed that the stationary energy levels persist in providing a basis for analyzing the quantum-mechanical dynamics of the charges subject to the electromagnetic interaction. Typically, small disintegration rates of the form $e^{-const/E}$ are obtained, for small values of the amplitude electric field E. The same approaches can be applied to ionization of molecules or atomic clusters. For proton emission, alpha-particle decay or ion fragmentation the tunneling through the Coulomb potential barrier must be included. As a technical point we analyze below the charge emission from a bound state, affected by the presence of an adiabatically-introduced electromagnetic radiation, in the presence of a Coulomb barrier; the problem may exhibit relevance for studies of proton emission or alpha-particle decay.

The spontaneous proton emission and alpha-particle decay proceed by tunneling through the Coulomb potential barrier, as a result of many "attempts" the charge makes to penetrate this barrier. The (high) frequency of this process is of the order $1/t_a$, where t_a corresponds, approximately, to the energy level spacing; in atomic nuclei this spacing, for the relevant eneryy levels, is of the order $200keV$, which gives $t_a \simeq 3\times10^{-21}s$.[28] The charge emission proceeds by a spontaneous rise above the zero level followed by the tunneling through the Coulomb potential barrier (we leave aside the so-called tunneling through the internal potential barrier); the order of magnitude of the energy of the charge is a few MeV, which ensures a quasi-classical tunneling. The broadening of the charge energy levels introduces an energy uncertainty which may take the charge energy above the level of zero energy, especially for charges lying on high-energy levels (we leave aside the so-called pre-formation factor of the alpha particle). In this situation the charge tunnels through the potential barrier, as a free particle. The same process for ion fragmentation is practicallly negligible (this would give an ion auto-fragmentation with an extremely low probability, due to very low rate of penetration "attempts"). The contribution of the electromagnetic radiation to setting the charge free (and ready for tunnneling) is much smaller than the spontaneous contribution, such that we may neglect it. Similarly, we consider a sufficiently weak electromagnetic radiation, such that we may neglect its effects upon the mean-field potential. We limit ourselves to the effect of the electromagnetic interaction on the tunneling rate.

Let us consider a charge q with mass m in the potential barrier $V(\mathbf{r})$ in the presence of an electromagnetic radiation with the vector potential $\mathbf{A}=\mathbf{A}_0\cos(\omega t-\mathbf{kr})$, where A_0 is the amplitude of the vector potential, ω is the radiation frequency and $\mathbf{k}$ is the radiation wavevector ($\omega = ck$, c being the speed of light in vacuum); we limit ourselves to linear polarization, but the calculations can be extended easily to general polarization; the electromagnetic field is transverse, *i.e.* $\mathbf{kA}=0$. Since the phase velocity of the non-relativistic charge is much smaller than the speed of light c in vacuum, we may neglect the spatial phase $\mathbf{kr}$ in comparison with the temporal phase ωt; consequently, the vector potential may be approximated by $\mathbf{A}\simeq\mathbf{A}_0\cos\omega t$. This approximation amounts to neglecting the effects of the magnetic field. It is assumed that this potential is introduced adiabatically. The charge is immersed in the radiation field, such that its hamiltonian is the standard non-relativistic hamiltonian

$$H=\frac{1}{2m}\left(\mathbf{p}-\frac{q}{c}\mathbf{A}\right)^2+V(\mathbf{r}) \quad , \tag{1}$$

where the momentum $\mathbf{p}$ includes the electromagnetic momentum $q\mathbf{A}/c$ beside the mechanical momentum $m\mathbf{v}$, where $\mathbf{v}$ is the velocity of the particle. We consider the Schrodinger equation

$$i\hbar\frac{\partial\psi}{\partial t}=H\psi \ . \tag{2}$$

In equation (2) we perform the well-known Kramers-Henneberger transform[29]-[32] (with a vanishing electromagnetic interaction for $t\rightarrow-\infty$)

$$\begin{gathered}\psi=e^{iS}\varphi \ , \\ S=\tfrac{q}{\hbar mc\omega}\mathbf{A}_0\mathbf{p}\sin\omega t-\tfrac{iqA_0^2}{8\hbar mc^2\omega}(2\omega t+\sin2\omega t) \ ;\end{gathered} \tag{3}$$

the Schrodinger equation becomes

$$i\hbar\tfrac{\partial\varphi}{\partial t}=\tfrac{1}{2m}p^2\varphi+\widetilde{V}(\mathbf{r})\varphi \ ,$$
$$\widetilde{V}(\mathbf{r})=e^{-iS}V(\mathbf{r})e^{iS}=V(\mathbf{r}-q\mathbf{A}_0\sin\omega t/mc\omega) \ ; \tag{4}$$

it is convenient to introduce the electric field $\mathbf{E}=\mathbf{E}_0\sin\omega t$, $\mathbf{E}_0=\omega\mathbf{A}_0/c$; we get

$$S=\frac{q}{\hbar m\omega^2}\mathbf{E}_0\mathbf{p}\sin\omega t-\frac{iqA_0^2}{8\hbar mc^2\omega}(2\omega t+\sin 2\omega t) \tag{5}$$

and

$$\widetilde{V}(\mathbf{r})=V(\mathbf{r}-q\mathbf{E}/m\omega^2) \ . \tag{6}$$

We can see that for high-intensity fields the potential (including the mean-field potential) is rapidly vanishing along the field direction; consequently, the charge is set free in a short time.[15] Now we assume that the field intensity is weak (in accordance with its adiabatic application); specifically we assume $qE_0/m\omega^2\ll a$, where a is the dimension of the region the charge moves in (the atomic nucleus); for protons this inequality means $E_0\ll 10^2 statvolt/cm$ $(I\ll 10^7 w/cm^2)$. Under these conditions the charge oscillates, radiates higher-order harmonics and tunnels through the potential barrier given by equation (6); the "attempt" frequency to penetrate the barrier and the energy uncertainty which rises the energy level above the zero level are practically not affected by the field.

We assume a Coulumb potential barrier $V(r)\simeq Zq^2/r$; in the absence of the field the tunneling proceeds from $r_1=a$ to $r_2=Zq^2/\mathcal{E}_r$, where $\mathcal{E}_r$ is the radial energy of the charge; it is convenient to introduce the parameter $\xi=qE_0/m\omega^2 a\ll 1$, which includes the effect of the field. In the presence of the field these limits become

$$\widetilde{r}_1=\left|\mathbf{a}-q\mathbf{E}/m\omega^2\right| \tag{7}$$

and $\widetilde{r}_2=r_2$, where $\mathbf{a}=a\mathbf{r}/r$. We expand $\widetilde{r}_1$ in powers of ξ and get

$$\widetilde{r}_1=a\left(1-\xi\sin\omega t\cdot\cos\theta+\frac{1}{2}\xi^2\sin^2\omega t\cdot\sin^2\theta\right)+... \ , \tag{8}$$

where θ is the angle the radius vector $\mathbf{r}$ makes with the electric field $\mathbf{E}_0$.

To continue we use a simplified nuclear model. The free charge attempting to penetrate the potential barrier has momentum $\mathbf{p}_n$ and kinetic energy $\mathcal{E}_n=p_n^2/2m$, where n is a generic notation for its state; we may leave aside the orbital motion and denote by $\mathbf{p}_{rn}$ the radial momentum and by $\mathcal{E}_{rn}$ the radial energy. Let $\mathbf{p}_r$ and $\mathcal{E}_r=p_r^2/2m$ be the highest radial momentum and, respectively, the highest radial energy; they correspond to the total momentum $\mathbf{p}$ and, respectiveley, total energy $\mathcal{E}=p^2/2m$ (a degeneration may exist, which can be included). This charge may tunnel through the potential barrier $V(r)$ from $\widetilde{r}_1$ to $\widetilde{r}_2$. The relevant factors in the wavefunction ψ given by equation (3) are

$$e^{\frac{iqE(t)}{\hbar m\omega^2}\cos\theta\cdot(p_2-p_1)+\frac{i}{\hbar}\int_{\widetilde{r}_1}^{\widetilde{r}_2}dr\cdot p_r(r)} \ , \tag{9}$$

where $p_r(r)=\sqrt{2m\left[\mathcal{E}-V(r)\right]}$, $p_{1,2}=p_r(\widetilde{r}_{1,2})=\sqrt{2m\left[\mathcal{E}-V(\widetilde{r}_{1,2})\right]}$; it is easy to see that $p_2=0$. It follows that the tunneling probability (transmission coefficient) is given by $w=e^{-\gamma}$, where

$$\gamma=-A\xi\sin\omega t\cdot\cos\theta+B \ ,$$
$$A=\tfrac{2a|p_1|}{\hbar} \ , \ \xi=\tfrac{qE_0}{m\omega^2 a} \ , \ B=\tfrac{2}{\hbar}\int_{\widetilde{r}_1}^{\widetilde{r}_2}dr\,|p_r(r)| \tag{10}$$

and $|p_1| = \sqrt{2m\left[V(\widetilde{r}_1) - \mathcal{E}\right]}$ and $|p_r(r)| = \sqrt{2m\left[V(r) - \mathcal{E}\right]}$; the condition $V(\widetilde{r}_1) > \mathcal{E}$ ensures the existence of the bound state. We expand the coefficient A in powers of ξ and take the average with respect to time; we get

$$\gamma = -\frac{Zq^2}{2\hbar}\sqrt{\frac{2m}{Zq^2/a - \mathcal{E}}}\xi^2\cos^2\theta + B... \ ; \tag{11}$$

the same procedure applied to the coefficient B leads to

$$B = \gamma_0 - \tfrac{a\xi^2}{2\hbar}\sqrt{2m(Zq^2/a - \mathcal{E})} + \tfrac{a\xi^2}{2\hbar}\sqrt{\tfrac{2m}{Zq^2/a-\mathcal{E}}}(3Zq^2/2m - \mathcal{E})\cos^2\theta \ \ , \tag{12}$$

where γ_0 corresponds to the absence of the radiation, and finally to

$$\gamma = \gamma_0 - \frac{a\xi^2}{2\hbar}\sqrt{2m(Zq^2/a - \mathcal{E})}\left[1 - \frac{Zq^2/2a - \mathcal{E}}{Zq^2/a - \mathcal{E}}\cos^2\theta\right] \ . \tag{13}$$

We can see that the effect of the radiation is to increase the rate of charge emission by a factor proportional to the square of the electric field and to introduce a slight anisotropy.

We can define a total disintegration probability

$$w_{tot} = \left\{1 + \frac{a\xi^2}{2\hbar}\sqrt{2m(Zq^2/a - \mathcal{E})}\left[1 - \frac{Zq^2/2a - \mathcal{E}}{3(Zq^2/a - \mathcal{E})}\right]\right\} w^0_{tot} \tag{14}$$

by integrating over angle θ, where $w^0_{tot} = e^{-\gamma_0}$. The disintegration rate per unit time is $(1/\tau)w_{tot} = w_{tot}$, where τ is related to the time t_a estimated above and the time introduced by the energy uncertainty.

The exponent γ_0 corresponding to the absence of the radiation is

$$\gamma_0 = \frac{Zq^2}{\hbar}\sqrt{2m/\mathcal{E}}\left(\arccos\sqrt{\mathcal{E}a/Zq^2} - \sqrt{\mathcal{E}a/Zq^2}\sqrt{1 - \mathcal{E}a/Zq^2}\right) \ ; \tag{15}$$

since $Zq^2/a \gg \mathcal{E}$ (for protons $q^2/a = 2.5MeV$) , we may use the approximate formulae

$$\gamma_0 \simeq \frac{\pi Zq^2}{2\hbar}\sqrt{2m/\mathcal{E}} \tag{16}$$

and

$$w_{tot} = \left(1 + \frac{5a\xi^2}{12\hbar}\sqrt{2mZq^2/a}\right) w^0_{tot} \ . \tag{17}$$

As it is well know the interplay between the very large values of τ and the very small values of $e^{-\gamma_0}$, makes the disintegration rate be very sensitive to the energy values, and to vary over a wide range.

After the emission of a charge, the mean-field potential suffers a reconfiguration (rearrangement) process and the potential $V(\mathbf{r})$ is modified; this is the well-known process of "core shake-up" (or "core excitation"); a new bound state is formed and a new transformation process may begin for the modified potential $V(\mathbf{r})$.

The tunneling probability w given above is a transmission coefficient (we can check that $w < 1$); with probability $1-w$ the charge electron is reflected from the potential barrier; in these conditions the bound state is "shaken-up" and the charge resumes its process of multi-photon absorption and emission of high-order harmonics, untill it may be rescattered back to the core; these are the well-known recollision processes.

In conclusion, we may say that in usual experiments of charge emission from bound states under the action of the electromagnetic radiation, the electromagnetic interaction is suddenly applied, especially for strong fields. This process has been analyzed in Ref. [15]. The adiabatic introduction of the electromagnetic interaction has, in this respect, only a technical interest. It was widely investigated for atom ionization, and the approach can be extended to molecular or atomic cluster ionization. It allows the use of the well-known technique of penetration through a potential barrier (mainly coulombian) for proton emission from atomic nuclei, nuclear alpha-particle decay and ion fragmentation of molecules or atomic clusters; in the later case the disintegration rate is extremely low, as a result of the low kinetic energy of the ions. We have analyzed above the disintegration rate for the charge emission from atomic nuclei in the case of the adiabatic introduction of (weak) electromagnetic interaction, with application to proton emission and nuclear alpha-particle decay. Under these circumstances, it has been shown in this paper that the tunneling rate (through Coulomb potential) is slightly enhanced by the presence of the radiation by corrections whose leading contributions are of second-order in the electric field, with a slight anisotropy.

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