

**Macroscopic quantum-mechanical scattering. Coherent scattering of neutrinos**

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**Abstract**

It is shown that in certain conditions a macroscopic quantum-mechanical scattering may occur, which may lead to a coherent cross-section on a macroscopic scale in a monocrystal. The conditions are satisfied by neutrinos, but not satisfied by other projectiles, with a higher cross-section. This may explain Weber-type experiments of neutrino detection by a perfect, stiff sapphire monocrystal. The occurrence of coherence domains for quantum-mechanical scattering and the formation of domains for classical diffraction are analyzed, and the force exerted upon a macroscopic target is estimated. It is concluded that neutrinos have a distinctive feature in this respect, due precisely to their very small cross-section.

**Introduction.** In two papers published in 1985 and 1988 Weber claimed that neutrinos (antineutrinos), from various sources like tritium, nuclear reactors and the Sun, could be detected by their coherent scattering by a perfect, stiff, sapphire monocrystal with a high Debye temperature (mounted on a torsion balance and equilibrated by a lead dummy).[1, 2] The coherent cross-section would be  $\sigma = N^2\sigma_0$ , where  $N$  is the number of unit cells in the target and  $\sigma_0$  is the cross-section of a single unit cell (particle, *e.g.*, atomic nucleus). Such a highly enhanced cross-section  $\sim N^2$  would give rise to a measurable force upon a torsion balance. Weber's claims have been criticized both on theoretical and experimental grounds, the main objection being that the form factor would reduce appreciably the cross-section, and, on the other hand, such a coherence effect is not observed in  $X$ -, gamma rays or neutron scattering[3]-[12] (see also Refs. [13, 14]). A discussion of the theoretical objections and negative experiments was given by Nicolescu, who presented a positive experiment;[15] indeed, there exists an experiment by Cruceru et al, which confirmed Weber's prediction for solar neutrinos.[15]-[17] The problem is still controversial. We show in this paper that a coherent scattering of neutrinos may appear in the conditions formulated by Weber and company, as a consequence of a quantum-mechanical treatment of the crystal as a whole (a macroscopic quantum-mechanical scattering). This is a distinctive condition of the neutrino scattering, which is not fulfilled by other projectiles, with a higher cross-section. The main reason for such a behaviour is precisely the extremely small cross-section  $\sigma_0$  ( $10^{-44}cm^2$ ) of the neutrinos. For  $\sigma_0 = 10^{-44}cm^2$  and  $N = 10^{22}$  (less than  $0.1 mol$ ) the coherent cross-section is  $\sigma = 1cm^2$ . For a neutrino flux density  $\Phi = 10^{12}/cm^2 \cdot s$  the time between two collisions is  $\tau = 1/\Phi\sigma = 10^{-12}s$ . On the other hand, an atom in thermal equilibrium at room temperature has a velocity of the order  $10^4cm/s$ . In an elementary act of collision the atom is perturbed from its equilibrium state and receives a momentum of the order  $p = E/c$ , on the average, where  $E$  is the energy of the neutrino projectile (and  $c = 3 \times 10^{10}cm/s$  is the speed of light). For  $E = 1MeV$  the momentum transfer is of the order  $p = 5 \times 10^{-17}g \cdot cm/s$ . Consequently, the energy perturbation of the atom is of

the order  $\Delta E = vp = 5 \times 10^{-13} \text{erg}$ . The time needed for this atom to recover its equilibrium is of the order  $\Delta t_{eq} = \hbar/\Delta E = 2 \times 10^{-15} \text{s}$  (where  $\hbar \simeq 10^{-27} \text{erg} \cdot \text{s}$  is Planck's constant). We can see that  $\tau \gg \Delta t_{eq}$ . We can say that the atoms recover quickly their equilibrium state between two successive collisions, and the incident neutrino beam sees the crystal as a whole. Therefore, we need to adopt a quantum-mechanical treatment for the entire crystal. It is worth noting that if the cross-section increases to, say,  $\sigma_0 = 10^{-24} \text{cm}^2$ , as for  $X$ -, gamma rays or neutrons, the "collision" time decreases to  $\tau = 10^{-32} \text{s}$ , which is much shorter than the equilibrium time  $\Delta t_{eq}$ , all the other conditions remaining the same. In that case the incident projectile beam sees the crystal as consisting of distinct, independent atoms, such that a coherent scattering ( $\sigma = N^2 \sigma_0$ ) for the entire crystal is not possible. The particularity of a coherent scattering suffered by neutrinos in the whole crystal resides precisely in their extremely small cross-section  $\sigma_0$ . On the other hand, a single-particle cross-section  $\sigma_0 = 10^{-24} \text{cm}^2$  increases considerably the total cross-section, such that we need to re-consider the scattering in this case.

**Macroscopic quantum-mechanical scattering.** Let us assume a macroscopic target consisting of  $N \gg 1$  identical "atoms" (atomic nuclei, molecules, unit cells in a crystal). The interaction with an incident beam of particles can be written as

$$H = a^3 h(\xi) \sum_{i=1}^N \delta(\mathbf{r} - \mathbf{r}_i) , \quad (1)$$

where  $a$  is the range of the single-particle interaction  $h(\xi)$ ,  $\xi$  denotes the internal coordinates of the atoms and  $\mathbf{r}_i$  are the atomic positions. The time between two successive collisions is  $\tau = 1/\Phi\sigma$ , where  $\Phi$  is the incident flux density and  $\sigma$  is the total cross-section. In an elementary act of collision an atom receives a momentum of the order of the momentum  $p$  of the incident particle. The atom has a thermal velocity  $v \simeq \sqrt{T/M}$ , where  $T$  is the temperature and  $M$  is the mass of the atom. The atomic motion is perturbed by an energy of the order  $\Delta E = vp$ , so it needs a time  $\Delta t_{eq} \simeq \hbar/\Delta E = \hbar/\sqrt{T/M}p$  to recover its equilibrium. Let us assume

$$\tau > \Delta t_{eq} ; \quad (2)$$

then, the incident particles see the macroscopic target as a whole, and we need to work with the wavefunction of the whole, macroscopic target.

A perfect monocrystal suffers two kinds of motion. One kind is the motion of the crystal as a whole, where all the laticial positions of the atoms  $\mathbf{r}_i^0$  move by the same distance. The wavefunction corresponding to this motion is

$$\Phi_{\mathbf{K}}(\xi; \mathbf{r}) = \sum_{i=1}^N e^{i\mathbf{K}\mathbf{r}_i^0} \varphi(\xi; \mathbf{r} - \mathbf{r}_i^0) , \quad (3)$$

where  $\mathbf{K}$  is the quasi-wavevector of the crystal (quasi-momentum  $\hbar\mathbf{K}$ ) and  $\varphi(\xi; \mathbf{r} - \mathbf{r}_i^0)$  are (orthonormal) wavefuctions localized on the positions  $\mathbf{r}_i^0$ . We can see that the wavefunction  $\Phi_{\mathbf{K}}$  has the translational symmetry of the crystal. We call  $\Phi_{\mathbf{K}}$  a coherent wavefunction. The other type of motion of the crystal is the thermal motion with atomic displacements  $\mathbf{v}_i$  ( $\mathbf{r}_i = \mathbf{r}_i^0 + \mathbf{v}_i$ ); the corresponding wavefunction is

$$\psi(\xi; \mathbf{r}) = \sum_{i=1}^N e^{i\chi_i} \varphi(\xi; \mathbf{r} - \mathbf{r}_i^0 - \mathbf{v}_i) , \quad (4)$$

where  $e^{i\chi_i}$  are random phase factors; we call  $\psi$  an incoherent wavefunction. Both these wavefunctions are normalized to  $N$ ; they are orthogonal to each other. The wavefunction of the crystal is

$$\Psi_{\mathbf{K}} = \sqrt{1 - g^2} \Phi_{\mathbf{K}} + g\psi , \quad (5)$$

where  $g$  is a weight coefficient. This coefficient is proportional to the square root of the relative number of vibration states of the crystal, properly normalized. This relative number of phonon states is proportional to

$$I = \frac{1}{\omega_D^3} \int_0^{\omega_D} d\omega \frac{\omega^2}{e^{\hbar\omega/T} - 1} , \quad (6)$$

where  $\omega_D$  is the Debye frequency and  $\omega$  is the phonon frequency; the Debye temperature is  $\Theta = \hbar\omega_D$ . For  $T/\Theta \ll 1$  the integral is  $I \simeq 2.4(T/\Theta)^3$ , while for  $T/\Theta \gg 1$  the integral is  $I \simeq \frac{1}{2}T/\Theta$ . A normalized expression for the relative number of states is  $4.8(T/\Theta)^2$  for  $T/\Theta \ll 1$  and 1 for  $T/\Theta \gg 1$ . An interpolation formula is  $I \simeq 4.8(T/\omega_D)^2 / [1 + 4.8(T/\omega_D)^2]$ , such that we can take for the weight coefficient

$$g \simeq \frac{2.2(T/\Theta)}{[1 + 4.8(T/\Theta)^2]^{1/2}} . \quad (7)$$

We can see that for high Debye temperatures the main contribution to  $\Psi_{\mathbf{K}}$  comes from the coherent wavefunction ( $g \ll 1$ ), while for low Debye temperatures the main contribution comes from the incoherent field ( $g \rightarrow 1$ ).

Let us assume a wavefunction  $\frac{1}{\sqrt{V}}e^{i\mathbf{k}\mathbf{r}}$  for an incident particle, and an initial ( $i$ ) wavefunction  $\frac{1}{\sqrt{V}}e^{i\mathbf{k}\mathbf{r}}\Psi_{\mathbf{K}}$ , where  $V$  denotes the volume. The normalization to unity of this wavefunction requires a factor  $\sqrt{a^3}$  in the wavefunctions  $\varphi$ , such that the scalar product is  $\langle \varphi(\xi; \mathbf{r}_j - \mathbf{r}_i), \varphi(\xi; \mathbf{r}_j - \mathbf{r}_i) \rangle^2 = \delta_{ij}$ . The matrix elements of the interaction between the wavefunctions  $\Phi_{\mathbf{K}}$  and  $\psi$  are zero. The matrix elements of the interaction between two wavefunctions  $\Phi_{\mathbf{K}}$  and  $\Phi_{\mathbf{K}'}$  (coherent scattering) lead to the momentum conservation

$$H_{fi} \sim \sum_{i=1}^N e^{i(\mathbf{K}-\mathbf{K}')\mathbf{r}_i^0} e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}_i^0} = N\delta_{\mathbf{K}'+\mathbf{k}', \mathbf{K}+\mathbf{k}} , \quad (8)$$

where  $\mathbf{k}'$ ,  $\mathbf{K}'$  are the wavevectors of the final state ( $f$ ). We can see that the difference in momentum of the incident particle is taken by the crystal, which moves as a whole. According to equation (8) the coherent cross-section is

$$\sigma_{coh} = N^2\sigma_0 , \quad (9)$$

where  $\sigma_0$  is the single-particle cross-section. A similar calculation for the incoherent matrix elements (wavefunctions  $\psi$ ) leads to

$$H_{fi} \sim \sum_{i=1}^N e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}_i^0} (1 + i(\mathbf{k} - \mathbf{k}')\mathbf{v}_i + ...) , \quad (10)$$

where

$$\mathbf{v}_i = \frac{1}{\sqrt{N}} \sum_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}_i^0} \mathbf{v}_{\mathbf{q}} \quad (11)$$

is the phonon field. The first term in equation (10) corresponds to a displacement of the crystal as a whole, so it is already included in the coherent scattering. We are left with

$$H_{fi} \sim \sqrt{N} (\mathbf{q}\mathbf{v}_{\mathbf{q}}) \delta_{\mathbf{k}', \mathbf{k}+\mathbf{q}} . \quad (12)$$

We can see that the difference in momentum of the incident particle is taken by phonons; the incoherent scattering excites phonons. In addition, the incoherent cross-section is proportional to  $N$ . For the cross-section we need to average  $(\mathbf{q}\mathbf{v}_\mathbf{q})^2$  over the thermal states. The maximum value of this average is of the order  $T/Mc_s^2$  (at room temperature), where  $c_s$  is the mean phonon velocity. This thermal factor is reminiscent of the Debye-Waller factor (and the diffuse scattering). The total incoherent scattering can be written as

$$\sigma_{incoh} = N\sigma_0^{ph} , \quad (13)$$

where the single-particle cross-section  $\sigma_0^{ph}$ , arising from phonons, is smaller than  $\sigma_0$  by the thermal factor.

The total cross-section of the crystal is

$$\sigma = \sqrt{1 - g^2}\sigma_{coh} + g^2\sigma_{incoh} . \quad (14)$$

For atoms placed randomly (like in amorphous solids, liquids, etc) the coherent wavefunction  $\Phi_{\mathbf{K}}$  is missing and the weight coefficient is  $g = 1$ .

**Neutrino scattering.** We adopt  $N \simeq 10^{22}$  for the number of unit cells in the sapphire monocrystal ( $\simeq 24g$ , density  $4g/cm^3$ ) used in Weber's experiments,[1, 2] and other similar experiments[15]-[17](the volume of the unit cell of sapphire is large). Making use of  $\sigma_0 = 10^{-44}cm^2$  we get a coherent cross-section  $\sigma_{coh} \simeq 1cm^2$ . For a Debye temperature  $\Theta = 10^3K$  the weight factor at room temperature is  $\sqrt{1 - g^2} \simeq 0.7$ . The total coherent cross-section is  $\sigma \simeq 0.7cm^2$ . We note that this cross-section is smaller than the area of the crystal. For a flux density  $\Phi = 10^{12}/cm^2 \cdot s$  the collision time is  $\tau = 1/\Phi\sigma_{coh} \simeq 10^{-12}s$ . At room temperature the thermal velocity of an atom is  $\simeq 10^4cm/s$ . For a neutrino energy  $E = 1MeV$  the momentum is  $p = E/c \simeq 5 \cdot 10^{-17}g \cdot cm/s$  and the equilibrium time is of the order  $\Delta t_{eq} = \hbar/vp \simeq 2 \times 10^{-15}s$ . Since  $\tau \gg \Delta t_{eq}$  the macroscopic quantum-scattering described above applies.

The force acting upon the target in the forward direction is  $F = \Phi\sigma_{coh}p \simeq 3.5 \times 10^{-5}dyn$ . This is a measurable force. We note that it is sensitive to the values of the input parameters. For instance, a Debye temperature  $\Theta = 100K$  leads to a weight coefficient  $\sqrt{1 - g^2} \simeq 0.15$  and a weaker force by a factor  $\simeq 5$ . Also, for an amorphous solid, although the conditions of a quantum-mechanical scattering may be fulfilled, the force is extremely weak, as a consequence of the very small incoherent cross-section.

For solar neutrinos the single-particle cross-section may have the same order of magnitude ( $\sigma_0 = 10^{-44}cm^2$ ), but the energy and the flux density are lower ( $E \simeq 300keV$ ,  $\Phi \simeq 10^{11}/cm^2 \cdot s$ ). The conditions of a coherent scattering are preserved, but the force is diminished by a factor  $1/30$  ( $\simeq 10^{-6}dyn$ ). For tritium neutrinos the decrease is appreciable, though a higher  $\sigma_0$  or a slightly greater number of unit cells  $N$  may compensate the decrease (while preserving the conditions of coherent scattering). We conclude that Weber-type experiments could exhibit a measurable force acting upon a sapphire crystal, at least for nuclear reactor neutrinos, or solar neutrinos.

**Other projectiles. Coherence domains.** We adopt the value  $\sigma_0 = 10^{-24}cm^2$  for other types of projectiles (like  $X$ -, gamma rays or neutrons). A coherent cross-section would be much larger than the area of the crystal. The crystal responds to this unphysical situation by developing coherence domains. Let us assume that  $n_d$  uncorrelated domains exist in the crystal, each with  $N_d$  unit cells (as a mean size), such that  $n_d = N/N_d$ . By a formal analogy with the high-purity crystals we use the fraction  $f = 1/N_d$ . This fraction varies between  $f = 1/N$ , when we have only one domain, *i.e.* the whole target, and  $f = 1$ , when the whole target is fragmented in "atomic" domains.

The scattering amplitude can be written as

$$S = \sum_{a=1}^{n_d} e^{i\chi_a} (H_{fi})_a , \quad (15)$$

where  $e^{i\chi_a}$  are random phase factors. By averaging the squared scattering amplitude over the phase factors, we get  $|S|^2 = \sum_a |(H_{fi})_a|^2$ , such that the cross-section becomes

$$\bar{\sigma} = n_d \sigma_d , \quad (16)$$

where  $\sigma_d$  is the cross-section of a domain.

According to this equation, the coherent cross-section  $\sigma_{coh} = \sigma_0 N^2$  is reduced by the coherence domains to

$$\bar{\sigma} = \sigma_0 n_d N_d^2 = \frac{\sigma_0 N}{f} ; \quad (17)$$

we can see that this formula gives the total coherent cross-section ( $\sigma_0 N^2$ ) for  $f = 1/N$  and the incoherent cross-section ( $\sigma_0 N$ ) for  $f = 1$ . In this latter case  $\sigma_0$  should be replaced by  $\sigma_0^{ph}$  (a similar procedure leaves the incoherent cross-section arising from phonons unchanged,  $\bar{\sigma}_{incoh} = n_d N_d \sigma_0^{ph} = N \sigma_0^{ph} = \sigma_{incoh}$ ).

In order to have a quantum-mechanical scattering the conditions  $\bar{\tau} > \Delta t_{eq}$  and  $\bar{\sigma} < A$  should be satisfied, where  $A$  is the area of the target; they lead to

$$f > \frac{\hbar \Phi}{vp} \sigma_0 N , \quad f > \frac{\sigma_0 N}{A} \quad (18)$$

and a number of unit cells  $N_d = 1/f < A/\sigma_0 N$  in each domain. For  $\sigma_0 = 10^{-24} \text{cm}^2$  this number is too small for any macroscopic target ( $N_d < 10^2 A$ ,  $N = 10^{22}$ ); the domains are not well defined, such that  $f$  approaches unity and the scattering tends to an incoherent scattering. We conclude that the quantum-mechanical scattering cannot appear for large single-particle cross-sections, like  $\sigma_0 = 10^{-24} \text{cm}^2$ . We note that for neutrinos ( $\sigma_0 = 10^{-44} \text{cm}^2$ )  $f > 10^{-22}/A$ ,  $N_d < 10^{22} A$  and we may have one domain in the whole target. The coherent scattering occurs for neutrinos in a crystal precisely due to the small neutrino cross-section  $\sigma_0$ . The above considerations apply also to a polycrystalline target, where  $f$  is limited, in addition, by the size of the crystallites and, consequently, the cross-section is much diminished.

**Classical scattering.** If inequation (2) is not satisfied (*i.e.*, if  $\tau < \Delta t_{eq}$ ), the incident particles see the target "atoms" (atomic nuclei, molecules, unit cells) as independent scatterers. We call this scattering a classical scattering. We can see that this condition implies low energies. The initial wavefunction is  $\frac{1}{\sqrt{V}} e^{i\mathbf{k}\mathbf{R}} \frac{1}{\sqrt{V}} e^{i\mathbf{K}\mathbf{R}_c}$  (up to wavefunctions corresponding to the internal degrees of freedom), where  $\mathbf{k}$  is the wavevector of the incident particle,  $\mathbf{K}$  is the wavevector of the center of mass,  $\mathbf{R}_c$  is the position of the center of mass and  $\mathbf{R} = \mathbf{r} + \mathbf{R}_c$ . The matrix elements of the interaction given by equation (1),

$$H_{fi} \sim \sum_{i=1}^N e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}_i} \delta_{\mathbf{K}'+\mathbf{k}', \mathbf{K}+\mathbf{k}} , \quad (19)$$

includes the form-factor

$$F(\mathbf{k} - \mathbf{k}') = \sum_{i=1}^N e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}_i} , \quad (20)$$

where  $\mathbf{k}'$  is the wavevector of the scattered particle and  $\mathbf{K}'$  is the final wavevector of the center of mass. We can see that the total momentum, including the momentum of the center of mass, is conserved. For a crystal  $F(\mathbf{k} - \mathbf{k}') = N\delta_{\mathbf{k}', \mathbf{k}+\mathbf{g}}$ , where  $\mathbf{g}$  is a reciprocal vector of the lattice. For an amorphous target  $\mathbf{g} = 0$ . It follows that we have diffraction peaks. For the cross-section of a peak we get  $d\sigma_{\mathbf{g}} \sim N^2 d\omega_{\mathbf{g}}$ , where the solid angle  $\omega_{\mathbf{g}}$  extends to the range  $\Delta\omega_{\mathbf{g}} \simeq N^{-2/3}(2\pi/dk')^2$ , where  $d$  is the mean distance between unit cells (scatterers). It follows  $\sigma_{\mathbf{g}} \sim N^{4/3}/(dk')^2$ . On the other hand the number of peaks is  $\simeq (dk')^2 \gg 1$ , such that the total cross-section is

$$\sigma = N^{4/3}\sigma_0, \quad (21)$$

where  $\sigma_0$  is the single-particle cross-section. For one peak  $\sigma$  should be divided by the number of peaks. As it is well known, this cross-section is affected by the Debye-Waller factor and diffuse scattering. According to equation (15) for  $n_d = N/N_d = fN$  domains the cross-section is

$$\bar{\sigma} = n_d N_d^{4/3} \sigma_0 = \frac{\sigma_0 N}{f^{1/3}}. \quad (22)$$

For  $f = 1/N$  we recover the total cross-section  $N^{4/3}\sigma_0$  of one domain, while for  $f = 1$  the cross-section reduces to  $N\sigma_0$  of an incoherent scattering.

The conditions  $\bar{\tau} = 1/\Phi\bar{\sigma} < \Delta t_{eq}$  and  $\bar{\sigma} < A$  lead to

$$\frac{\sigma_0 N}{A} < f^{1/3} < \frac{\hbar\Phi}{vp} \sigma_0 N, \quad (23)$$

which implies  $N_d = 1/f < (A/\sigma_0 N)^3$  ( $\bar{\sigma} < A$ ). For  $\sigma = 10^{-24} \text{cm}^2$  and  $N = 10^{22}$  the number of unit cells  $N_d < 10^6 A^3$  in a domain may indicate well-defined domains for macroscopic targets. The force is bounded by above according to the inequality  $F < \Phi A p$ . Since  $p < \frac{\hbar\Phi}{v} A$  (from equations (23)), this upper bound is given by  $F < \frac{\hbar}{v} (\Phi A)^2 \simeq 10^{-7} A^2 \text{dyn}$  ( $\Phi = 10^{12} / \text{cm}^2 \cdot \text{s}$ ,  $v = 10^4 \text{cm/s}$ ). For any reasonably large area  $A$  and flux density  $\Phi$  it is difficult to satisfy these conditions ( $p < \hbar\Phi A/v$ ) and to measure such a force in current experimental situations.

**Concluding remarks.** A quantum-mechanical scattering is identified in certain conditions in macroscopic targets, which may lead to a coherent cross-section in high-purity, stiff monocrystals. This coherent scattering may explain the Weber-type experiments of neutrino detection by using sapphire monocrystals. The coherent-scattering conditions are not fulfilled by other types of projectiles, with a higher single-particle cross-sections (like  $X^-$ , gamma or neutrons). In these cases a classical diffraction may occur in crystals, which generates a weak force, at the limit of detection.

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