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Earthquake correlations and Bath's law

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Abstract

Within the framework of the geometric-growth model of energy accumulation in the focal region we derive the single-event (Gutenberg-Richter) and the two-event (bivariate, pair) statistical distributions of earthquakes. In earthquake clusters, consisting of foreshock-main shock-aftershock sequences, we identify dynamical correlations, associated, mainly, with high-magnitude main shocks, and purely statistical correlations, associated with moderate-magnitude main shocks. It is shown that the dynamical correlations may account, at least partially, for the roff-off effect in the Gutenberg-Richter distributions. The appropriate tool of approaching the accompanying seismic activity (foreshocks and aftershocks) is the pair distribution function for the difference in magnitude, where the magnitude difference is allowed to take negative values. The seismic activity which accompanies a main shock can be viewed as fluctuations in magnitude, with a vanishing mean value of the magnitude difference and the standard deviation as a measure of the average difference in magnitude between the main shock and the greatest aftershock or foreshock (Bath's law). Making use of the magnitude-difference distribution we derive the Bath's law and discuss statistical correlations in earthquake distributions. Deterministic time-magnitude correlations are also presented.

1 Introduction

The Bath's law states that the average difference ΔM between the magnitude of a main shock and the magnitude of its largest aftershock is independent of the magnitude of the main shock.[1, 2] The reference value of the average magnitude difference is $\Delta M = 1.2$. Deviations from this value have been reported (see, for instance, Refs. [3]-[5]), some being discussed in Ref. [1].

The earliest advance in understanding the origin of the empirical Bath's law was made in Ref. [6], where the main shock and its aftershocks were viewed as members of the same statistical ensemble, distributed in magnitude. The magnitude-difference distribution introduced in Ref. [6] may include correlations, which are viewed sometimes as indicating that the main shocks are statistically distinct from the aftershocks, or the foreshocks.[7, 8] The Bath's law enjoyed many discussions and attempts of elucidation.[9]-[15] The prevailing opinion ascribes the variations in ΔM to the bias in selecting data and the insufficiency in the realizations of the statistical ensemble. This standpoint was substantiated by means of the binomial distribution.[3, 5, 15] In order to account for the deviations of ΔM the ETAS (epidemic-type aftershock sequence) model for the differences in the selection procedure of the mainshocks and the aftershocks was employed.[15]

According to this model, the variations in the number ΔM are related to the realizations of the statistical ensemble and the values of the fitting parameters (see also Refs. [3, 5]).

We show in this paper that the appropriate tool of approaching the accompanying seismic activity (foreshocks and aftershocks) of the main shocks is the distribution function of the difference in magnitude. This distribution is derived herein by using the pair (two-event, bivariate) distribution, as well as by means of the conditional probabilities (the Bayes theorem). Making use of the pair distribution, we identify dynamical correlations in the foreshock-main shock-aftershock sequences, apart from purely statistical correlations. The dynamical correlations arise from an "earthquake interaction", *i.e.* an exchange (transfer) of energy (*e.g.* a static stress) between focal regions. The dynamical correlations may account, at least partially, for the roll-off effect in the small-magnitude region of the Gutenberg-Richter statistical distributions. By using the pair distribution we are led to extend the difference in magnitude to negative values, thus obtaining a symmetric distribution for the foreshocks and aftershocks. Such a magnitude-difference distribution has a vanishing mean value for the magnitude difference. This suggests to view the accompanying seismic activity as representing fluctuations in magnitude, and to take their standard deviation as a measure for the Bath's average difference ΔM between the magnitude of the main shock and its largest aftershock (foreshock). This way, the Bath's law is derived. In addition, it is suggested that moderate-magnitude doublets may be viewed as "Bath partners". Also, deterministic time-magnitude correlations in the associated seismic activity are presented.

2 Single-event distributions

According to the geometric-growth model of energy accumulation in a localized focal region,[16] the accumulated energy E is related to the accumulation time t by

$$1 + t/t_0 = (1 + E/E_0)^r \quad , \tag{1}$$

where t_0 and E_0 are time and energy thresholds and r is a geometrical parameter. This parameter is related to the reciprocal of the number of effective dimensions of the focal region and to the anisotropic strain accumulation rate. The parameter r varies in the range $1/3 < r < 1$. An average parameter r may take any value in this range.

The threshold parameters should be viewed as very small, such that t/t_0, $E/E_0 \gg 1$; equation (1) may be written as

$$t/t_0 \simeq (E/E_0)^r \quad . \tag{2}$$

A uniform frequency of events $F(t) = t_0/t$ in time t indicates that $1/t_0$ is the seismicity rate. The time and energy distributions are

$$P(t) = -\frac{\partial F(t)}{\partial t} = \frac{t_0}{t^2} \ , \ P(E) = \frac{rE_0^r}{E^{1+r}} \ . \tag{3}$$

Making use of $E/E_0 = e^{bM}$, $t/t_0 = e^{\beta M}$, where M is the magnitude, $\beta = br$ and $b = \frac{3}{2} \cdot \ln 10 = 3.45$ (according to Ref. [17]), we get the Gutenberg-Richter magnitude distribution[18]

$$P(M) = \beta e^{-\beta M} \quad . \tag{4}$$

Equation (4) is used to fit the empirical distribution $P(M) = \Delta N/N_0 \Delta M$ of ΔN earthquakes with magnitude in the range $(M, M + \Delta M)$ out of the total number of earthquakes $N_0 = T/t_0$ which occurred in time $T \gg t_0$. Also, the cumulative distribution $P_{ex}(M) = e^{-\beta M}$ of all the earthquakes with magnitude greater than M is used io fit the empirical excedence rate $P_{ex}(M) = N_{ex}/N_0$, usually in the logarithmic form $\ln N_{ex} = \ln N_0 - \beta M$. The fiting parameter β (the slope of the logarithmic excedence rate) varies from region to region and depends on the period of time T and the size of the data set. Its variation range is $1.15 < \beta < 3.45$ (in decimal logarithms $0.5 < \beta < 1.5$), in agreement with the theoretical range given by $\beta = br$, $1/3 < r < 1$. Usually, the average value $\beta = 2.3$ ($r = 2/3$) is taken as the reference value.[19, 20]

3 Bivariate distribution

Apart from regular (background) earthquakes, there exist earthquakes which are preceded by foreshocks and followed by aftershocks. Such earthquake clusters consist of a main shock, with magnitude M_s, and accompanying (associated) foreshocks and aftershocks with magnitude M smaller than M_s ($M < M_s$). Very likely, since such sequence earthquakes are associated in time and space, they are correlated. In general, correlations are included in bivariate (two-event, pair) distributions, which are given by the mixed second-order derivative of a generating function of two variables. Let us assume that two successive earthquakes may occur in time t, one after time $t_1 = t_0 e^{\beta M_1}$, another after time $t_2 = t_0 e^{\beta M_2}$ from the occurrence of the former. Using the partition $t = t_1 + t_2$ and the generating function $F(t_1, t_2) = t_0/(t_1 + t_2)$ given above, we get the distribution

$$P(t_1, t_2) \sim \frac{\partial^2 F}{\partial t_1 \partial t_2} = \frac{2t_0}{(t_1 + t_2)^3} \ , \tag{5}$$

or, properly normalized,

$$P(M_1, M_2) = 4\beta^2 \frac{e^{\beta(M_1+M_2)}}{\left(e^{\beta M_1} + e^{\beta M_2}\right)^3} \ . \tag{6}$$

This distribution is different from $P(M_1)P(M_2) = \beta^2 e^{-\beta(M_1+M_2)}$, which indicates that the two events $M_{1,2}$ are correlated. Making use of the notation $M_1 = M_2 + m$, we get

$$P(M_1, M_2) = 4\beta^2 \frac{e^{-\beta max(M_1,M_2)} e^{-\beta|m|}}{\left(1 + e^{-\beta|m|}\right)^3} \ , \tag{7}$$

where $\mid m \mid < max(M_1, M_2)$. Equation (7) highlights the magnitude-difference distribution in the variable m. If we integrate this distribution with respect to the variable M_2 (and redefine $M_1 = M$), we get the marginal distribution

$$P^{mg}(M) = \beta e^{-\beta M} \frac{2}{\left(1+e^{-\beta M}\right)^2} \tag{8}$$

and the corresponding cumulative distribution

$$P^{mg}_{ex}(M) = e^{-\beta M} \frac{2}{1+e^{-\beta M}} \ , \tag{9}$$

We can see in these equations the presence of the single-event distribution$\sim e^{-\beta M}$.

4 Dynamical correlations

The bivariate distribution derived above exhibits an interesting particularity: in the limit of small magnitudes the cumulative marginal distributions can be written as

$$P^{mg}_{ex}(M) = e^{-\beta M} \frac{2}{1+e^{-\beta M}} \simeq e^{-\beta M} \frac{1}{1-\frac{1}{2}\beta M} \simeq e^{-\frac{1}{2}\beta M} \ . \tag{10}$$

We can see that the slope β of the logarithmic cumulative Gutenberg-Richter distribution is changed into the smaller slope $\beta/2$ in the region of small magnitudes. Such a deviation (called the roll-off effect) is well known in empirical studies;[21, 22] it is attributed usually to an insufficient determination of the small-magnitude data. We can see that it is due, at least partially, to correlations. Since the great majority of earthquakes is concentrated on small magnitudes, we can say that there exists a sub-set of earthquakes governed by the single-event distribution

$$P_c(M) = \frac{1}{2}\beta e^{-\frac{1}{2}\beta M} \ . \tag{11}$$

This distribution can be derived from a time-energy accumulation law $t/t_0 = (E/E_0)^{r/2}$ (where the parameter r is changed into $r/2$, equation (2)); according to this law, the same energy is accumulated in a shorter time (in comparison with the r-law). Very likely, these correlations imply an interaction between focal regions (an "earthquake interaction"), as, for instance, an exchange (transfer) of static stress; we call this type of correlations dynamical correlations. Since the large-magnitude main shocks have a large productivity of accompanying small-magnitude seismic events, the dynamical correlations belong, mainly, to clusters with high-magnitude main shocks. Making use of the empirical distributions, it is easy to find the relationship $N_c^2 = (4\Delta N_c^2/\Delta N \Delta M)N_0$, where N_c and N_0 is the total number of dynamically-correlated earthquakes and the rest of earthquakes (governed by the Gutenberg-Richter distribution $\sim e^{-\beta M}$), respectively. We can see that $N_c \sim \sqrt{N_0}$, like the statistical deviation. Since $N_c \ll N_0$, the dynamically correlated earthquakes do not affect much the Gutenberg-Richter distribution, except for small magnitudes.

The bivariate distribution given above can be written both for the earthquakes governed by the Gutenberg-Richter distribution $\sim e^{-\beta M}$ and for the sub-set of dynamically-correlated earthquakes governed by the distribution $\sim e^{-\frac{1}{2}\beta M}$. The procedure of extracting dynamically-correlated earthquakes can be iterated, passing from $\beta/2$ to $\beta/4$, etc; however, the number of affected earthquakes tends rapidly to zero, and the procedure becomes irrelevant.

5 Bath's law

Let M_s and M be the magnitudes of the main shock and an acompanying earthquake (foreshock or aftershock), respectively. We define the ordered magnitude difference $m = M_s - M > 0$ for foreshocks and $m = M - M_s < 0$ for aftershocks, such that $\mid m \mid < M_s$. According to equation (7), the bivariate distribution of the pair consisting of the main shock and an acompanying event is

$$P(M_s, m) = 4\beta^2 e^{-\beta M_s} \frac{e^{-\beta|m|}}{\left(1 + e^{-\beta|m|}\right)^3} \ . \tag{12}$$

This distribution is symmetric with respect to the change aftershocks ($m < 0$)-foreshocks ($m > 0$).

First, we apply this distribution to the dynamically-correlated earthquakes; to this end, we replace β in equations (12) by $\beta/2$. Since the exponential $e^{-\frac{1}{2}\beta|m|}$ falls off rapidly to zero with increasing $\mid m \mid$, we may discard it in the denominator in equation (12). More, for large M_s we may also discard the condition $\mid m \mid < M_s$ and let $\mid m \mid$ go to infinity. We get

$$P(M_s, m) \simeq \tfrac{1}{8}\beta^2 e^{-\frac{1}{2}\beta M_s} e^{-\frac{1}{2}\beta|m|} \ . \tag{13}$$

We can see that that the events M_s and $\mid m \mid$ are independent in this distribution. The only correlations left in equation (13) are the dynamical correlations;.we may use the independent magnitude-difference distribution

$$p(m) = \frac{1}{4}\beta e^{-\frac{1}{2}\beta|m|} \ . \tag{14}$$

It is worth noting that this distribution can also be derived from conditional probabilities (and Bayes theorem). Indeed, since $M_1 = M_1 - M_2 + M_2$ and $M_2 = M_2 - M_1 + M_1$, the law $\sim e^{-\frac{1}{2}\beta M}$ suggests a magnitude-difference distribution $\sim e^{-\frac{1}{2}\beta(M_1-M_2)}$ for $M_1 > M_2$ and fixed M_2, and a distribution $\sim e^{-\frac{1}{2}\beta(M_2-M_1)}$ for $M_2 > M_1$ and fixed M_1. In both cases, these distributions can be written as $\sim e^{-\frac{1}{2}\beta|m|}$, where $m = M_1 - M_2$ (or $m = M_2 - M_1$), $\mid m \mid < max(M_1, M_2)$, irrespective of which $M_{1,2}$ is fixed.

Making use of the distribution $p(m)$, the mean value $\overline{m}$ is zero ($\overline{m} = 0$). The next correction to this mean value, *i.e.* the smallest deviation of m, is the standard deviation $\Delta m = \sqrt{\overline{m^2}}$. Using equation (14) we get $\overline{m^2} = 8/\beta^2$. We may conclude that the average difference in magnitude between the main shock and its largest aftershock (or foreshock) is given by

$$\Delta M = \Delta m = \frac{2\sqrt{2}}{\beta} \ . \tag{15}$$

This is the Bath's law. The number $2\sqrt{2}/\beta$ does not depend on the magnitude M_s (but it depends on the parameter β, corresponding to various realizations of the statistical ensemble). It is worth noting that Δm given by equation (15) implies an averaging (of the squared magnitude differences). Making use of the reference value $\beta = 2.3$ we get $\Delta M = 1.23$, which is the Bath's reference value for the magnitude difference. In the geometric-growth model the reference value $\beta = 2.3$ corresponds to the parameter $r = 2/3$. We can check that higher-order moments $\overline{m^{2n}}$, $n = 2, 3, ...$ are larger than $\overline{m^2}$ (for any value of β in the range $1.15 < \beta < 3.45$).

If we extend the dynamical correlations to moderate-magnitude mainshocks, we keep the condition $\mid m \mid < M_s$; this condition accounts for purely statistical correlations; we get $\Delta M = \sqrt{2}/\beta$, which leads to $\Delta M \simeq 0.61$ for the reference value $\beta = 2.3$. Such a variablility of ΔM can often be found in empirical studies. For instance, from the analysis made in Ref. [3] of Southern California earthquakes 1990-2001 we may infere $\beta \simeq 2$ and an average $\Delta M \simeq 0.45$ (with large errors); from Ref. [5], New Zealand catalog (1962-1999) and Preliminary Determination of Epicentres catalog (1973-2001), we may infere $\beta \simeq 2.5 - 2.3$ and an average $\Delta M = 0.43 - 0.54$, respectively, while $\Delta M = \sqrt{2}/\beta$ gives $0.56 - 0.61$. In other cases, like the California-Nevada data analyzed in Ref. [4], the parameters are $\beta = 2.3$ and $\Delta M \simeq 1.2$, in agreement with the formula given in equation (15). We note that $\Delta M = \sqrt{2}/\beta$ given here is an over-estimate, because it extends, in fact, the dynamical correlations (equation (11)) to small-magnitude main shocks.

Leaving aside the dynamical correlations we are left with purely statistical correlations for clusters with moderate-magnitude main shocks. Purely statistical correlations may appear as a result of "unknown causes". For instance, an earthquake may produce changes in the neighbourhood of its focal region (adjacent regions), and these changes may influence the occurrence of another earthquake. Similarly, an associated seismic activity may be triggered by a "dynamic stress", not a static one.[23] "Unknown causes" is used here in the sense that the model employed for describing these earthquakes does not account for such causes. For moderate-magnitude main shocks we may still discard the exponential in the denominator in equations (12), but keep the condition $\mid m \mid < M_s$. We get

$$P(M_s, m) \simeq \beta^2 e^{-\beta M_s} e^{-\beta|m|} \tag{16}$$

and $\Delta m = 1/\sqrt{2}\beta$. For the reference vale $\beta = 2.3$ we get $\Delta m \simeq 0.31$. The Bath partner for such a small value of the magnitude difference looks rather as a doublet.[24, 25]

The correlation coefficient (covariance) $R = \overline{M_s M}/\Delta M_s \Delta M$ between the main shock and an accompanying event $M = M_s - \mid m \mid$ ($\mid m \mid < M_s$) can be computed by using the distribution given in equation (16). We get $R = 2/\sqrt{5}$. For the correlation coefficient beteen two accompanying events M_1 and M_2 we need the three-events distribution (which includes $M_{1,2}$ and M_s). This distribution can be derived in the same manner as the pair distribution given by equation (5).

6 Time-magnitude correlations

Let us assume that an amount of energy E accumulated in time t is released by two successive earthquakes with energies $E_{1,2}$, such as $E = E_1 + E_2$. Since, according to equation (2),

$$\begin{gathered} t/t_0 = (E/E_0)^r = (E_1/E_0 + E_2/E_0)^r < \\ < (E_1/E_0)^r + (E_2/E_0)^r = t_1/t_0 + t_2/t_0 \ , \end{gathered} \tag{17}$$

where $t_{1,2}$ are the accumulation times for the energies $E_{1,2}$, we can see that the time corresponding to the pair energy is shorter than the sum of the independent accumulation times of the members of the pair. This indicates correlations, which may be expected in earthquake clusters. This is another type of correlations, different from dynamical or purely statistical correlations in magnitude. They are deterministic time-magnitude correlations, arising from the non-linearity of the accumulation law given by equation (2). The time interval τ between the two successive earthquakes,

$$\tau = t_1 \left[(1 + E_2/E_1)^r - 1 \right] \ , \tag{18}$$

given by $t = t_1 + \tau$, depends on the accumulation time t_1. If we introduce the magnitudes $M_{1,2}$ in equation (18), we get

$$\tau = t_1 \left[\left(1 + e^{-bm} \right)^r - 1 \right] \ , \tag{19}$$

where $m = M_1 - M_2$. We can see that this equation relates the time τ to the magnitude difference. The same equation can be applied to dynamically-correlated earthquakes, by replacing r by $r/2$. We get

$$\tau = t_1 \left[\left(1 + e^{-bm} \right)^{r/2} - 1 \right] \ , \tag{20}$$

These correlations can be called time-magnitude correlations.

We apply this equation to a main shock-aftershock sequence, where M_1 is the magnitude of the main shock ($m > 0$); similar results are valid for the foreshock-main shock sequence. For the largest aftershock, where m may be replaced by $\Delta m = 2\sqrt{2}/\beta$, we get

$$\begin{gathered} \tau_0 = t_1 \left[\left(1 + e^{-b\Delta m} \right)^{r/2} - 1 \right] \simeq \\ \simeq \frac{1}{2} r t_1 e^{-b\Delta m} = \frac{1}{2} r t_1 e^{-2\sqrt{2}/r} \end{gathered} \tag{21}$$

(for $b\Delta m \gg 1$). This is the occurrence time of the Bath partner, measured from the occurrence of the main shock. The ratio τ_0/t_1 varies between 3.5×10^{-5} ($r = 1/3$) and 3×10^{-2} ($r = 1$); for $r = 2/3$ we get $\tau_0/t_1 = 5 \times 10^{-3}$.

It is worth noting, according to equation (20), that a partner close to the main shock in magnitude ($bm \ll 1$) occurs after a lapse of time

$$\Delta t \simeq t_1 \left(2^{r/2} - 1 \right) \ , \tag{22}$$

which is much greater than τ_0 ($\Delta t/t_1$ varies between 0.12 and 0.41 for $1/3 < r < 1$). We can see that, even if the pair probability $p(m) = (\beta/4)e^{-\frac{1}{2}\beta|m|}$ is greater for $m = 0$, an earthquake

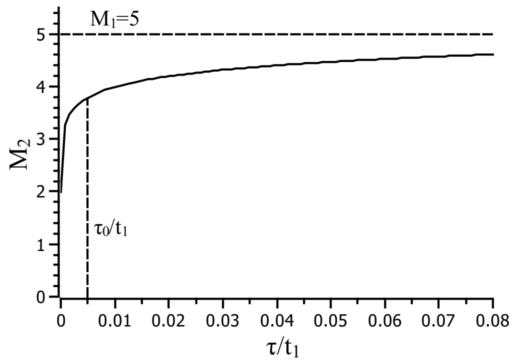

Figure 1: The magnitude M_2 of the accompanying seismic events *vs* the time τ elapsed from the main event with magnitude M_1 and accumulation time t_1 (equation (23) for $M_1 = 5$, $b = 3.45$, $r = 2/3$). The Bath partner $M_2 \simeq 3.8$ corresponds to $\tau_0/t_1 \simeq 5 \times 10^{-3}$. Higher values of the magnitude M_2 occur at much longer times, where the correlations are unlikely.

close in magnitude to the main shock occurs much later, where it may be difficult to view it as an aftershock. Since $\left(1 + e^{-bm}\right)^{r/2}$ (in equation (20)) is a decreasing function of m, we can say, indeed, that the largest aftershock is farther in time with respect to the main shock in comparison with aftershocks lower in magnitude. The duration τ_0 given by equations (21) for the occurrence of the largest aftershock may be taken as a measure of the extension in time of the aftershock (and the foreshock) activity. It may serve as a criterion for defining the accompaning seismic activity.

From equation (20) we can get the distribution of the magnitudes M_2 of the acompanying earthquakes with respect to the time τ, measured from the occurrence of the main shock with magnitude M_1, either in the future (aftershocks) or in the past (foreshocks). Indeed, we get from equation (20)

$$M_2 = M_1 + \frac{1}{b} \ln \left[(1 + \tau/t_1)^{2/r} - 1 \right] \ , \tag{23}$$

where t_1 $(= t_0 e^{\frac{1}{2}\beta M_1})$ is the accumulation time of the main shock; M_2 in equation (23) is defined for $\left(1 + e^{-bM_1}\right)^{r/2} - 1 < \tau/t_1 < 2^{r/2} - 1$ $(0 < M_2 < M_1)$. The function M_2 is plotted in Fig. 1 *vs* τ/t_1 for $b = 3.45$, $r = 2/3$ $(\beta = 2.3)$ and $M_1 = 5$. For τ/t_1 very close to zero the magnitude M_2 is vanishing, and for $\tau/t_1 \longrightarrow 2^{r/2} - 1$ the magnitude M_2 tends to M_1; the Bath partner occurs at $\tau_0/t_1 \simeq \frac{1}{2} r e^{-2\sqrt{2}/r} \simeq 5 \times 10^{-3}$ with the magnitude $M_2 = 3.8$. The function $M_2(\tau/t_1)$ is a very steep function, for the whole (reasonable) range of parameters; the whole accompanying seismic activity is, practically, concentrated in the lapse of time τ_0. On the scale τ/t_1 the pair probability of this activity is an abruptly increasing function of M_2. If we use M_2 given by equation (23) in the distribution $\frac{1}{2}\beta e^{-\frac{1}{2}\beta M_2}$ for small values of τ/t_1, we get an Omori-type law, as expected.

Finally, we note that the aftershock (foreshock) magnitude M_2 given by equation (23) can be approximated by

$$M_2 \simeq \left(1 - \frac{1}{2} r\right) M_1 + \frac{1}{b} \ln \left(\frac{2\tau}{r t_0}\right) \tag{24}$$

for

$$\frac{1}{2} r e^{-(1-r/2)bM_1} < \tau/t_0 < \left(2^{r/2} - 1\right) e^{\frac{1}{2}\beta M_1} \ , \tag{25}$$

where t_0 is the cutoff time. The lower bound $\frac{1}{2} r t_0 e^{-(1-r/2)bM_1}$in equation (25) corresponds to a (very small) quiescence time elapsed from the occurence of the main shock $(M_2 = 0)$. This time is much shorter than the cutoff time t_0, so, in fact, it is irrelevant. The cumulative fraction of aftershocks (foreshocks) with magnitude from zero to M_2 is $N_{cum}/N = 1 - e^{-\frac{1}{2}\beta M_2}$, where N is the total number of aftershocks (foreshocks). Making use of equation (24) we get the cumulative fraction for time τ

$$N_{cum}/N = 1 - \left(\frac{rt_0}{2\tau}\right)^{r/2} e^{-\frac{1}{2}(1-r/2)\beta M_1} \ . \tag{26}$$

This fraction is a rapidly increasing function of time, as it was pointed out recently.[26] The cutoff time, which is necessary in equation (26), remains an empirical parameter.

7 Concluding remarks

In foreshock-main shock-aftershock sequences of associated (accompanying) earthquakes we can discerne two types of correlations. One type, which we call dynamical correlations, imply an "interaction between earthquakes", *i.e.* an interaction between their focal regions (*e.g.*, a static stress). Another type consists of purely statistical correlations. The single-event distribution for the dynamical correlations is derived by analyzing the bivariate (two-event, pair) distribution. It is a Gutenberg-Richter-type exponential law with the parameter β changed into $\beta/2$. This change reflects a roll-off effect in the small-magnitude region of the Gutenberg-Richter distribution, related, mainly, to clusters with high-magnitude main shocks.

The correlations are discussed by means of the magnitude-difference distribution for earthquake pairs, where the difference in magnitude is extended to negative values. This distribution has a vanishing mean value of the magnitude difference, such that the foreshock-aftershock seismic activity appears as fluctuations in magnitude. The corresponding standard deviation is the average diference in magnitudes between the main shock and its greatest aftershock (foreshock). This difference in magnitude is given by $\Delta M = 2\sqrt{2}/\beta$ for dynamically correlated earthquakes, which leads to $\Delta M = 1.2$ for the reference value $\beta = 2.3$ (1 for decimal logarithms). If the purely statistical correlations are included, the difference in magnitude is, approximately, $\Delta M = \sqrt{2}/\beta$. The difference between these two formulae and the variability of the parameter β may explain the variability in the results of the statistical analysis of empirical data for ΔM. The purely statistical correlations for moderate-magnitude main shocks lead to smaller values $\Delta M = 1/\sqrt{2}\beta$, where the Bath partner looks rather as a doublet.

Also, deterministic time-magnitude correlations are discussed within the framework of the geometric-growth model of energy accumulation. The time delay between the main shock and its largest aftershock (foreshock) is estimated; it is suggested to use this time interval as a criterion of estimating the temporal extension of the aftershock (foreshock) activity. The magnitude distribution in time of the accompanying seismic activity is also presented.

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