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Surface plasmon-polaritons

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Abstract

The surface plasmon-polaritons are identified on a circular, straight, long wire (either metallic or dielectric). They are radial magnetic-type dispersive modes, excited by the unipolar currents, except for the symmetric mode, which is non-dispersive. This latter mode may guide an electromagnetic wave along the surface, dispersionless and with the speed of light in vacuum. A similar situation occurs for a half-space.

Internal field. Polarization. Let us assume the presence of a free electromagnetic field $\boldsymbol{E}_0$ (electric field) and $\boldsymbol{H}_0$ (magnetic field). It obeys the Maxwell equations

$$\begin{gathered} div\boldsymbol{E}_0 = 0 \ , \ div\boldsymbol{H}_0 = 0 \ , \\ curl\boldsymbol{E}_0 = -\frac{1}{c}\frac{\partial \boldsymbol{H}_0}{\partial t} \ , \ curl\boldsymbol{H}_0 = \frac{1}{c}\frac{\partial \boldsymbol{E}_0}{\partial t} \ , \end{gathered} \tag{1}$$

where c is the speed of light in vacuum.

In matter, the mobile charges q, with concentration n, are displaced by the external field, by a displacement field $\boldsymbol{u}$. It generates a charge density imbalance $-nqdiv\boldsymbol{u}$ and a current density $nq\frac{\partial \boldsymbol{u}}{\partial t}$. In addition, in magnetic matter a magnetic current may appear, given by $c \cdot curl\boldsymbol{M}$, where $\boldsymbol{M}$ is called magnetization. These charge and curent densities should be added to the right side of the first and fourth Maxwell equations (Gauss and Maxwell-Ampere equations), and additional internal field also should be added.[1] By historical tradition, $\boldsymbol{E}_0$ is denoted by $\boldsymbol{D}$ (electric displacement) and the total magnetic field $\boldsymbol{H}$ is denoted by $\boldsymbol{B}$ (magnetic induction). Let us assume a non-magnetic matter ($\boldsymbol{M} = 0$) and preserve the notation $\boldsymbol{H}$ for the total magnetic field (magnetic induction). In addition, we denote by $\boldsymbol{E} = \boldsymbol{E}_0 + \boldsymbol{E}_i$ the total electric field, $\boldsymbol{E}_i$ being the internal electric field, and use $\boldsymbol{E}_0$ or $\boldsymbol{D}$ for the external electric field. The Maxwell equations become

$$\begin{gathered} div\boldsymbol{E} == -4\pi nqdiv\boldsymbol{u} \ , \ div\boldsymbol{H} = 0 \ , \\ curl\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} \ , \ curl\boldsymbol{H} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi}{c}nq\frac{\partial \boldsymbol{u}}{\partial t} \ , \end{gathered} \tag{2}$$

where

$$div\boldsymbol{E}_i = -4\pi nqdiv\boldsymbol{u} \ . \tag{3}$$

From equation (3) we get $\boldsymbol{E}_i = -4\pi nq\boldsymbol{u}$. This solution is valid for infnite matter. $\boldsymbol{P} = nq\boldsymbol{u}$ is called polarization; the first equation (2) becomes $div(\boldsymbol{E}+4\pi\boldsymbol{P}) = 0$, whence $\boldsymbol{E}+4\pi\boldsymbol{P} = \boldsymbol{D} = \boldsymbol{E}_0$, $div\boldsymbol{D} = 0$; the fourth equation (2) becomes $curl\boldsymbol{H} = \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t}$.

The equation of motion of an elementary mobile charge with mass m is

$$m\ddot{\boldsymbol{u}}+m\gamma\dot{\boldsymbol{u}}=q\boldsymbol{E}_0+q\boldsymbol{E}_i \ , \tag{4}$$

where γ is the attenuation coefficient. In equation (4) we neglect internal forces and view the charge velocity as being sufficiently small, such that we may neglect also the Lorentz force. In finite bodies Coulomb interaction is an internal force.[1, 2] Equation (4) can be written as

$$\ddot{\boldsymbol{u}}+\omega_0^2\boldsymbol{u}+\gamma\dot{\boldsymbol{u}}=\frac{q}{m}\boldsymbol{E}_0 \ , \tag{5}$$

where ω_0 given by $\omega_0^2=4\pi nq^2/m$ is the plasma frequency. Equation (5) is suitable for metals; for dielectrics an additional important internal (elastic) force appears.[1] The solution of equation (5) for a monochromatic wave with frequency ω is

$$\boldsymbol{u}=-\frac{q\boldsymbol{E}_0}{m}\frac{1}{\omega^2-\omega_0^2+i\omega\gamma} \ ; \tag{6}$$

we get

$$\begin{gathered}\boldsymbol{E}_i=\tfrac{\omega_0^2}{\omega^2-\omega_0^2+i\omega\gamma}\boldsymbol{E}_0 \ , \ \boldsymbol{P}=-\tfrac{1}{4\pi}\tfrac{\omega_0^2}{\omega^2-\omega_0^2+i\omega\gamma}\boldsymbol{E}_0 \ , \\ \boldsymbol{E}=\boldsymbol{E}_0+\boldsymbol{E}_i=\tfrac{\omega^2+i\omega\gamma}{\omega^2-\omega_0^2+i\omega\gamma}\boldsymbol{E}_0\end{gathered} \tag{7}$$

and, from $\boldsymbol{D}=\boldsymbol{E}_0=\varepsilon\boldsymbol{E}$, $\boldsymbol{P}=\chi\boldsymbol{E}$, $\varepsilon=1+4\pi\chi$,

$$\varepsilon=1-\frac{\omega_0^2}{\omega^2+i\omega\gamma} \ , \ \chi=-\frac{1}{4\pi}\frac{\omega_0^2}{\omega^2+i\omega\gamma} \ , \tag{8}$$

where ε is the dielectric function and χ is the electric susceptibility. In addition, since the current density $nq\frac{\partial\boldsymbol{u}}{\partial t}$ is $\boldsymbol{j}=-i\omega nq\boldsymbol{u}=\sigma\boldsymbol{E}$, we get the conductivity

$$\sigma=\frac{1}{4\pi}\frac{i\omega_0^2}{\omega+i\gamma} \ , \ \varepsilon=1+\frac{4\pi i\sigma}{\omega} \ . \tag{9}$$

We can see that the internal field has a resonance at the plasma frequency ω_0 (the plasmon).

By equation (4) the first Mawell equation (2) (Gauss law) is solved. In the fourth Maxwell equation (2)

$$\begin{gathered}curl\boldsymbol{H}=\tfrac{1}{c}\tfrac{\partial\boldsymbol{E}}{\partial t}+\tfrac{4\pi}{c}nq\tfrac{\partial\boldsymbol{u}}{\partial t}= \\ =-\tfrac{i\omega}{c}\left(\boldsymbol{E}_0+\boldsymbol{E}_i+4\pi nq\boldsymbol{u}\right)\end{gathered} \tag{10}$$

(Maxwell-Ampere equation) $\boldsymbol{E}_i+4\pi nq\boldsymbol{u}=0$, such that this equation reduces to

$$curl\boldsymbol{H}=-\frac{i\omega}{c}\boldsymbol{E}_0 \ ' \tag{11}$$

therefore, $\boldsymbol{H}=\boldsymbol{H}_0$, as expected for non-magnetic matter. The second Maxwell equation (magnetic Gauss law) is satisfied. The third Maxwell equation (Faraday law) can be written as

$$curl\boldsymbol{E}=curl\boldsymbol{E}_0+curl\boldsymbol{E}_i=\tfrac{i\omega}{c}\boldsymbol{H}_0 \ , \tag{12}$$

which shows that the internal field $\boldsymbol{E}_i=-4\pi nq\boldsymbol{u}$ is given by a gradient, as expected when charges are present. It is worth noting that the internal field and the total electric field are complex, due to the presence of the attenuation coefficient γ. It is worth noting that these fields propagate in matter with the velocity $c/\sqrt{\varepsilon}$, where $\sqrt{\varepsilon}$ is the (complex) refractive index; usually, its large positive imaginary part givesva high dissipation in metals.

Surface plasmon-polaritons. We consider a straight wire with a circular cross-section with radius a in cylindrical cordinates (r, φ, x). We introduce the notation $\boldsymbol{u} = \boldsymbol{v}\theta(a-r)$. We recall that the function $\theta(a-r)$ is not defined for $r = a$. In the charge density $-nqdiv\boldsymbol{u}$ we have

$$div\boldsymbol{u} = div\boldsymbol{v} \cdot \theta(a-r) - v_r^s \delta(a-r) \ , \tag{13}$$

where $v_r^s = v_r(a, \varphi, x)$ is the r-th component of the displacement $\boldsymbol{v}$. This equation is valid for $r \leq a$, according to the definition of the charge density. Therefore, the δ-function in equation (13) should be viewed as $\frac{1}{2}\delta$. Due to this circumstance v_r^s should be viewed as a distinct (surface) displacement denoted by $w(r, \varphi)$. The Gauss law for the internal electric field reads

$$div\boldsymbol{E}_i = -4\pi nqdiv\boldsymbol{v} \cdot \theta(a-r) + 2\pi nqw\delta(a-r) \ . \tag{14}$$

the solution of this equation is $E_i^v = -4\pi nq\boldsymbol{v}$ for $r < a$ and

$$E_{ir}^s = -2\pi nqw \ . \tag{15}$$

The term E_i^v is the bulk contribution. The Maxwell equations are solved as in the preceding section for the bulk contribution. The term E_{ir}^s is the surface contribution, arising from the surface charge density $-\frac{1}{2}nqw$. This contribution is decoupled from the bulk term. We write the Maxwell equations for the surface fields, denoted by $\boldsymbol{e}$ and $\boldsymbol{h}$; for instance, $E_{ir}^s = e_{ir}$. Wherever we encounter the derivative $\partial/\partial r$ we integrate it out over the thickness of the surface; in $curl's$ it appears only tangential fields associated with this derivative, which are continuous, while h_r in the magnetic Gauss law is also continuous. We get the two-dimensional Maxwell equations

$$\begin{gathered} (curl\boldsymbol{e})_\varphi = \frac{\partial e_r}{\partial x} = \frac{i\omega}{c}h_\varphi \ , \ (curl\boldsymbol{e})_x = -\frac{1}{a}\frac{\partial e_r}{\partial \varphi} = \frac{i\omega}{c}h_x \ , \\ (curl\boldsymbol{h})_r = \frac{1}{a}\frac{\partial h_x}{\partial \varphi} - \frac{\partial h_\varphi}{\partial x} = -\frac{i\omega}{c}e_{0r} + \frac{i\omega}{c} \cdot 2\pi nqw \end{gathered} \tag{16}$$

and

$$(curl\boldsymbol{h})_{\varphi,x} = -\frac{i\omega}{c}e_{0\varphi,x} \ , \ (div\boldsymbol{h})_{\varphi,x} = 0 \ . \tag{17}$$

We note the occurence of the so-called uni-polar current density $\frac{1}{2}nq\dot{w}$ on the right of the radial Maxwell-Ampere equation (16).

The equation of motion the radial surface displacement w is

$$m\ddot{w} + m\gamma\dot{w} = qe_{0r} + qe_{ir} \ , \tag{18}$$

or

$$\ddot{w} + \frac{1}{2}\omega_0^2 w + \gamma\dot{w} = \frac{q}{m}e_{0r} \ ; \tag{19}$$

the other components of the displacement are those of the bulk displacement. From equation (19) we get

$$w = -\frac{qe_{0r}}{m}\frac{1}{\omega^2 - \frac{1}{2}\omega_0^2 + i\omega\gamma} \ . \tag{20}$$

We can see that the surface electric field has a resonance at $\omega_0/\sqrt{2}$ (surface plasmon). If the Coulomb interaction is included the plasmon (bulk and surface) branches become dispersive and the plasmons pass over gradually into polaritons.[3] A similar result $(\omega_0/\sqrt{2})$ is well known for a half-space.

Making use of equation (20), the system of equations (16) and (17) can be solved. We assume a dependence $\sim e^{ikx}e^{il\varphi}$ for the external field $\boldsymbol{e}_0$, $\boldsymbol{h}_0$, where l is an integer, and get

$$\begin{gathered} e_r = e_{0r} + e_{ir} = e_{0r} - 2\pi nqw = \\ \tfrac{\omega^2+i\omega\gamma}{\omega^2-\frac{1}{2}\omega_0^2+i\omega\gamma}e_{0r} = \tfrac{1}{\varepsilon_s}e_{0r} \ , \end{gathered} \tag{21}$$

where

$$\varepsilon_s = 1 - \frac{\frac{1}{2}\omega_0^2}{\omega^2 - \frac{1}{2}\omega_0^2 + i\omega\gamma} \tag{22}$$

is the surface (radial) dielectric function, and

$$\begin{gathered} h_\varphi = \tfrac{ck}{\omega\varepsilon_s}e_{0r} \ , \ h_x = -\tfrac{lc}{a\omega\varepsilon_s}e_{0r} \ , \\ h_r = h_{0r} \ . \end{gathered} \tag{23}$$

From the second equation (16) we get the dispersion relation

$$\omega^2 = c^2\left(\frac{l^2}{a^2} + k^2\right) \ ; \tag{24}$$

it defines the surface polaritons, associated with the magnetic field $h_{\varphi,x}$; since this field is resonant for the surface plasmon frequency $\omega = \omega_0/\sqrt{2}$, they are called surface plasmon-polaritons. We can see that there are many branches of plasmon-polaritons, and all are dispersive for $l \neq 0$ (asymmetric modes). There exists only one non-dispersive branch $l = 0$ (symmetric mode), with $h_x = 0$ and $h_\varphi \neq 0$. The symmetric surface plasmon-polariton mode may guide the electromagnetic field along the wire, non-dispersively and with the speed of light in vacuum.[5]-[8] Noteworthy, it is a magnetic-type mode, excited by a transverse external field (e_{0r}). It is different from Sommerfeld "surface wave", which is a bulk, electric-type mode, excited by a longitudinal electric field.[9] A completely analogous situation occurs for a half-space.

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