

On the local magnitude scale of earthquakes

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Abstract

A local magnitude scale is defined for earthquakes by means of the amplitudes of the P and S seismic waves and their inversion for the seismic-moment tensor. The seismic waves are derived by solving the elastic wave equation with a localized shearing fault source. The inversion is achieved by means of the dyadic representation of the focus and the energy conservation. The scale includes the distance from the focus to the observation point.

Key words: magnitude of earthquakes; local scale of magnitude; seismic-moment magnitude

Introduction. By means of the seismographs we are able to locate the position of earthquakes' focus and the position of the epicentre. For typical earthquakes the focus is localized in space, in a region with dimensions much smaller than the distances of interest. For instance, with a reasonable accuracy, the dimension l of the focus is beyond $1km$. More accurate estimations give $40m - 400m$. It seems that in Vrancea there exist two active focal regions, one placed around $130km$ depth, another at $90km$ depth; in addition, there exist crustal earthquakes in Vrancea, with a focal depth around $5-20km$. Many of the surface earthquakes exhibit an extended focal rupture. The seismographs are placed on Earth's surface, at distances much larger than the dimension of the focal region, such that it is reasonable to assume that the earthquake focus is represented by a function $\delta(\mathbf{R} - \mathbf{R}_0)$, or its derivatives, where \mathbf{R} is the position of the observation point and \mathbf{R}_0 is the position of the focus.

Moreover, the seismic activity in the focus, during an earthquake, is much shorter than the durations of interest. Very likely, the seismic stress accumulated in the focus overcomes the rocks resistance, and it is discharged by a sudden rupture. Consequently, it is reasonable to describe the seismic activity in the focus by a function $T\delta(t)$, where T is a measure of the duration of the seismic activity.

The force density acting in the focus during an earthquake is

$$f_i = M_{ij}T\delta(t)\partial_j\delta(\mathbf{R} - \mathbf{R}_0) , \quad (1)$$

where $i, j = 1, 2, 3$ are cartesian labels and the symmetric tensor M_{ij} is the tensor of the seismic moment. This tensor indicates that the focus is a shearing fault. Explosions proceed by an isotropic dilatation, where $M_{ij} = -M\delta_{ij}$. The two mechanisms are exclusive to one another. We call the earthquakes with the focal mechanism described by equation (1) elementary earthquakes.

The elastic wave equation with the source given by equation (1) has been solved in Ref. [2] for a homogeneous isotropic elastic medium (also, the static deformations produced by the force

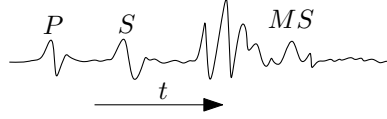


Figure 1: The P and S seismic waves and the mainshock (MS).

density $M_{ij}\partial_j\delta(\mathbf{R} - \mathbf{R}_0)$ have been solved[3]). In the far-field region the solutions (*i.e.* the elastic displacement) are two spherical-shell waves, with thickness $c_{l,t}T$, propagating with the longitudinal velocity c_l and the transverse velocity c_t of the elastic waves. These waves are the seismic P and S waves, visible on any seismogram. The typical values of the two velocities for earth are $c_l = 3km/s$, $c_t = 7km/s$. The circular wavefronts of these waves propagate on Earth's surface with a greater velocity. They leave behind secondary-waves sources on Earth's surface, which generate secondary waves. The effect of these waves on Earth's surface is cumulative, such that the P and S waves are followed by a mainshock with an abrupt front and a long tail. This feature appears on any seismogram. A typical seismogram is shown in Fig. 1.

The earth is not a homogeneous and isotropic elastic medium; it includes inhomogeneities. There exist a few inhomogeneities with large dimensions; they affect the direction and the velocity of the elastic waves. This is why, the position of the focus and the epicentre are determined only approximately. The dimensions of the most numerous inhomogeneities are small, of the order of the focal dimensions. They affect the content of the waves in the spherical-shell waves with wavelengths comparable with this dimension order. Consequently, the seismic waves exhibit a small-amplitude dispersion with short wavelengths (short period of oscillation). This dispersion is visible on seismograms. It is compounded with the seismographs' eigenoscillations. In estimating the amplitude of the displacement from seismograms we should use the envelope of the oscillations. It may happen that the amplitude is not the same on the two sides of the oscillations; in that case we should use either the maximum amplitude, or a mean amplitude. It is more difficult to establish a direct connection between the displacement in the mainshock and the characteristics of the focal region. The amplitudes of the longitudinal (l) wave (P wave) and the transverse (t) wave (S wave) are given by[1, 2]

$$\mathbf{v}_l = \frac{1}{4\pi\rho c_l^3 TR} M_4 \mathbf{n} \ , \ \mathbf{v}_t = \frac{1}{4\pi\rho c_t^3 TR} (\mathbf{M} - M_4 \mathbf{n}) \ , \quad (2)$$

where ρ is the earth's density, $\mathbf{n} = \mathbf{R}/R$ is the unit vector from the focus to the observation point (position \mathbf{R}) and $M_i = M_{ij}n_j$, $M_4 = M_i n_i$; hence, the seismic-moment vector is

$$\mathbf{M} = 4\pi\rho TR \left(c_l^3 \mathbf{v}_l + c_t^3 \mathbf{v}_t \right) \ . \quad (3)$$

Moment magnitude. Equations (2) have been inversed to get the seismic-moment tensor.[1] To this end, the Kostrov representation of the fault has been used and the energy conservation. Only three components of the tensor are independent. The solution leads to the determination of the duration T from

$$T^2 = 2R \frac{c_l v_l^2 + c_t v_t^2}{(c_l^6 v_l^2 + c_t^6 v_t^2)^{1/2}} \ , \quad (4)$$

the magnitude of the tensor $(M_{ij}^2)^{1/2} = \sqrt{2}M$, the energy released by the earthquake $E = M/2$ and the magnitude of the seismic-moment vector

$$M = 2\pi\rho(2R)^{3/2} (c_l v_l^2 + c_t v_t^2)^{1/2} (c_l^6 v_l^2 + c_t^6 v_t^2)^{1/4} \ . \quad (5)$$

By using the moment magnitude in Hanks-Kanamori relation[4, 5]

$$\lg(M_{ij}^2)^{1/2} = \frac{3}{2}M_w + 16.1, \quad (6)$$

we can determine the moment magnitude M_w from measurements of the amplitude of the P and S seismic waves on Earth's surface (M_{ij} measured in $\text{dyn} \cdot \text{cm}$). In addition, the solution of the equations (2) gives the orientation of the fault, the dimension of the focus and the slip along the fault.

It is convenient to introduce a mean displacement v and a mean velocity c , such that equation (4) gives $cT = \sqrt{2Rv}$; hence, we get immediately an estimate of the focal volume $V \simeq \pi(cT)^3 = \pi(2Rv)^{3/2}$. Also, from equation (5) the magnitude of the seismic-moment vector is $M \simeq 2\pi\rho c^2(2Rv)^{3/2}$ and the energy is $E \simeq \pi\rho c^2(2Rv)^{3/2} (= \mu V, \text{ where } \mu \text{ is the Lamé coefficient})$. The moment magnitude can be estimated immediately from equation (6).

Local magnitude. The use of mean displacement v and the mean velocity c does not introduce large errors in the logarithm. Let us compute

$$\begin{aligned} \lg(M_{ij}^2)^{1/2} &= \frac{3}{2}\lg R + \frac{3}{2}\lg v + \lg(8\pi\rho c^2) = \\ &= \frac{3}{2}M_w + 16.1 \end{aligned} \quad (7)$$

by using equations (5) and (6), or

$$\lg v = M_w - \lg R - \frac{2}{3}\lg(8\pi\rho c^2) + 10.73. \quad (8)$$

For $\rho = 5\text{g/cm}^3$ and $c = 5\text{km/s}$ we get $8\pi\rho c^2 = 10^{13.54}$ and

$$\lg v = M_w - \lg R + 1.8. \quad (9)$$

We can see that $\lg v + \lg R$ may be taken as a magnitude related to the moment magnitude, *i.e.* to the energy released by earthquakes. Since the focal distance R appears explicitly in this equation, it defines a local magnitude. Therefore, the definition

$$M_l = \lg v + \lg R - 4.8 \quad (10)$$

is suggested for a local scale of magnitudes, calibrated by using cm as unit, such that $M_l = 0$ for $v = 10^{-2.2}\text{cm}$ and $R = 100\text{km}$. The relation with the moment magnitude is

$$M_l = M_w - 3 \quad (11)$$

($M_l = 0$ for $M_w = 3$). Equation (11) is valid only for an approximate estimation of the moment magnitude M_w , given above by $M \simeq 2\pi\rho c^2(2Rv)^{3/2}$.

Equation (10) is similar with the great variety of local magnitude scales in use. In particular, the original Richter local magnitude[6] is defined as

$$M_L = \lg v - 2.48 + 2.76 \lg \Delta, \quad (12)$$

where v is measured in m and Δ is the epicentral distance measured in km ; it is calibrated to $M_L = 0$ for $v = 10^{-3}\text{m}$ and $\Delta = 100\text{km}$. [7] The relevance the distance R in the local magnitude was recognized in Ref. [8]

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