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## On the relevance of the foreshocks in forecasting seismic mainshocks

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### Abstract

We analyze the usefulness of the foreshocks in forecasting seismic mainshocks. The analysis is based on possible correlations which may exist between foreshocks and mainshocks. Such correlations are expressed by a time-magnitude relationship, previously established in Ref. [5], which indicates the presence of an abrupt magnitude-descending sequence of correlated foreshocks in the proximity of a mainshock. By fitting this formula, we are able to derive the occurrence time of the main shock. Also, we can forecast the magnitude of the mainshock, providing we know the parameters of the background seismicity of the seismic region. We report here on the application of this procedure to three Vrancea mainshocks and the strong l'Aquila earthquake. The limitations of the procedure are discussed.

Key words: time-magnitude correlations; foreshocks; mainshocks; forecasting

**Theoretical background.** It is well known that seismic mainshocks are accompanied by foreshocks and aftershocks, which are localized in the spatial and temporal relative proximity of the mainshocks. After a strong mainshock the focal region and its surroundings may be modified, and smaller aftershocks may appear. Similarly, the energy accumulated in the focal region may be released in advance by smaller foreshocks, which may announce the occurrence of a mainshock. Obviously, such seismic events are correlated, in the sense that the characteristic parameters of an event depend on the characteristic parameters of other. According to the epydemic-type aftershock sequence (ETAS) model every seismic event in the sequence foreshocks-mainshock-aftershocks is correlated to every other seismic event in the sequence.[1]-[4] It is reasonable to assume that such correlations are hierarchical in the degree of magnitude. We focus here only on the main correlations which connect the foreshocks and the aftershocks to the mainshock (and, of course, the mainshock to the foreshocks and the aftershocks).

The correlated foreshocks may have a potential of forecasting the occurrence of a mainshock. However, not all the precursory events are correlated foreshocks, nor all the subsequent seismic events are correlated aftershocks. Although correlations are very likely to be present, it is difficult to distinguish the correlations from the regular, background seismic activity.

Also, although very likely, the forecasting potential of the foreshocks remains elusive until a quantitative description of the correlations is not available. Such a quantitative description should relate foreshocks characteristics to the time left up to the occurrence of the main shock. Recently, it has been shown that the foreshock (moment) magnitude $M$ is related to the time $\tau$ until the mainshock by[5]

$$M \simeq \frac{1}{b}\ln(\tau/\tau_0) \ , \tag{1}$$

where $b = 3.45$ (3/2 in decimal logarithms) is the well-known Hanks-Kanamori parameter and $\tau_0$ is a cutoff time which depends on the magnitude of the mainshock and the parameters of the background seismic activity. The small threshold time $\tau_0$ indicates a very short quiescence time[6] before the occurrence of the main shock ($\tau > \tau_0$). In addition, the time $\tau$ should be cut off by an upper threshold, at least for $M$ not to be greater than the magnitude $M_0$ of the main shock ($M < M_0$, $\tau < \tau_0 e^{bM_0}$).

As it is well known, the background seismic activity is governed by the Gutenberg-Richter (GR) statistical law. Its standard cumulative (excedence) form is $P_{ex}(M) = e^{-\beta M}$, where $P_{ex}(M)$ is the probability of occurence of an earthquake with magnitude greater than $M$ and the GR parameter $\beta$ varies in the range 1.15 to 3.45 (in decimal logarithms 0.5 to 1.5); the mean value $\beta = 2.3$ (in decimal logarithms $\beta = 1$) is usualy accepted as a reference value.[7]-[10]. If the number of earthquakes with magnitude greater than $M$ is $N(M)$ out of a total number $N_0$ of earthquakes in a given seismic region in a given long time interval $T$, we may write $P_{ex}(M) = N(M)/N_0 = N(M)t_0/T$, where $t_0$ is the inverse of a mean seismicity rate. The law is applied with a small-magnitude cutoff which accounts for the completeness magnitude of the catalog and the well-known roll-off effect occurring at small magnitudes.[11, 12] Consequently, the parameter $t_0$ is a fitting parameter, like $\beta$. In its linear-logarithmic form the law reads

$$\ln\left[N(M)/T\right] = -\ln t_0 - \beta M \ . \tag{2}$$

By fitting this law we can extract the parameters $t_0$ and $\beta$ of the background seismicity. We performed such a fit for a set of 3640 earthquakes with magnitude $M \geq 3$ which occurred in Vrancea during $1981 - 2018$. The resulting parameters are $-\ln t_0 = 11.32$ ($t_0$ measured in years) and $\beta = 2.26$ (with an estimated 15% error). The data for Vrancea have been taken from the Romanian Earthquake Catalog 2018, http://www.infp.ro/data/romplus.txt.[13] A completeness magnitude $M = 2.2$ to $M = 2.8$ is usually accepted for Vrancea (a more conservative figure would be $M = 3$),[14] and the magnitude average error is $\Delta M = 0.1$. A similar fit, with slightly modified parameters, is valid for 8455 Vrancea earthquakes with magnitude $M \geq 2$ (period $1980 - 2019$).

It has been shown that the ratio $r = \beta/b$ is related to the mechanism of energy accumulation in the seismic focus.[15] We do not stop to discuss here the relevance of this parameter for the accumulation time and the statistical distributions of the earthquakes, but rather we use $r$ as a convenient parameter. We can see that this parameter is easily derived from the background seismic activity. By making use of the data given above, we get $r = 2.26/3.45 = 0.65$. The cutoff time is given by

$$\tau_0 = r t_0 e^{-b(1-r)M_0} \ , \tag{3}$$

where $M_0$ is the magnitude of the mainshock.[5]

Equation (1) can also be written as

$$M(t) = \frac{1}{b} \ln \frac{t_{ms} - t}{\tau_0} \ , \tag{4}$$

where $t_{ms}$ is the occurrence time of the mainshock. This equation can be fitted to the foreshock magnitudes for the parameters $t_{ms}$ and $\tau_0$ ($b = 3.45$). From $\tau_0$ and equation (3), by making use of the parameters of the bakground seismicity ($t_0$, $\beta$, $r$), we can get the magnitude $M_0$ of the mainshock. Equation (4) is limited by $t_{ms} - t > \tau_0$ and a higher cutoff which acounts for $M < M_0$ at sleast ($t_{ms} - t < \tau_0 e^{bM_0}$).

It is worth noting that the time $t_{ms}$ depends on the magnitude of the main shock, as expected. For instance, a magnitude $M$ indicates a time $\tau = \tau_0 e^{bM}$ up to the main shock. Let us assume
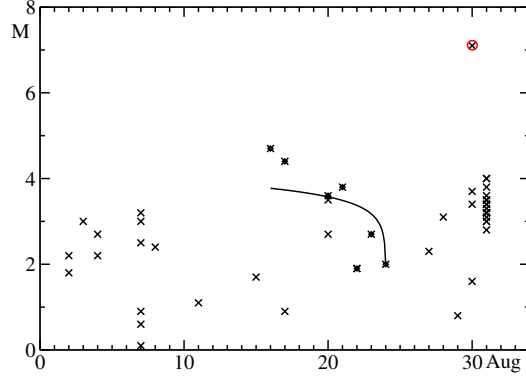
Figure 1: Vrancea seismic activity in the period 1 August - 31 August 1986 (Romanian Earthquake Catalog, 2018). The curve is the fit of Eq. (4) to data from 16 August to 24 August (fitting parameters $t_{ms} = 24$ August and $\tau_0 = 10^{-4.76}$ days; see text).

that we are interested in a main shock with magnitude $M_0 = 7$; then by using $t_0 = e^{-11.32}$ (years, for Vrancea) and $r = 0.65$ given above, we get $\tau_0 = \frac{2}{3}10^{-8.42}$ (years); a foreshock with magnitude $M = 5$ would indicate that we are at $\tau = \frac{2}{3}10^{-8.42}10^{7.5} = 0.079$ years, *i.e.* $\simeq 29$ days, from that main shock.

It is worth noting that we use the background-seismicity parameters for estimating $M_0$. This is justified as long as we limit ourselves to moderate magnitudes (which is the most frequent situation encountered in practice). However, equation (4) has a very high slope in the neighbourhood of $t_{ms}$, such that a reliable estimation of the fitting parameters $t_{ms}$ and $\tau_0$ can only be achieved by a special data set, which would include, ideally, many small-magnitude foreshocks with magnitudes falling rapidly to zero.

**Applications and results.** Vrancea is the main seismic region of Romania. Three strong earthquakes occurred in Vrancea, since we have reliable recordings: magnitude $M = 7.1$, 30 August 1986; magnitude $M = 6.9$, 30 May 1990; magnitude $M = 6.4$, 31 May 1990 (Romanian Earthquake Catalog 2018, http://www.infp.ro/data/romplus.txt). We have applied the fitting procedure described above to the 7.1-earthquake (depth $131km$). This earthquake and all its precursory events since 1 August are shown in Fig. 1. All these earthquakes occurred in an area with dimensions $\simeq 100km \times 80km$ ($45° - 46°$ latitude, $26° - 27°$ longitude), at various depths in the range $30km - 170km$, except for the events of 7-8 August and the 1.6-event of 30 August, whose depth was $5km - 20km$. As shown in Fig. 1, we can identify a magnitude-descending sequence from 16 August to 24 August, which we fitted by equation (4). The fitting parameters were $t_{ms} = 24$ August, $\tau_0 = 10^{-4.76}$ days and a large rms relative error 0.32. For earthquakes which occurred in the same day we have used the maximum magnitude. The fitting parameters indicate the occurence of a main shock with magnitude 4.4 on 24 August. Other magnitude-descending sequences may be analyzed in Fig. 1, with a larger error.

We cannot identify magnitude-descending sequences for the earthquake pair of 30-31 May 1990 (depth $87 - 91km$).

The same procedure has been applied to the Vrancea earthquake with magnitude 3.8 (local magnitude 4.1), viewed as a main shock, which occurred on 30 November 2021, where we used the foreshock sequence from 24 November to 29 November (6 earthquakes). We forecasted a main-shock with magnitude 4.5 on 1 December (all the data are taken from Romanian Earthquake Catalog 2018, http://www.infp.ro/data/romplus.txt).[16] All these earthquakes occurred within $45° - 46°$ latitude, $26° - 27°$ longitude, at depths in the range $90km - 180km$.

Also, we have analyzed the set of precursory events of the l'Aquila earthquake, 6 April 2009 (magnitude 6.3, local magnitde 5.9), where we identified two magnitude-descending sequences, with earthquakes succeeding rapidly at intervals of hours. The first sequence, consisting of 7 earthquakes with local magnitudes from 2.1 to 1.0, occurred on 2 April. The fitting of these data indicates a main shock approximately 5 hours before the earthquake with magnitude 3.0 of 3 April (with a large rms relative error 0.4). The second sequence consists of 5 earthquakes with magnitudes from 1.9 to 1.1, which occurred on 6 April. The fit, with a similar large error, indicates the occurrence of a main shock at the time $01 : 35$; the l'Aquila earthquake occurred at $01 : 32$ (UTC; the last foreshock was recorded at $01 : 20$). The data used in this analysis are taken from the Bollettino Sismico Italiano, 2002-2012, in $\pm 25 km$ an area around the epicentre of the l'Aquila earthquake ($42.342°$ latitude, $13.380°$ longitude). The lack of the background seismicity parameters $\beta$ and $- \ln t_0$ prevents us from estimating the magnitude of the main shocks for l'Aquila. We note that the use of local magnitudes in equation (4) generates (small) errors.

**Discussion and conclusions.** We have presented above a procedure which can be used in short-term forecasting of seismic mainshocks. The procedure is based on the correlations which may be present between foreshocks and the mainshock. The presence of the correlations has a statistical character. It is not necessary that they exist always, and we do not know apriori when they exist or not. According to the theory, these correlations produce an abrupt magnitude-descending sequence of foreshocks in the proximity of a mainshock, but not all magnitude-descending precursory events are necessarily correlated foreshocks. Prior to a mainshock (as well as in the subsequent lapse of time) the seismic conditions of the focal region may suffer changes, which are unknown. The theoretical considerations on which the present procedure is based assume that all the factors which may intervene remain the same. In particular, one component of the procedure - the determination of the magnitude of the mainshock - assumes that the background seismicity preserves its statistical parameters. Consequently, the procedure presented above may exhibit important limitations. For instance, between the moment of forecasting and the predicted occurrence moment of a mainshock the local seismic conditions may change, such that we may have a false positive. Also, a similar change may lead to a false negative. Nevertheless, if correlations exist and nothing else changes, we can forecast the occurrence time and the magnitude of a mainshock, by using the abrupt magnitude-descending sequence of foreshocks which occur in its both spatial and temporal proximity.

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