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Entropy of earthquakes

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Abstract

The Gutenberg-Richter statistical distributions are discussed and the entropy of earthquakes is derived. Both canonical and microcanonical earthquake distributions are given, and Einstein's fluctuation formula is deduced for earthquakes. These results are applied to the seismic activity of Vrancea in the period $1980 - 2019$, for 8498 earthquakes with magnitude $M \geq 2$. It is shown that the parameter β of the magnitude distribution exhibits a tendency of increasing with time, due to the accumulation of small-magnitude earthquakes, interrupted from time to time by ruptures towards smaller values of β, caused by earthquakes with greater magnitudes. These variations of the parameter β do not obey the normal distribution of the fluctuations. Therefore, the seismic activity analyzed here cannot be identified with a statistical ensemble at (quasi-) equilibrium.

Introduction. The entropy of earthquakes has been introduced in Seismology from Statistical Physics, as a measure of the disorder produced by the seismic activity in a given region and a given, relatively long, time interval.[1]-[8] In contrast to the equilibrium Statistical Physics, and closer to the Physical Kinetics, the time dependence of the entropy of earthquakes is of primary interest. In this connection, it is more related to the information entropy.[9, 10]

As hazardous as it may be, the prediction of the seismic activity in a given region is still very attractive. We analyze in this paper the possibility offered by the changes in the entropy of earthquakes to assess changes in the seismic activity. Such an analysis should be based on a few certain general hypotheses. First, we should view the seismic activity as a sequence of random, independent earthquakes with various magnitudes. We should assume that the earthquake occurrence may be described by a statistical distribution in magnitude. We should leave aside any possible correlations between earthquakes, like triggerring or blocking effects, as well as possible correlations between foreshocks, aftershocks and main shocks. As in any statistical description, the earthquake distribution should exhibit fluctuations over a short-time scale. It is claimed that such fluctuations have been identified in yearly variations of global earthquake populations,[7] though they may look rather as quasi-periodic variations. The existence of a statistical distribution implies a statistical equilibrium. A statistical equilibrium is characterized by quasi-stationarity, *i.e.* the statistical distribution may exhibit only slow variations, at most, in long periods of time. The parameter of the fluctuation distribution changes correspondingly in such a long-time seismic activity. The slow variations of the equilibrium may give the possibility to identify the variation tendency by relatively short-time sampling of the data. This short-time seismic activity is affected by fluctuations, so we are led to analyze the fluctuation distribution. Unfortunately, the analysis of a large number of earthquakes with magnitude $M \geq 2$ (8498 earthquakes), which occurred in

Vrancea during the time interval (years) $1980-2019$ does not gives support to the assumptions formulated above. In particular, the variations of the statistical distribution do not obey the fluctuations normal distribution. The reason for this behaviour originates in the fact that the seismic activity is feeded continuously by the tectonic energy, such that the statistical ensemble is not at equilibrium.

The technical means of analyzing the statistical equilibrium and its fluctuations is the entropy. We derive in this paper the Gutenberg-Richter statistical distribution in magnitude for a canonical ensemble by using the standard method of maximizing the entropy under the constraint of a fixed mean magnitude. This constraint corresponds to a quasi-stationary tectonic loading and a quasi-stationary energy release by seismic activity. Similarly, by using the standard method of a microcanonical statistical ensemble, we derive the fluctuations normal distribution (known as Einstein's fluctuation formula). The parameter of this formula is precisely the parameter of the equilibrium distribution. It follows that, by fitting the fluctuation formula to a data sampling collected over a relatively short time, we would have access to the equilibrium distribution and to its tendency in such relatively short times. The prediction would be based on the expected variation of the mean magnitude; consequently, we might only say that in the next period higher-magnitude earthquakes may, or may not, be expected. If the fluctuations do not obey Einstein's normal distribution, then we are in a non-equilibrium state. As said above, the above (quasi-)stationarity scenario is not supported by empirical studies, at least for Vrancea region. We give a few examples to this end, by analyzing a large set of Vrancea earthquakes.

Gutenberg-Richter statistical distributions. The Gutenberg-Richter magnitude probability

$$P(M)=\beta e^{-\beta M} \tag{1}$$

is well-known in Seismology; in equation (1) M denotes the earthquake magnitude and β is a parameter. Originally, this law was derived from empirical observations. Indeed, it is well documented by statistical analysis[11]-[14] that the number of earthquakes $N(M)$ with magnitude greater than M is given by

$$\ln N(M)=\ln N(0)-\beta M \tag{2}$$

(cumulative, or excedence, distribution); hence, $dN/N(M)=-\beta dM$, and the density of magnitude probability[15] is

$$\frac{\Delta N}{N(0)\Delta M}=\beta e^{-\beta M} \quad . \tag{3}$$

If the total, large, number of earthquakes $N(0)$ occurs in the long time T, we can define the seismicity rate $1/t_0$, where $N(0)=T/t_0$, and equation (3) becomes

$$\frac{t_0\Delta N}{T\Delta M}=\beta e^{-\beta M} \ , \ \frac{\Delta N}{T}=\frac{\beta\Delta M}{t_0}e^{-\beta M} \quad , \tag{4}$$

or

$$\ln\left(\frac{\Delta N}{T}\right)=\ln\left(\frac{\beta\Delta M}{t_0}\right)-\beta M \ ; \tag{5}$$

hence, we may define a mean recurrence time

$$t_r=\frac{T\beta\Delta M}{\Delta N}=t_0e^{\beta M} \quad . \tag{6}$$

for the earthquakes with magnitude M.

Equations (1), (3) and (5) define statistical distributions. They may be called Gutenberg-Richter statistical distributions. According to the theory of fluctuations of the statistical distributions, a measure of the fluctuations, *i.e.* of the deviation from the mean value of the statistical variable (magnitude M) is the standard deviation

$$\delta M=\left(\overline{M^2}-\overline{M}^2\right)^{1/2}=\left(-\frac{\partial\overline{M}}{\partial\beta}\right)^{1/2}=\frac{1}{\beta}=\overline{M}\ ; \tag{7}$$

it follows that the error in determining the mean recurence time by equation (6) is 100%, or, at least,
$\left[\left(\overline{M^2}\right)^{1/2}-\overline{M}\right]/\overline{M}=\sqrt{2}-1$; it is a large error.

At the same time, the magnitude was introduced in Seismology as a logarithmic measure of the energy released by an earthquake. More precisely, it is assumed that the energy E of an earthquake with magnitude M is given by $E/E_0=e^{bM}$, or

$$\ln E=\ln E_0+bM\ \ , \tag{8}$$

where E_0 is an energy cutoff and $b=\frac{3}{2}\ln 10=3.45$, by convention ($\ln 10\simeq 2.3$).[16, 17] Later, the energy was related to the magnitude of the seismic moment, and the earthquake magnitude entering equation (8) was called moment magnitude.[18, 19] Equations of the type (8) may be called Hanks-Kanamori (or Gutenberg-Richter) law. As we can see, they have a definition character.

The statistical Gutenberg-Richter distributions in time, energy and magnitude have been derived from a geometrical-growth model of accumulation of energy in the focal region[20, 21] (the exact relationship between energy and the magnitude of the seismic moment was established in Ref. [22]). The relation between the parameters β and b is $\beta=br$, where the parameter r is related to the number of effective dimensions of the focal region and the rate of energy accumulation. For a uniform pointlike focal region $r=1/3$, for a two-dimensional focal region $r=1/2$, while for a one-dimensional region r tends to unity. Very likely, the parameter r varies in the range $1/3<r<1$, which entails a variation $\frac{1}{2}\ln 10<\beta<\frac{3}{2}\ln 10$ (*i.e.*, $1.15<\beta<3.45$). According to the theory of energy accumulation in the focal region, the relationship between the accumulation time t and the accumulated energy E is

$$1+t/t_0=\left(1+E/E_0\right)^r\ \ . \tag{9}$$

This relationship leads to a frequency of events $1/(1+t/t_0)$, a time probability

$$P(t)dt=\frac{1}{\left(1+t/t_0\right)^2}\frac{dt}{t_0}\ ,\ 0<t<\infty \tag{10}$$

and energy and magnitude probabilities

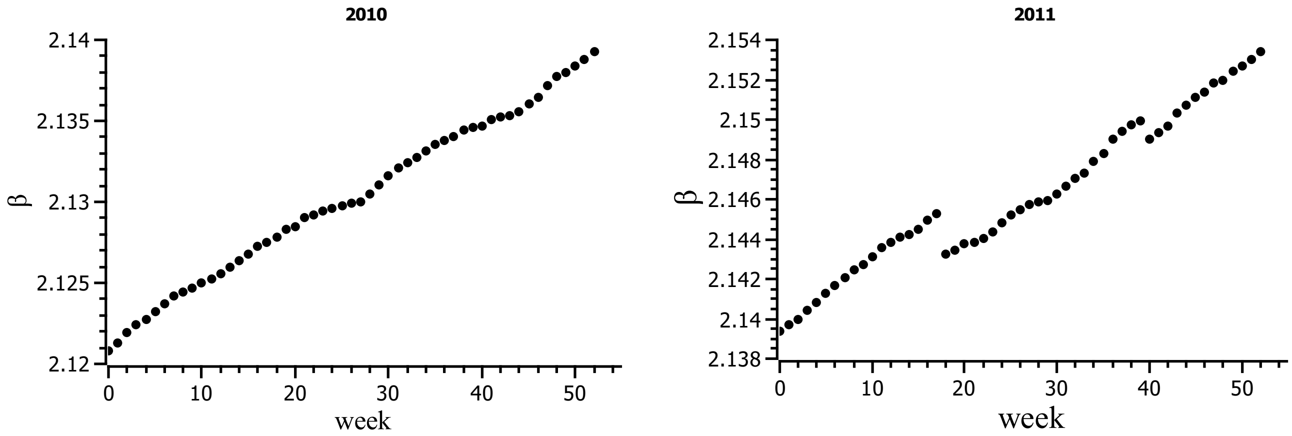
$$\begin{aligned}P(E)dE&=\tfrac{r}{(1+E/E_0)^{1+r}}\tfrac{dE}{E_0}\ ,\ 0<E<\infty\\ P(M)dM&=2^r\tfrac{\beta e^{bM}}{\left(1+e^{bM}\right)^{1+r}}dM\ ,\ 0<M<\infty.\end{aligned} \tag{11}$$

For high energies ($E\gg E_0$, $M\gg 1$, $t\gg t_0$) we may use the approximation

$$t/t_0\simeq\left(E/E_0\right)^r=e^{\beta M}\ \ , \tag{12}$$

which leads to the Gutenberg-Richter statistical distributions (equation (1)).

Fitting the data. The Gutenberg-Richter magnitude distribution given by equations (1), (2) and (5) are widely used to fit data. By such a fitting we derive the parameters β (and, therefore,



(a) (b)

Figure 1: Short-time (weeks) continuous (panel a, year 2010) and discontinuous (ruptures, panel b, year 2011) variations of the parameter β for Vrancea (Romanian Earthquake Catalog ROMPLUS, 2018 (updated)).

r) and t_0 (seismicity rate). An important problem in such fitting procedures is the choice of the data. The data and the fitting parameters depend on the seismic region, the time period and the cutoff parameters.

In the region of small magnitudes, the data has a smaller slope, *i.e.* a smaller parameter β. This roll-off effect in the Gutenberg-Richter distribution is usually assigned to the insufficiency in the determination of the small-magnitude data. The problem of the small-magnitude cutoff (completeness of earthquake catalogs) enjoys many discussions, especially in connection with the aftershocks and foreshocks sequences.[23]-[33] It was shown recently that the roll-off effect arises, at least partially, from earthquake dynamical correlations.[34]

On the other hand, for large magnitudes, the data distribution is uncertain. It is difficult to include high-magnitude earthquakes in a statistical analysis, because they are rare events and may not belong to a statistical ensemble. However, their minor weight in the statistical ensemble does not affect the results too much.

An analysis of a large set of global earthquakes with $5.8 < M < 7.3$ ($\Delta M = 0.1$) indicates $\beta = 1.38$ (and $1/t_0 = 10^{5.5}$ per year), corresponding to $r = 0.4$, a value which suggests an intermediate two/three-dimensional focal mechanism.[14] Equations (1), (2) and (5) have been fitted to a set of 1999 earthquakes with magnitude $M \geq 3$ ($\Delta M = 0.1$), which occurred in Vrancea between $1974 - 2004$ (31 years).[20, 21] The mean values of the fitting parameters are $-\ln t_0 = 9.68$ and $\beta = 1.89$ ($r = 0.54$). The same fit have been done for a set of 3640 earthquakes with magnitude $M \geq 3$ which occurred in Vrancea during $1981 - 2018$ (38 years). The fitting parameters for this set are $-\ln t_0 = 11.32$ and $\beta = 2.26$ ($r = 0.65$).[35] The fitting values given above have an error of approximately 17%. The data for Vrancea have been taken from Ref. [36].

In decimal logarithms the parameter β reads $\beta = br = \frac{3}{2}r$. Since $1/3 < r < 1$, this parameter varies in the range $1/2 < \beta < 3/2$. Usually, the average value $\beta = \frac{3}{2}r = 1$ ($\beta = 2.3$ in natural logarithms) is currently used as a reference value, corresponding to $r = 2/3$.[37]-[40]

Entropy of earthquakes. Let us assume that a region is loaded with seismic energy. This energy is released in time, by a sequence of earthquakes with various magnitudes M. We may associate a statistical distribution $\rho(M)$ to this seismic activity. The probability density $\rho(M)$

should be normalized,

$$\int dM\rho(M) = 1 \ , \tag{13}$$

and the mean magnitude should be a constant,

$$\int dM \cdot M\rho(M) = \overline{M} \ ; \tag{14}$$

this condition corresponds to the original load of energy, which is well determined. We may view a continuous loading, and a continuous energy release. Also, we may expect this continuous regime to be stationary (or quasi-stationary), such that the condition (14) is fulfilled. If the region is "free", or "isolated", *i.e.* it is not subject to external influences, and the earthquakes are independent events, it is reasonable to assume that the release of the seismic energy will be completed in a sufficiently long period of time, *i.e.,* in the conditions given above, the amount of released energy is maximal (and equal with the load). We seek a functional of ρ which attains its maximum value for a certain function ρ; this ρ corresponds to the seismic activity in that region. Let us introduce the functional

$$S = -\int dM \cdot \rho \ln \rho \ , \tag{15}$$

and look for its maximum value under the conditions (13) and (14); to this end we need a vanishing first-order variation of the (Lagrange) functional

$$-\int dM \cdot \rho \ln \rho - \alpha \int dM\rho(M) - \beta \int dM \cdot M\rho(M) \ , \tag{16}$$

where α and β are parameters which will be determined from equations (13) and (14). From equation (16) we get immediately the first-order variation

$$\int dM \left(-\ln \rho - 1 - \alpha - \beta M\right) \delta\rho(M) = 0 \ ; \tag{17}$$

hence,

$$\rho(M) = \beta e^{-\beta M} \tag{18}$$

and, by equation (14),

$$\overline{M} = \frac{1}{\beta} \tag{19}$$

(and $-1-\alpha = \ln \beta$). We can see (equation (18)) that we recover the Gutenberg-Richter magnitude distribution given by equation (1). The quantity S is called the entropy of the earthquakes. It characterizes the seismic activity of a seismic region in a long interval of time. This seismic activity proceeds in such a manner as to maximize the entropy; indeed, the second-order variation of equation (16) is

$$\int dM \cdot (-1/\rho) \left[\delta\rho(M)\right]^2 < 0 \ , \tag{20}$$

which shows that S is maximal with respect to ρ, for ρ given by equation (18). We may say, according to Statistical Physics, that this maximal S corresponds to the statistical equilibrium of the seismic activity in that region and in that (long) period of time. In Statistical Physics the distribution given by equation (18) is called canonical distribution, or Gibbs distribution.[41]

From equation (15) the entropy is given by

$$S = 1 - \ln \beta \ , \tag{21}$$

or (equation (16)),

$$S = \beta\overline{M} - \ln \beta \ . \tag{22}$$

we can see that

$$\left(\frac{\partial S}{\partial \beta}\right)_{\overline{M}} = 0 \ . \tag{23}$$

In equation (22) we can view β and $\overline{M}$ as independent variables, such that $\left(\partial S/\partial\overline{M}\right)_{\beta} = \beta$ and $dS = \beta d\overline{M}$. This latter relation (which defines an equilibrium transformation) indicates the physical meaning of the entropy: its changes are proportional to the changes in the mean magnitude.

The definition (15) may lead to another viewpoint as regards the entropy. We may view the seismic activity as consisting of a sequence M_i, $i = 1, 2, ...N$, of magnitudes, each realized by N_i random, independent processes (in Statistical Physics this is Boltzmann's hypothesis of the so-called "molecular chaos"); the probability ρ_i of each of these processes is $\rho_i = 1/N_i$. We may define the entropy $s_i = -\ln\rho_i = \ln N_i$ and the average entropy $\overline{s}_i = -N_i\rho_i\ln\rho_i = \ln N_i = s_i$. It follows that the probability ρ_i is given by $\rho_i = e^{-s_i} = e^{-\overline{s}_i}$. In Statistical Physics this distribution is called the microcanonical distribution, and $s_i = \overline{s}_i$ is called microcanonical entropy.[42] Obviously, this entropy is maximal under the condition of normalized probabilities ρ_i: ρ_i is a constant for each M_i. We may use $\rho_i = \beta e^{-\beta M_i}$, and get the microcanonical entropy

$$s_i = \overline{s}_i = \beta M_i - \ln\beta \ . \tag{24}$$

Since the processes are independent and random, irrespective of their magnitude, their probability ρ_m is given by

$$\rho_m^N = \prod_i \rho_i = e^{-\sum_i \overline{s}_i} \ , \tag{25}$$

whence the entropy

$$S = -\ln\rho_m = \frac{1}{N}\sum_i \overline{s}_i = \beta\overline{M} - \ln\beta \tag{26}$$

(where N is the number of the i-processes). We can see that S, given by equation (26) is the canonical entropy given by equation (22). The microcanonical probability

$$\rho_m = e^{-S} = \beta e^{-\beta\overline{M}} \tag{27}$$

is the value of the canonical probability for the mean magnitude.

At equilibrium the entropy is maximal ($(\partial S/\partial\beta)_{\overline{M}} = 0$, equation (23)); therefore, we may have small variations $\delta S = \frac{1}{2}(\partial^2 S/\partial\beta^2)_{\overline{M}}(\delta\beta)^2 = (\delta\beta)^2/2\beta^2$. This variation is positive. It indicates the tendency of departing from equilibrium. The probability ρ_m gives the fluctuation probability

$$\rho_f = \frac{1}{\sqrt{2\pi}\beta} e^{-\frac{(\delta\beta)^2}{2\beta^2}} \tag{28}$$

(properly normalized). We can see that the equlibrium may have fluctuations, whose measure is the standard deviation

$$\Delta\beta = \left[\overline{(\delta\beta)^2}\right]^{1/2} = \beta \tag{29}$$

given by the normal distribution in equation (28). This is Einstein's fluctuation formula.[43] If in a distribution $\overline{M}e^{-\beta\overline{M}}$ we may view the parameter β as a statistical variable, then we get the standard deviation $\Delta\beta = 1/\overline{M}$, which coincides with equation (29) for $\beta\overline{M} = 1$. A similar analysis can be made for the canonical distribution $\beta e^{-\beta M}$, leading to fluctuations $\Delta M = 1/\beta = \overline{M}$. Various realizations of the statistical equilibrium (in the same conditions) exhibit fluctuations, *i.e.* for another sampling of equilibrium, *i.e.* for another region, or another (long) period of time, with the same mean magnitude, *i.e.* for another realization of the statistical ensemble at equilibrium, the entropy may fluctuate.

Standard deviations as large as the mean value indicate a serious limitation of the information we may get from statistical analysis of earthquake distributions. In addition, the "same conditions" requirement of the fluctuation formula (known as the "null hypothesis") may not be fulfilled; for instance, over a similarly (long) period of time, the geological conditions of the seismic region may change, or the accuracy of the measured magnitudes may differ. While the information about the evolution of the entropy over long times are not very useful, the changes in the entropy over shorter times are expected to be more relevant in short-term prediction of earthquakes.

Short-term prediction scenario. Let us assume that we fitted the Gutenberg-Richter distribution (*e.g.*, the cumulative distribution, equation (2)) to data gathered over a long period of time t_0 for a given region. At the moment of time t_0 we have the fitting parameter β_0. For a sufficiently long period of time t_0 we may assume that this seismic activity is statistically well-defined. Let us take (at random) the next moments of time t_i, $i = 0, 1, 2, ...N$ and update the fitting to get the parameters β_i. If the quasi-equilibrium is preserved, for time intervals $t_{i+1} - t_i$ sufficiently small (but still as large as to have a measurable seismic activity in each) we may assume the the variations $\delta\beta_i = \beta_{i+1} - \beta_i$ are fluctuations. For a sufficiently large N we may fit their distribution with the normal law given by equation (28). Thus, we get the fitting parameter β. If $\beta = \beta_0$ (within the fitting errors), the seismic equilibrium has not changed. If $\beta \neq \beta_0$ the equilibrium has changed over the period $t_N - t_0$. Consequently, we may expect a tendency to recover the equilibrium over the next period of time $t_{2N+1} - t_{N+1}$. Such an information may be regarded as a short-time prediction. For instance, if $\beta < \beta_0$, we may expect in the next time interval a decrease in the mean magnitude, *i.e.* the number of earthquakes with low magnitudes will increase, and high-magnitude earthquakes are not likely. On the contrary, if $\beta > \beta_0$, then we may expect an increase in the number of earthquakes with higher magnitude. We note that an increase (decrease) in β amounts to a decrease (increase) in equilibrium entropy (equation (21)).

It may happen that the distribution of the parameter changes $\delta\beta_i$ is not a normal distribution. Then, the ensemble is not at equilibrium in the time period $t_N - t_0$. Under the equilibrium hypothesis, we may expect an evolution towards equilibrium in the next period. For instance, if the distribution of the variations $\delta\beta_i$ is shifted towards higher values, *i.e.*, if the parameters β_i show a tendency to increase, we may expect a decrease of these parameters in the next period, *i.e.* an increase in the mean magnitude.

It is worth noting that the prediction scenario described above is valid under the main asumption of independent, random seismic events. If correlations exist, the entropy formulae derived above do not apply. A special case in this connection is the short-term foreshock (and aftershock) activity, which can be viewed as fluctuations in magnitude. We may say that a certain sequence of seismic events is a foreshock sequence only aposteriori, when these events are followed by a main shock. On the other hand, the accompanying seismic activity still obeys (approximately) the Gutenberg-Richter magnitude distribution,[44] and a decrease in the parameter β observed for the foreshock activity was interpreted as an increase in equilibrium entropy.[8] However, such an observation cannot be used as a (very) short-time prediction, without an apriori knowledge. In general, we may say that the particular character of the high-magnitude seismic events and their accompanying (associated) seismic activiy of foreshocks and aftershocks is not very relevant for the prediction method described herein.

Also, we note that the practical application of the prediction procedure described above depends on the choice of the (long) time period t_0, the (short) time intervals $t_{i+1} - t_i$ and the (large) number N of these intervals. This choice can only be made in close connection with the particular character of the seismic activity in the given region and in the given (long) time period.

Unfortunately, the scenario described above is not supported by data, at least for Vrancea region. The variations in the parameter β are quasi-uniformly increasing in time, due to the accumulation of small-magnitude earthquakes. This quasi-uniform tendency is interrupted from time to time by large-magnitude earthquakes, which decrease appreciably the parameter β. Two examples of short-time variations of beta are given in Fig. 1 for Vrancea seismic activity. The time t_0 is from January 1*st* 1980 to December 31*st* 2009, with 5396 earthquakes with magnitude greater than $M \geq 2$. The Gutenberg-Richter fit to these data gives $\beta_0 = 2.121$ (error 17%). We have updated the parameter β for each week of the year 2010 (Fig. 1, panel a). We can see that this parameter increases continuously over the whole this year (due to the accumulation of small-magnitude earthquakes). A similar procedure was used for each of the next years up to 2019 (8498 earthquakes with $M \geq 2$ in the whole period $1980 - 2019$, taken from Ref. [36]). In some years the continuous increase of the parameter β is disrupted by the occurrence of greater-magnitude earthquakes, like in the year 2011 (Fig. 1, panel b). Such variations of the parameter β cannot be fitted by a normal distributions, and, therefore, they cannot be viewed as fluctuations. We can only say, very imprecisely and qualitatively, that after a period of small-magnitude seismic activity it is likely to follow a few earthquakes with greater magnitude, and viceversa, which is a useless, common-sense expectation. Rigorously speaking, the seismic activity is not at (quasi-) equilibrium, because the tectonic energy source feeds it continuously.

Concluding remarks. The Gutenberg-Richter statistical distribution in magnitude is derived for a canonical ensemble by the standard procedure of maximizing the quasi-equilibrium entropy. By assuming the seismic activity as consisting of a sequence of random, independent earthquakes with various magnitudes, the corresponding microcanonical ensemble is examined by standard methods and the fluctuations normal distribution is derived for earthquakes (Einstein's fluctuation formula). The existence of fluctuations may induce the idea of a possible short-term prediction, based on the analysis of these fluctuations. This scenario is examined here for the seismic activity in Vrancea, and found inconsistent. The seismic activity is feeded continuously by the tectonic energy and, consequently, it cannot be viewed as a quasi-equilibrum statistical ensemble. The Gutenberg-Richter distribution is shifted continuously towards small-magnitude earthquakes, with random re-arrangements caused by higher-magnitude earthquakes.

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