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Electric pulse on a metallic wire

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Abstract

An idealized model of propagation of electric pulses along metallic wires is presented. Within this model, a short electric pulse produced at a point on a long metallic wire may propagate along the wire with the speed of light in vacuum over long distances, practically dispersionless and without energy loss. The pulse excites in wire a tail of plasmon-polaritons, which propagate along the wire with the speed of light in vacuum c, plasma frequency ω_p (wavelength $\lambda = c/\omega_p$) and wavefronts at $\mid x \mid= ct$, where x the coordinate along the wire and t denotes the time. This is a non-thermal (adiabatic) process, with a very low rate of energy dissipation and, practically, radiationless. The reason for such a lossless propagation resides in the high values of the plasma frequency and the restoring force of the charges, which lead to a charge displacement much smaller than the mean freepath.

Key words: electric pulse, metallic wires; plasmon-polaritons; lossless propagation

It is well known that electromagnetic waves may be guided along metallic wires and focused in metallic structures with a restricted geometry, beyond the diffraction limit.[1]-[12] These phenomena have been associated with Sommerfeld surface waves[1] and surface plasmon-polaritons.[5]-[8] Recently, there is an increasing interest in propagating electric pulses along metallic wires, especially in the terahertz frequency range.[13]-[21] Their propagation is almost dispersionless and their velocity is close to the speed of light in vacuum.

We present in this paper an idealized model of plasmon-polaritons excited in a long metallic wire by electric pulses, which may propagate over a long distance, practically without energy loss. The effects of an electric pulse propagating in a metallic wire are different from those associated with the propagation of a surface wave. The propagation of the pulse and its associated effects are not restricted to the surface; they occur inside the wire.

We consider a metallic wire, with a shape close to a straight line (*i.e.*, without many tight bends). The wire has a circular cross-section with radius r_0, much smaller than its length. We denote by x the coordinate along the wire. An electric pulse is applied at $x = 0$, uniformly distributed over the cross-section. The charge density of the pulse may be represented as

$$\rho_0 = Q\tau\delta(t)\delta(x)\theta(r_0 - r) \ , \tag{1}$$

where Q is the surface charge density, τ is the short duration of the pulse, t denotes the time and r is the radius in the plane of the cross-section. Similarly, the current density of the pulse may be represented as

$$j_{0x} = Qa\delta(t)\delta(x)\theta(r_0 - r) \ , \tag{2}$$

where a is the small spatial extension of the pulse along the x-direction. The δ-functions in equations above should be viewed as equal to $1/\tau$, $1/a$ over a duration τ or a distance a, respectively. Outside the pulse the charge and the current are zero, while inside the pulse the charge density is $\rho_0 = Q/a$ and the current density is $j_{0x} = Q/\tau = v\rho_0$, where $v = a/\tau$ is the velocity of charge injection (pulse building). We can see that the charge conservation is verified. The charge and the current densities given above determine the scalar potential Φ_0 and the x-component $A_{0x} = \frac{a}{c\tau}\Phi_0$ of the vector potential, where c is the speed of light in vacuum. The scalar potential satisfies the wave equation

$$\frac{1}{c^2}\ddot{\Phi}_0 - \Delta\Phi_0 = 4\pi\rho_0 = 4\pi Q\tau\delta(t)\delta(x)\theta(r_0 - r) \; ; \tag{3}$$

by means of the Fourier transformations we get the solution

$$\Phi_0 = 2\pi Qc\tau r_0 \int d\kappa dk \frac{J_1(kr_0)J_0(kr)}{K} \sin cKt \cdot e^{i\kappa x} \;\; , \tag{4}$$

where κ is the x-component of the wavevector $\boldsymbol{K} = (\kappa, \boldsymbol{k})$, $\boldsymbol{k}$ is the transverse wavevector and $J_{0,1}$ are the Bessel functions of zeroth and first order, respectively. It is easy to see that the potential Φ_0 has a slow variation with r, except for $r \simeq r_0$ where the surface effects appear. Inside the wire ($r < r_0$), and leaving aside the surface effects, we may approximate the potential given by equation (4) by

$$\begin{aligned} \Phi_0 &\simeq 2\pi Qc\tau \textstyle\int d\kappa dz J_1(z)\frac{\sin c\kappa t}{\kappa}e^{i\kappa x} = \\ &= 2\pi Qc\tau\theta(t- \mid x \mid /c) \; . \end{aligned} \tag{5}$$

Indeed, in the conditions given above equation (3) becomes the one-dimensional wave equation

$$\frac{1}{c^2}\ddot{\Phi}_0 - \frac{\partial^2\Phi_0}{\partial x^2} = 4\pi Q\tau\delta(t)\delta(x) \;\; , \tag{6}$$

whose solution is the one-dimensional Green function $4\pi Q\tau \times \frac{c}{2}\theta(t- \mid x \mid /c))$.

It is easy to check that the potentials Φ_0 (given by equation (5)) and $A_{0x} = \frac{a}{c\tau}\Phi_0$ satisfy the Lorenz gauge; for instance, outside the pulse the derivatives are zero, while inside the pulse $\partial\Phi_0/c\partial t$ and $\partial A_{0x}/\partial x$ may be replaced by $2\pi Qc\tau/a$ and $-2\pi Qc\tau/a$, respectively. The solution given by equation (5) is valid inside the wire, close to the axis of the wire. Near the surface the function θ in equation (5) spreads out over a distance of the order r_0 (in all directions), such that, for thin wires, we may still view the solution as close to a θ-pulse. According to equation (4), the potentials are continuous with continuous derivatives at the surface of the wire. In the outside region, far away from the (thin) wire, we may view the pulse as being localized at a point in space ($x = 0$, $r = 0$), in a very short time τ, such that, at large distances, it generates the well-known spherical waves. We can estimate the energy radiated in space by such a pulse, and find out that the total energy radiated in time τ is extremely small. The assumptions implied by equations (1) and (2) and the approximation given by equation (5) define the idealized model of pulse propagation presented herein.

Making use of equation (5), the electric field along the wire ($E_{0x} = -\partial A_{0x}/c\partial t - \partial\Phi_0/\partial x$) is given by

$$E_{0x} = 2\pi Q\tau\left[sgn(x) - v/c\right]\delta(t- \mid x \mid /c) \simeq 2\pi Q\tau \cdot sgn(x)\delta(t- \mid x \mid /c) \tag{7}$$

(we assume $v \ll c$); the other two components of the electric field and the magnetic field are zero. This is a pulse of electric field which propagates (dispersionless) along the wire with the speed of light in vacuum. It may viewed as an external electric field. We can see that the pulse of the electric field is much narrow in time (a/c) than the source pulse (τ); indeed, two charges separated by distance a emit radiation with a time delay a/c, and $v = a/\tau \ll c$.

We note that the above results are obtained by using the charge and the current densities given by equations (1) and (2), which may be viewed as corresponding to charges injected into the wire from the outside. If a pulse-like charge and current imbalance is created (uniformly) by an external field (without charge transfer), the densities may be taken as $\rho_0 = Qa\tau\delta(t)\delta'(x)$ and $j_{0x} = -Qa\tau\delta'(t)\delta(x)$ (for $r < r_0$). The results obtained for such pulses are similar with those given by a δ-representation for ρ_0 and j_{0x}.

The electric field E_{0x} generates an external force acting on the mobile charges in the wire (electrons). In general, a displacement $\boldsymbol{u}$ of the charges generates an imbalance of charge density $-nqdiv\boldsymbol{u}$, where n is the charge concentration in the wire and q is the electron charge ($\simeq 4.8 \times 10^{-10} statC$). At the same time, an internal electric field $\boldsymbol{E}_i$ appears in the wire, given by $div\boldsymbol{E}_i = -4\pi nqdiv\boldsymbol{u}$ (polarization $\boldsymbol{P} = nq\boldsymbol{u}$); the internal field generates a force $-4\pi nq^2\boldsymbol{u}$. The equation of motion for the displacement of a charge along the wire is

$$m\ddot{u} + m\omega_p^2 u + m\gamma\dot{u} = qE_0\tau \cdot sgn(x)\delta(t- \mid x \mid /c) \ , \tag{8}$$

where $\omega_p = (4\pi nq^2/m)^{1/2}$ is the plasma frequency, γ is a damping coefficient ($\gamma \ll \omega_p$) and $E_0 = 2\pi Q$ is a notation introduced from equation (7). By a temporal Fourier transformation of equation (8) we get

$$u(\omega, x) = -\frac{qE_0\tau}{m} \cdot \frac{sgn(x)}{\omega^2 - \omega_p^2 + i\gamma\omega} e^{i\frac{\omega}{c}|x|} \tag{9}$$

and the displacement along the wire

$$u(t, x) = \frac{1}{2\pi}\int d\omega u(\omega)e^{-i\omega t} = \frac{qE_0\tau}{m\omega_p} sgn(x)\sin\omega_p(t- \mid x \mid /c)e^{-\frac{\gamma}{2}(t-|x|/c)} \ , \ t > \mid x \mid /c \tag{10}$$

and $u(t, x) = 0$ for $t < \mid x \mid /c$ (both poles in equation (9) are placed in the lower half-plane).

The displacement in equation (10) represents a wave of charge densiy (and internal electric field, polarization), which propagates inside the wire with the speed of light in vacuum; it may be called a polariton. The frequency of this polariton is the plasma frequency, such that we may call this wave a plasmon-polariton. Its wavelength is $\lambda = c/\omega_p$. Typical plasma frequency ω_p in metals is a few eV, *e.g.* $\omega_p \simeq 5 \times 10^{15} s^{-1}$ for $n = 10^{22} cm^{-3}$ (electron mass $\simeq 10^{-27} g$), such that λ is of the order $10^{-5} cm$ ($0.1\mu m$); it is comparable with the mean freepath of electrons in typical metals at room temperature, $\Lambda \simeq 0.1 - 1\mu m$. We note that this wave affects all the charges in the cross-section of the wire. We can see that the electric pulse propagating along the wire excites a tail of plasmon-polaritons, which are propagating along the wire with the speed of light in vacuum and wavefronts placed at $t = \mid x \mid /c$. The fact that the wavelength λ is much smaller than the diameter of the wire is in accordance with our assumption that the propagation does not depend appreciably on the transverse dimensions of the wire (and surface effects may be neglected).

Let us introduce the notation $u(t, x) = u_0 sgn(x)\sin\omega_p(t- \mid x \mid /c)e^{-\frac{\gamma}{2}(t-|x|/c)}$, where $u_0 = qE_0\tau/m\omega_p$ is the amplitude of the displacement (equation (10)). In usual conditions this is a very small displacement; for instance, for a field $E_0 = 10^{-2} statV/cm$ ($300V/m$) and $\tau = 1ns$ ($10^{-9}s$) we get $u_0 \simeq 0.1$Å. The maximal velocity of the displacement $u_0\omega_p$ is of the order $10^6 cm/s$ (much smaller than the Fermi velocity in metals).

The plasmon-polariton solution given above is valid outside the pulse ($t- \mid x \mid /c > 0$). Let us now estimate the motion of a charge inside the pulse. The period of oscillation of a charge ω_p^{-1} ($\simeq 2 \times 10^{-16}s$) is much shorter than the time a/c spent by the pulse upon a charge; for instance, for a pulse width $a = 3 \times 10^{-2} cm$ ($300\mu m$) we get $a/c = 10^{-12}s \gg \omega_p^{-1}$. Consequently, we may estimate the displacement of a charge inside the pulse by $w = qE_0\tau c/ma\omega_p^2$ (from equation (8); we note that the effective field inside the pulse is increased by the factor $\tau c/a$). Making use of the numerical data given here, we get $w \simeq 2 \times 10^{-13} cm$. We may see that the restoring force ($m\omega_p^2 w$) is very strong and the energy spent by the field to set the charge in motion is extremely small; practically, the field does not lose energy in this process.

However, we must include the damping force in the energy balance. The energy conservation derived from equation (8) reads

$$\frac{\partial}{\partial t}\left(\frac{1}{2}m\dot{u}^2 + \frac{1}{2}m\omega_p^2 u^2\right) + m\gamma\dot{u}^2 = qE_0\tau \cdot sgn(x)\delta(t- \mid x \mid /c)\dot{u} \ . \tag{11}$$

If we integrate equation (11) over a small interval about the point $t- \mid x/c$ (across the pulse), we can see that the first term on the left side (time derivative of the energy) may be neglected and the velocity of the displacement inside the pulse is of the form $\dot{u} = \dot{s}\tau \cdot sgn(x)\delta(t- \mid x \mid /c)$; it follows that the energy loss of the pulse (per charge and per unit time) is $m\gamma\dot{s}^2(\tau c/a)^2$, equal with the work done by the field upon a charge $qE_0\dot{s}(\tau c/a)^2$ per unit time; hence, we get $\dot{s} = qE_0/m\gamma$ and the energy loss $(q^2E_0^2/m\gamma)(\tau c/a)^2$ (per charge and unit time). Since the duration of the pulse on a charge is a/c, we may estimate this displacement as $s = qE_0a/mc\gamma$. In order to compute this displacement we need the damping coefficient γ. In cuasi-static conditions, the damping coefficient is given by $\gamma = \omega_p^2/4\pi\sigma$, where σ is the static conductivity; in typical metals $\gamma \simeq 10^{13} - 10^{14}s^{-1}$ (at room temperature). Using $\gamma = 10^{13}s^{-1}$ and the numerical data given here, we get $s \simeq 5\times 10^{-10}cm$. We can see that $s \ll \Lambda$ ($\Lambda \simeq 0.1 - 1\mu m$), such that the damping is ineffective. Therefore, we may say that the pulse is propagated practically without loss, over large distances.

Outside the pulse, equation (11) is the energy conservation for a damped oscillator, with the rate of energy loss γ. In general, the damping coefficient is diminished in oscillations, in comparison with the static conditions. We may view the damping coefficient as being given by $\gamma = 1/t_{lf}$, where t_{lf} is the lifetime of the electron excitations; it defines the mean freepath by $\Lambda = v_F t_{lf}$, where v_F is the Fermi velocity. The electron-phonon interaction brings, usually, a comparable contribution (besides the effects of impurities, etc). The electron lifetime can be estimated by $\gamma = 1/t_{lf} \simeq T^2/\hbar\mu$, where T is the temperature and μ is the Fermi level. For typical metals at room temperature we get $\gamma \simeq 10^{12}s^{-1}$ ($\mu = 1eV$) and $\Lambda \simeq 10^{-4}cm$ ($v_F = 10^8 cm/s$). Since the displacement amplitude u_0 is much smaller than the mean freepath Λ, the plasmon-polariton is a non-thermal (adiabatic) process, where the damping coefficient is, practically, not effective. The energy transferred by the electric pulse to the plasmon-polaritons may be transported by the latter over large distances.

It is worth analyzing the energy radiated by plasmon-polaritons. The electromagnetic potentials are given by the Kirchhoff formula for the charge density $\rho = -nq div\boldsymbol{u}$ and the current density $\boldsymbol{j} = nq\dot{\boldsymbol{u}}$. The potentials of the field generated by the plasmon-polaritons are given by

$$\Phi = A_x = nqu_0\frac{\omega_p}{c}S\int_0^{ct} dx' \frac{\cos\omega_p\left(t - \sqrt{r^2 + (x-x')^2}/c - x'/c\right)}{\sqrt{r^2 + (x-x')^2}} \ , \tag{12}$$

where r is the distance from the wire, $(x, \boldsymbol{r})$ is the position of the observation point and S denotes the area of the cross-section of the wire (for a semi-infinite wire). We introduce the new variables $z = \omega_p(x - x')/c$, $y = \omega_p r/c$ and note that the limit $\omega_p x/c$ may be taken to infinity, since the wavelength $\lambda = c/\omega_p$ is much smaller than any relevant distance. Then, the integral in equation (12) becomes a function of $t - x/c$, which leads to vanishing fields (and a vanishing radiated energy).

In conclusion, an electric pulse localized both in space and time on a thin metallic wire may propagate along the wire with the speed of light in vacuum, practically without energy loss and without dispersion; the pulse excites behind a tail of plasmon-polaritons which, also, propagate along the wire with the speed of light in vacuum and plasma frequency without energy loss. The reason for such a lossless propagation is the high plasma frequency, which leads to charge displacements much smaller than the mean freepath. The results presented here can be extended to dielectric wires.

The model discussed here is an idealized model. The idealization resides in assuming δ-functions for the pulse, which depend only on the coordinate x along the wire (and time t). Also, the building of the pulse is assumed to be achieved by a uniform injection of charges, or a uniform imbalance of charges over the cross-section of the wire, and the surface effects are neglected. These are important elements of idealization. In experiments, the pulse has a finite extension in all three directions and in time; usually, in terahertz experiments the pulses are radially polarized. In these conditions, the propagation should be considered in all three space dimensions and surface effects should be included, as in cylindrical wires. Then, we may have dispersion, energy loss and contributions from surface waves and surface plasmon-polaritons. The main effect of such more realistic conditions is an attenuation of the pulse with distance.

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References

[1] A. Sommerfeld, "Ueber die Fortpflanzung elektrodynamischer Wellen laengs eines Drahtes", Ann. Phys. Chem. **67** 233 (1899); *Lectures on Theoretical Physics*, vol. 3, *Electrodynamics*, Academic Press, London (1964).

[2] J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill, NY (1941).

[3] G. Gobau, "Surface waves and their application to transmission lines", J. Appl. Phys. **21** 1119 (1950).

[4] F. Sobel, F. L. Wentworth and J. C. Wiltse, "Quasi-optical surface waveguide and other components for the 100 to 300-Gc region", IRE Trans. Microwave Theor. Tech. **9** 512 (1961).

[5] C. Miziumski, "Utilization of a cylindrical geometry to promote radiative interaction with slow surface excitations", Phys. Lett **A40** 187 (1972).

[6] C. A. Pfeiffer, E. N. Economou and K. L. Ngai, "Surface polaritons in a circularly cylindrical interface: surface plasmons", Phys. Rev. **B10** 3038 (1974).

[7] J. C. Ashley and L. C. Emerson, "Dispersion relations for non-radiative surface plasmons on cylinders", Surf. Sci. **41** 615 (1974).

[8] H. Raether, *Surface Plasmons on Smooth and Rough Surfaces and on Gratings*, Springer (1988).

[9] R. M. Dickson and L. A. Lyon, "Unidirectional plasmon propagation in metallic nanowires", J. Phys. Chem. **B104** 6095 (2000).

[10] W. L. Barnes, A. Dereux and T. W. Ebbesen, "Surface plasmon subwavelength optics", Nature **24** 824 (2003).

[11] M. I. Stockman, "Nanofocusing of optical energy in tapered plasmonic waveguides", Phys. Rev. Lett. **93** 137404 (2004) (Erratum Phys. Rev. Lett. **106** 019901 (2011)).

[12] J.-C. Weeber, Y. Lacroute, A. Dereux, E. Devaux, T. Ebbesen, C. Girard, M. U. Gonzales and A.-L. Baudrion, "Near-field characterization of Bragg mirrors engraved in surface plasmon waveguides", Phys. Rev. **B70** 235406 (2004).

[13] J. Takahara, S. Yamagishi, H. Taki, A. Morimoto and T. Kobayashi, "Guiding of a one-dimensional optical beam with nanometer diameter", Opt. Lett. **22** 475 (1997).

[14] K. V. Nerkararyan, "Superfocusing of a surface polariton in a wedge-like structure", Phys. Lett. **A237** 103 (1997).

[15] K. Wang and D. M. Mittleman, "Metal wires for terahertz wave guiding", Nature **432** 376 (2004).

[16] T.-I. Jeon, J. Zhang and D. Grischkowsky, "THz Sommerfeld wave propagation on a single metal wire", Appl. Phys. Lett. **86** 161904 (2005).

[17] M. Wachter and M. Nagel and H. Kurz, "Sommerfeld wires at terahertz frequencies", IEEE 0-07803-9542–5 1299 (2006).

[18] K. Wang and D. M. Mittleman, "Dispersion of surface plasmon-polaritons on metal wires in terahertz frequency range", Phys. Rev. Lett. **96** 157401 (2006).

[19] Z. Zheng, N. Kanda, K. Konishi and M. Kuwata-Gonokami, "Efficient coupling of propagating broadband terahertz radial beams to metal wires", Opt. Express **21** 10.1364/OE 21.010642 (2013).

[20] W. P. E. M. o'pt Root, G. J. H. Brussaard, P. W. Smorenburg and O. J. Luiten, "Single-cycle surface plasmon polaritons on a bare metal wire excited by relativistic electrons", Nature Commun. 7:13769, 10.038/ncomms/13769 (2016).

[21] K. Teramoto, S. Inoue, S. Tokita, R. Yasuhara, Y. Nakamiya, T. Nagashima, K. Mori, M. Mashida and S. Sakabe, "Induction of subterahertz surafce waves on a metallic wire by intense laser interacting with a foil", Phys. Rev. **E97** 023204 (2018).