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**Penetration of oscillating uniform electric fields in metals**

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**Abstract**

It is shown that the electrons in metals, in spite of being a quantum, "cold" plasma, exhibit a much higher plasmon damping than classical plasmas. The overwhelming factor in this circumstance is the high electron density in metals. On the other hand, it is shown that the penetration depth of an oscillating uniform electric field in metals is of the order $\lambda_e \simeq v_F/\omega$, where $v_F$ is the Fermi velocity and $\omega$ is the frequency of the external field. This result should be compared with the penetration depth $\lambda_e \simeq v_{th}/\omega$ in classical plasmas (Ref. [1]), where $v_{th}$ is the thermal velocity of the electrons in plasma. Since the Fermi energy in metals is comparable with the temperature of the electrons in classical ("hot") plasmas, the two velocities $v_F$ and $v_{th}$ are comparable, and so are the two penetration depths.

The penetration depth of an oscillating, uniform, longitudinal electric field in a semi-infinite classical plasma is computed in Ref. [1]. This is a classical problem, identifed by Landau in the context of what was called later the Landau damping. According to the original Landau's result and the subsequent calculations (see the References in Ref. [1]), the attenuation law of the field would have the form $\sim e^{-(x/\lambda_e)^{2/3}}$, where $x$ is the distance from the wall of the plasma, $\lambda_e \simeq v_{th}/\omega$ is the penetration depth (attenuation length), $v_{th}$ is the thermal velocity of the electrons in plasma and $\omega$ is the frequency of the external field. This result is corrected in Ref. [1], where the attenuation law is shown to be $\sim e^{-x/\lambda_e}$.

A classical plasma is a dilute, weakly ionized gas, consisting of electrons, ions and neutral atoms; it is electrically neutral. In general, the electron-electron, the ion-ion and the electron-ion "collisions" are made through the long-range Coulomb interaction, such that they are, practically, ineffective; the state of motion of the electrons and the ions is not affected, practically, by these processes. The collisions with the atoms (electron-atom, ion-atom and atom-atom collisions), apart from being, mainly, elastic, are very rare, since the plasma is dilute. Therefore, we may conclude that a classical plasma is collisionless. The collision frequency is much smaller than the frequency of the external field. In these conditions we may use the Boltzmann equation without the collision term. Moreover, since the external field is weak, it will determine small changes in the motion of the particles, such that we may limit ourselves to zeroth order contributions to the perturbation; this is called the "linearized" Boltzmann equation, and, in the presence of the electric field, the resulting equation (linearized and collisionless) is termed the Boltzmann-Vlasov equation.

In the present Note we tackle the penetration problem in metals, where the electrons are governed by the Fermi-Dirac distribution, in contrast with the classical plasma, where the electrons are governed by the Maxwell distribution. This circumstance brings interesting features.

First, we note that, in general, under the action of an external electric field there exist two types of electronic motion. In one motion the electrons move as a whole; this is the plasmonic motion. Second, the changes in the statistical distribution are brought by the elementary electron excitations. In classical plasmas these excitations are the electrons themselves, in metals these excitations are the electron quasiparticles. The plasma frequency and the plasmon damping are governed by the electron density, which is very low in classical plasmas and very high in metals; while the attenuation length is governed by the thermal velocity of the electrons in classical plasmas and the Fermi velocity in metals. Since these velocities are comparable for the "hot" electrons in classsical plasmas and the electrons in the "cold" plasma in metals, the attenuation lengths are comparable.

As regards the linearized version of the Boltzmann equation, it holds in metals, since the effects of the external perturbation (electric field) are weak. The frequency of the electron-ion collisions in metals is relatively high, especially in static conditions ($\simeq 10^{13}s^{-1} - 10^{14}s^{-1}$ at room temperature; a comparable contribution is brought by the electron-phonon interaction, besides the contribution of the impurities, etc); it is comparable with the inverse of the lifetime of the electron elementary excitations ($\simeq 10^{12}s^{-1}$ at room temperature). These collisions are important in transport phenomena. On the other hand, they secure the thermal equilibrium. The oscillations of the external field are much slower (the opposite limit in comparison with classical plasmas), such that the changes brought by such external perturbations are not affected by collisions. The slow motion determined by the external field superposes over the rapid thermal motion. It follows that we may use the Boltzmann-Vlasov equation for electrons in metals without the collision term.[2]

As it is well known the statistical properties of the electrons in metals is governed by the Fermi-Dirac distribution

$$dN = \frac{2}{(2\pi\hbar)^3}\frac{1}{e^{\beta(p_x^2/2m+p_y^2/2m+p_z^2/2m-\mu)}+1}dVdp_xdp_ydp_z \ \ , \tag{1}$$

where $dN$ is the number of electrons in the volume $dV$ placed at any position $\boldsymbol{r}$ and in the momentum volume $dp_xdp_ydp_z$ placed at any momentum $\boldsymbol{p} = (p_x, p_y, p_z)$; $\mu$ is the chemical potential, $m$ is the electron mass and $\beta = 1/T$ is the inverse of the temperature $T$. The chemical potential $\mu$ is fixed by the total number of electrons $N$ in the volume $V$. Since the uniform applied field $E_0$ is directed along the $x$-direction, it is convenient to integrate over the momenta $p_{y,z}$. Equation (1) leads to

$$dN = \frac{m}{2\pi^2\hbar^3\beta}\ln\left[1+e^{\beta(\mu-p_x^2/2m)}\right]dVdp_x \ \ , \tag{2}$$

or, making use of $p_x = mv_x$ and denoting the velocity $v_x$ by $v$,

$$dN = \frac{m^2}{2\pi^2\hbar^3\beta}\ln\left[1+e^{\beta(\mu-mv^2/2)}\right]dVdv \ . \tag{3}$$

It follows that the unperturbed distribution is

$$F(v) = \frac{m^2}{2\pi^2\hbar^3\beta}\ln\left[1+e^{\beta(\mu-mv^2/2)}\right] \ ; \tag{4}$$

we note that

$$\int_{-\infty}^{+\infty} dvF(v) = n \ \ , \tag{5}$$

where $n$ is the electron density.

All the equations from (1) to (12) in Ref. [1] remain valid. In particular the denominator $A$ in equation (11) is given by

$$A \simeq k(1-\omega_0^2/\omega^2) - i\frac{4\pi^2 q^2}{mk}\frac{\partial F}{\partial v} \mid_{v=\omega/k} \ , \tag{6}$$

where $\omega_0 = (4\pi nq^2/m)^{1.2}$ is the plasma frequency and $\Gamma \simeq -2\pi^2 q^2\omega_0/mk^2)(\partial F/\partial v) \mid_{v=\omega_0/k}$ is the plasmon lifetime (Landau damping); in these equations $q$ is the electron charge, $k$ is the wavevector along the $x$-direction and $\omega$ is the frequency of the external field (and all the oscillating quantities). The only difference with respect to Ref. [1] is the distribution function given by equation (4) in place of the Maxwell distribution.

It is convenient to introduce the variable $\xi = \sqrt{\beta m/2}\omega/k$; the damping coefficient, as a function of $\xi$ $(\omega/k)$, can be written as

$$\Gamma = \frac{3\pi}{\sqrt{2}}\frac{\omega_0^2}{\omega}(v_{th}/v_F)^3\frac{\xi^2 \mid \xi \mid}{Ce^{\xi^2}+1} \ , \tag{7}$$

where $v_F = \hbar k_F/m = \frac{\hbar}{m}(3\pi^2 n)^{1/3}$ is the Fermi velocity, $v_{th} = 1/\sqrt{\beta m}$ is the thermal velocity and $C = e^{-\beta\mu}$; at room temperature $C \ll 1$. Since $v_{th}/v_F = 1/\sqrt{2\beta\mu}$, the damping coefficient can also be written as

$$\Gamma = \frac{3\pi}{4}\frac{\omega_0^2}{\omega}\frac{1}{(\beta\mu)^{3/2}}\frac{\xi^2 \mid \xi \mid}{Ce^{\xi^2}+1} \ . \tag{8}$$

The function $\xi^2 \mid \xi \mid / \left(Ce^{\xi^2}+1\right)$ has a maximum $\simeq (\beta\mu)^{3/2}$ for

$$\xi^2 \simeq \beta\mu - \ln\left(\frac{2}{3}\beta\mu - 1\right) \ ; \tag{9}$$

it follows that the maximum value of the damping coefficient is of the order $\Gamma \simeq \omega_0^2/\omega$. The damping coefficient for a classical plasma (with Maxwell distribution $F = n(\beta m/2\pi)^{1/2}e^{-\frac{1}{2}\beta mv^2}$, Ref. [1]) is

$$\Gamma_p = \sqrt{\pi}\frac{\omega_0^2}{\omega}\xi^2 \mid \xi \mid e^{-\xi^2} \ ; \tag{10}$$

it has a maximum of the same formal order $\Gamma_p \simeq \omega_0^2/\omega$. Since the plasma frequency in metals is much higher than the plasma frequency in classical plasmas, the damping of the plasmons is much greater in metals than in classical plasmas. The plasmon damping in metals occurs for $k_0 \simeq \omega_0/v_F$, while in classical plasmas it appears at $k_0 \simeq \omega_0/v_{th}$. Since $v_F$ and $v_{th}$ are comparable, the damping appears at much longer wavelengths in classical plasmas, as expected.

The zeros of the denominator in equation (11), Ref. [1], are the roots of the equation $A = 0$, which leads to

$$\frac{\xi^2 \mid \xi \mid}{Ce^{\xi^2}+1} = -i\frac{\mid \varepsilon \mid}{3\sqrt{2}\pi(1-\varepsilon)}\left(\frac{v_F}{v_{th}}\right)^3 \tag{11}$$

(compare with equation $\xi^2 \mid \xi \mid e^{-\xi^2} = -i\alpha$ in Ref. [1], where $\alpha =\mid \varepsilon \mid /2\sqrt{\pi}(1-\varepsilon))$. We note that $\omega \ll \omega_0$, so that $\varepsilon \longrightarrow -\infty$ and equation (11) becomes

$$\frac{\xi^2 \mid \xi \mid}{Ce^{\xi^2}+1} \simeq -\frac{i}{3\sqrt{2}\pi}\left(\frac{v_F}{v_{th}}\right)^3 \ . \tag{12}$$

Also, we note that $\beta\mu$ in $C = e^{-\beta\mu}$ is $\beta\mu = v_F^2/2v_{th}^2$, such that the solution of equation (12) is

$$\xi = \sqrt{\frac{\beta m}{2}}\frac{\omega}{k_{1,2}} \simeq \pm\frac{v_F}{(12\pi)^{1/3}v_{th}}(-1+i) \ , \tag{13}$$

where

$$k_1 \simeq \frac{(12\pi)^{1/3}\omega}{2\sqrt{2}v_F}(1+i) \ . \tag{14}$$

All equations (13) to (15) in Ref. [1] remain valid, providing $2\alpha^{1/3}$ is replaced by $(12\pi)^{1/3}/2\sqrt{2}$ ($\simeq$ 1) and $v_{th}$ is replaced by $v_F$. In particular, the penetration depth of the electric field is $\lambda_e \simeq v_F/\omega$. For typical metals $v_F \simeq 10^8 cm/s$, so that $\lambda_e \simeq 10cm$ for $\omega = 10^7 s^{-1}$. The penetration depth in classical plasmas is of the order $\lambda_e \simeq v_{th}/\omega$, where $v_{th}$ is the thermal velocity of the electrons in plasma. We note that the penetration depths in metals are comparable with those in classical plasmas, since the temperature of the electrons in classical ("hot") plasmas (*e.g.*, $10^4K$) is comparable with the Fermi energy in metals (a few $eV$).

## References

[1] M. Apostol, "Electric fields in a semi-infinite classical plasma", J. Theor. Phys. **260** (2019).

[2] M. Apostol, *Physical Kinetics*, Cambridge Scholars, Newcastle (2019).