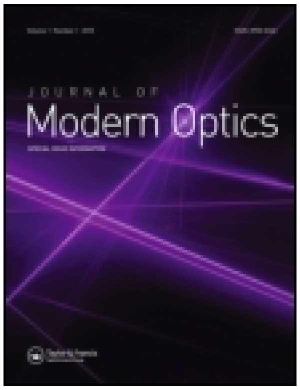

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Scattering of the electromagnetic waves from a rough surface

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Scattering of the electromagnetic waves from a rough surface

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The electromagnetic field scattered by a rough surface of a semi-infinite body is computed up to the second order of a perturbation scheme with the surface roughness as a perturbation parameter. The calculations are based on the equation of motion of the polarization within the Lorentz–Drude (plasma) model of polarizable, non-magnetic, homogeneous matter. The surface roughness contributes both to the main (specularly) reflected and refracted fields and diffuse scattering, or gives rise to secondary (second-order) diffraction peaks for a regular grating. The calculations are performed both for the *s*- and *p*-waves. Two-dimensional modes, resonant at certain frequencies, are identified, confined to and propagating only on the surface, as a consequence of the surface roughness.

Keywords: surface roughness; scattered electromagnetic field; Lorentz–Drude model; polarization motion

1. Introduction

Recently, there has been a great deal of interest in the role played by the surface roughness (corrugation) in a large variety of physical phenomena, including the dispersive properties of the surface plasmon-polariton in nanoplasmonics [1], terahertz-wave generation and detection [2] and electronic microstructures [3,4]. Enhanced, or suppressed, optical transmission in the subwavelength regime is associated with surface corrugation [5], which induces a highly-directional optical emission [6]. Multiple scattering has been emphasized, both experimentally and theoretically, in light scattered diffusely by a randomly rough surface [7], and the scattering theory within the Born approximation was applied early to the surface roughness modelled by a dispersive (position dependent) dielectric function [8]. A recent review can be found in [9].

The main difficulty in getting more definite results in such problems resides in modelling conveniently the surface roughness, so as to arrive at more mathematically operational approaches [10]. We present here a theoretical-perturbation scheme which can lead conveniently to an estimation of the effects of a rough surface in the scattering of electromagnetic waves. Perturbation theory is applied for a long time to this problem, by means of the so-called reduced Rayleigh equations, or, equivalenty, by using the extinction optical theorem, or other equivalent techniques [9]. The approach presented here is different. It delineates the surface roughness as a scattering entity distinct from the bulk and introduces explicitly the dynamics of the polarization. Beside offering more manageable analytic results, the present approach contributes to clarifying the physical content of the scattering of the electromagnetic waves by a rough surface.

The interest in the effect of rough surfaces on wave propagation has a long history, arising probably for the first time with Wood's [11] and Lord Rayleigh's works [12,13], where the approximation, called later the Rayleigh hypothesis, was introduced. For a sufficiently smooth surface, the scattered fields are represented as a superposition of plane waves with different in-plane (parallel to the surface) wavevectors. While such a representation is accepted as being reasonable at large distance from the surface, it may appear incomplete near a surface, especially one with a complex roughness, where multiple, internal scattering events, taking place within the rough structure, may invalidate it. For a surface roughness which is sufficiently small and smooth the hypothesis may be valid [14,15]. Great progress has been made with recognizing the surface roughness function as a small parameter in a perturbation theory [16], both a first-order theory and a systematic expansion one [17–26]. On the other hand, the role played by the polarization of matter in producing reflected, refracted and scattered electromagnetic waves was emphasized earlier [8]. The extent to which multiple scattering from the surface roughness or the constructive interference of the multiply scattered surface plasmon-polariton modes contribute to the enhancement of the (retroreflection) backscattering is still a matter of investigation.

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An expansion of the dielectric function in powers of surface roughness, beside obscuring the polarization charge and current sources, leads to technical difficulties related to localized delta-functions acting upon discontinuous functions, which may plague the results [8]. Higher-order expansions in the surface roughness function have been carried out [23–28], up to third and fourth order, using the reduced Rayleigh equations, in search of resonant-like phenomena that may occur in higher orders. Beside being rather cumbersome, such expansions are still of limited validity, on the one hand, especially when using the Rayleigh hypothesis (as usually done), and bring only very small contributions, on the other hand, leaving the convergence of the series expansion an open question. This is one of the reasons why most studies resort to numerical computations, which, unfortunately, require high-level accuracy [28].

If one assumes that the surface roughness is a small parameter in the problem, and its effects can be estimated by perturbation theory, then it is legitimate to leave aside a possible complex internal structure of the roughness, and view the surface roughness as an independent scattering entity, with its own global dynamics. Such a model is suitable not only for rough surfaces, but it can be generalized to superficial thin films, or coatings, or other structures grown on surfaces. We present here a perturbation-theoretical scheme, with the surface roughness as a perturbation parameter, which allows the computation of the electromagnetic field scattered by a rough surface of a semi-infinite body. The scheme is based on the equation of motion of the polarization, whose degrees of freedom are explicitly introduced, within the well-known Lorentz–Drude (plasma) model of polarizable, non-magnetic, homogeneous matter. Such an approach allows the explicit identification of the polarization charges and currents in the body, which produce the reflected, refracted and scattered fields. These fields can be computed straightforwardly, via integral equations, where the contribution of the surface roughness can be included as a perturbation. Supplemented by the equation of motion for the polarization, the integral equations can be solved, leading to explicit, analytical results. Not in the least, the structure of the integral equations allows the use of a reference frame bound to the wavevectors, which, technically, simplifies appreciably the calculations. The scattered field is computed up to the second order of the perturbation theory. Contributions pertaining to the main (specularly) reflected and refracted field and the diffuse scattering, as well as secondary peaks for regular grating, are clearly delineated by this technique. It is shown that the surface roughness generates surface modes, i.e. modes confined strictly to and propagating only on the surface (two-dimensional waves), resonant at certain frequencies.

2. Semi-infinite solid with a rough surface

We consider a polarizable homogeneous body with a density n of mobile charges q (e.g. electrons) moving in a uniform rigid neutralizing background. A small displacement $\mathbf{u}(\mathbf{R}, t)$ of these charges, where $\mathbf{R} = (\mathbf{r}, z)$, $\mathbf{r} = (x, y)$ is the position vector and t denotes the time, produces a change density $\rho = -nq\,\mathrm{div}\,\mathbf{u}$ and a current density $\mathbf{j} = nq\dot{\mathbf{u}}$, corresponding to a polarization $\mathbf{P} = nq\mathbf{u}$. The vector potential is given by

$$\mathbf{A}(\mathbf{R}, t) = \frac{1}{c}\int \mathrm{d}\mathbf{R}' \frac{\mathbf{j}(\mathbf{R}', t - |\mathbf{R} - \mathbf{R}'|/c)}{|\mathbf{R} - \mathbf{R}'|}, \tag{1}$$

or, taking the temporal Fourier transform,

$$\mathbf{A}(\mathbf{R}, \omega) = \frac{1}{c}\int \mathrm{d}\mathbf{R}' \frac{\mathbf{j}(\mathbf{R}', \omega)}{|\mathbf{R} - \mathbf{R}'|} \exp\left(\mathrm{i}\lambda|\mathbf{R} - \mathbf{R}'|\right), \tag{2}$$

where $\lambda = \omega/c$. The scalar potential Φ is obtained from $\mathrm{div}\,\mathbf{A} = \mathrm{i}\lambda\Phi$ (Lorenz gauge) and the fields are given by $\mathbf{E} = \mathrm{i}\lambda\mathbf{A} - \mathrm{grad}\Phi$ (electric field), $\mathbf{H} = \mathrm{curl}\,\mathbf{A}$ or $\mathrm{curl}\,\mathbf{E} = \mathrm{i}\lambda\mathbf{H}$ (magnetic field). We use the well-known decomposition [29]

$$\frac{\exp\left(\mathrm{i}\lambda|\mathbf{R} - \mathbf{R}'|\right)}{|\mathbf{R} - \mathbf{R}'|} = \frac{\mathrm{i}}{2\pi}\int \mathrm{d}\mathbf{k}\frac{1}{\kappa}\exp[\mathrm{i}\mathbf{k}(\mathbf{r} - \mathbf{r}')]\exp\left(\mathrm{i}\kappa|z - z'|\right) \tag{3}$$

for the spherical wave (Green function) in Equation (2), where $\kappa = (\lambda^2 - k^2)^{1/2}$, as well as Fourier transforms of the type

$$\mathbf{u}(\mathbf{r}, z) = \frac{1}{(2\pi)^2}\int \mathrm{d}\mathbf{k}\mathbf{u}(\mathbf{k}, z)\exp(\mathrm{i}\mathbf{k}\mathbf{r}) = \frac{1}{S}\sum_{\mathbf{k}}\mathbf{u}(\mathbf{k}, z)\exp(\mathrm{i}\mathbf{k}\mathbf{r}), \tag{4}$$

where S is the surface area. For simplicity, the argument ω is omitted in such formulae, as well as, occasionally, the wavevector argument $\mathbf{k}$.

Next, we consider a semi-infinite body extending over the region $z > h(\mathbf{r})$, where $h(\mathbf{r})$, with $\int \mathrm{d}\mathbf{r}\, h(\mathbf{r}) = 0$, is the surface roughness function, to be further specified. The polarization for this body is taken as

$$\mathbf{P} = nq(\mathbf{u}, u_z)\theta(z - h(\mathbf{r})), \tag{5}$$

where $\mathbf{u}$ lies in the $\mathbf{r}$-plane, u_z is directed along the z-direction and θ is the step function ($\theta(z) = 1$ for $z > 0$, $\theta(z) = 0$ for $z < 0$). The electromagnetic potentials

(Fourier transforms) can easily be written as

$$\mathbf{A} = \frac{2\pi\lambda}{\kappa}\int \mathrm{d}\mathbf{r}'\int_{h(\mathbf{r}')} \mathrm{d}z'(\mathbf{u}, u_z)\exp\left(\mathrm{i}\kappa|z-z'|\right)\exp(-\mathrm{i}\mathbf{k}\mathbf{r}'),$$
$$\Phi = \frac{2\pi}{\kappa}\int \mathrm{d}\mathbf{r}'\int_{h(\mathbf{r}')} \mathrm{d}z'(\mathbf{k}\mathbf{u}, \kappa\,\mathrm{sgn}(z-z')u_z) \times \exp\left(\mathrm{i}\kappa|z-z'|\right)\exp(-\mathrm{i}\mathbf{k}\mathbf{r}'). \quad (6)$$

In order to compute the electric field it is convenient to refer the in-plane vectors (i.e. vectors parallel with the surface of the half-space) to the vectors $\mathbf{k}$ and $\mathbf{k}_\perp = e_z \times \mathbf{k}$, where e_z is the unit vector along the z-direction; for instance, we write

$$\mathbf{u} = u_1\frac{\mathbf{k}}{k} + u_2\frac{\mathbf{k}_\perp}{k} \quad (7)$$

and a similar representation for the electric field parallel with the surface of the half-space. The components u_1 and u_z correspond to the p-wave (parallel wave), while the component u_2 corresponds to the s-wave (from the German 'senkrecht' which means 'perpendicular'). In performing the calculations, it is worth paying attention to the derivative of the modulus function, according to the equation

$$\frac{\partial^2}{\partial z^2}\exp\left(\mathrm{i}\kappa|z-z'|\right) = -\kappa^2\exp\left(\mathrm{i}\kappa|z-z'|\right) + 2\mathrm{i}\kappa\delta(z-z'). \quad (8)$$

We get the electric field

$$E_1 = 2\pi\mathrm{i}\kappa\int \mathrm{d}\mathbf{r}'\int_{h(\mathbf{r}')} \mathrm{d}z' u_1\exp\left(\mathrm{i}\kappa|z-z'|\right)\exp(-\mathrm{i}\mathbf{k}\mathbf{r}') - \frac{2\pi k}{\kappa}\frac{\partial}{\partial z}\int \mathrm{d}\mathbf{r}'\int_{h(\mathbf{r}')} \mathrm{d}z' u_z\exp\left(\mathrm{i}\kappa|z-z'|\right)\exp(-\mathrm{i}\mathbf{k}\mathbf{r}'),$$
$$E_2 = \frac{2\pi\mathrm{i}\lambda^2}{\kappa}\int \mathrm{d}\mathbf{r}'\int_{h(\mathbf{r}')} \mathrm{d}z' u_2\exp\left(\mathrm{i}\kappa|z-z'|\right)\exp(-\mathrm{i}\mathbf{k}\mathbf{r}'),$$
$$E_z = -\frac{2\pi k}{\kappa}\frac{\partial}{\partial z}\int \mathrm{d}\mathbf{r}'\int_{h(\mathbf{r}')} \mathrm{d}z' u_1\exp\left(\mathrm{i}\kappa|z-z'|\right)\exp(-\mathrm{i}\mathbf{k}\mathbf{r}') + \frac{2\pi\mathrm{i}k^2}{\kappa}\int \mathrm{d}\mathbf{r}'\int_{h(\mathbf{r}')} \mathrm{d}z' u_z\exp\left(\mathrm{i}\kappa|z-z'|\right)\exp(-\mathrm{i}\mathbf{k}\mathbf{r}') - 4\pi\int \mathrm{d}\mathbf{r}'\int_{h(\mathbf{r}')} \mathrm{d}z'\theta(z-h(\mathbf{r}'))u_z\exp(-\mathrm{i}\mathbf{k}\mathbf{r}'). \quad (9)$$

It is easy to check the identities

$$\mathrm{i}kE_1 + \frac{\partial E_z}{\partial z} = -4\pi\int \mathrm{d}\mathbf{r}'\theta(z-h(\mathbf{r}'))\left(\mathrm{i}ku_1 + \frac{\partial u_z}{\partial z}\right) \times \exp(-\mathrm{i}\mathbf{k}\mathbf{r}') - 4\pi\int \mathrm{d}\mathbf{r}'\delta(z-h(\mathbf{r}'))u_z\exp(-\mathrm{i}\mathbf{k}\mathbf{r}'),$$
$$k\frac{\partial E_1}{\partial z} + \mathrm{i}\kappa^2 E_z = -4\pi\mathrm{i}\lambda^2\int \mathrm{d}\mathbf{r}'\theta(z-h(\mathbf{r}'))u_z\exp(-\mathrm{i}\mathbf{k}\mathbf{r}'), \quad (10)$$

which are the expression of Maxwell equations for this geometry (the first equation above represents the Gauss' law, while the second equation arises from the combination of the Faraday and Maxwell–Ampere equations). We note that by the change of variable $z' \to z' + h(\mathbf{r}')$ the factor $\exp[\pm\mathrm{i}\kappa h(\mathbf{r}')]$ appears in Equations (9), which is familiar in the reduced Rayleigh equations and other techniques [9,19].

We assume that the magnitude of the roughness function $h(\mathbf{r})$ is much smaller than the relevant wavelengths of the electromagnetic field and use the approximation

$$\mathbf{P} = \mathbf{P}^{(0)} + \mathbf{P}^{(1)} + \mathbf{P}^{(2)} + \cdots,$$
$$\mathbf{P}^{(0)} = nq(\mathbf{u}, u_z)\theta(z), \quad \mathbf{P}^{(1)} = -nqh(\mathbf{r})(\mathbf{u}, u_z)\delta(z), \quad (11)$$
$$\mathbf{P}^{(2)} = \frac{1}{2}nqh^2(\mathbf{r})(\mathbf{u}, u_z)\delta'(z),$$

where $\delta(z)$ is the Dirac delta-function and $\delta'(z)$ its derivative. It is worth noting the specific conditions of validity for such an approximation (which leads to a second-order perturbation-theoretical approach). First, we note that $\theta(z-h(\mathbf{r}))$ should be treated as a distribution (e.g. products like $\theta^2(z-h(\mathbf{r})) = \theta(z-h(\mathbf{r}))$ should be avoided when using the expansion (11)). Next, the surface coordinate $z = h(\mathbf{r})$ should have a smooth variation; therefore, we restrict ourselves to the limit $\mathbf{q} \to 0$ in the Fourier transform $h(\mathbf{q})$, i.e. the in-plane surface modulation length must be longer than the relevant wavelength of the incident field. In addition, we note that the expansion given by Equation (11) simplifies appreciably the internal structure of the surface roughness. In particular, we lose the multiple and depolarizing scattering. The expansion given by Equation (11) transforms a semi-infinite body with a rough surface into a half-space with a plane surface plus an additional surface layer of zero thickness, localized on the plane surface. This surface layer brings its own contribution to the energy flux of the scattered waves, which adds to the (conserving) energy flux corresponding to the half-space [17,19,30]. On the other hand, this later simplification opens the possibility of treating surface layers with a structure different from the bulk, like coatings, deposited thin films, or other surface structures built on a plane surface. The polarization $\mathbf{P}^{(0)}$ corresponds to a half-space extending over the region $z > 0$, while the polarizations $\mathbf{P}^{(1,2)}$ are surface polarizations localized on the surface $z = 0$. They describe the surface roughness as a two-dimensional independent, additional structure, related to (superposed on), but distinct from the main body (half-space).

We expand the electromagnetic potentials and fields given above (Equations (6) and (9)) in a similar manner. The calculations are straightforward.

Leaving aside the factor nq (it is restored in the final formulae), we get (for the Fourier transforms)

$$\mathbf{A}^{(0)} = \frac{2\pi\lambda}{\kappa}\int_0 dz'(\mathbf{u},\, u_z)\exp\left(i\kappa|z-z'|\right),$$
$$\Phi^{(0)} = \frac{2\pi}{\kappa}\int_0^\infty dz'\mathbf{ku}\exp\left(i\kappa|z-z'|\right) - \frac{2\pi i}{\kappa}\frac{\partial}{\partial z}\int_0^\infty dz' u_z\exp\left(i\kappa|z-z'|\right), \qquad (12)$$

$$\mathbf{A}^{(1)}(\mathbf{k},z) = -\frac{2\pi\lambda}{\kappa}(\mathbf{g}(\mathbf{k}),\, g_z(\mathbf{k}))\exp(i\kappa|z|),$$
$$\Phi^{(1)}(\mathbf{k},z) = -\frac{2\pi}{\kappa}[\mathbf{kg}(\mathbf{k}) + \kappa\,\mathrm{sgn}(z)\,g_z(\mathbf{k})]\exp(i\kappa|z|) \qquad (13)$$

and

$$\mathbf{A}^{(2)}(\mathbf{k},z) = -\frac{\pi\lambda}{\kappa}\left(\mathbf{s}'(\mathbf{k}) - i\kappa\,\mathrm{sgn}(z)\mathbf{s}(\mathbf{k}),\; s'_z(\mathbf{k}) - i\kappa\,\mathrm{sgn}(z)s_z(\mathbf{k})\right)\exp(i\kappa|z|),$$
$$\Phi^{(2)}(\mathbf{k},z) = -\frac{\pi}{\kappa}[\mathbf{ks}'(\mathbf{k}) - i\kappa\,\mathrm{sgn}(z)\mathbf{ks}(\mathbf{k})]\exp(i\kappa|z|) - \frac{\pi}{\kappa}\left[\kappa\,\mathrm{sgn}(z)s'_z(\mathbf{k}) - i\kappa^2 s_z(\mathbf{k})\right]\exp(i\kappa|z|) + 2\pi s_z(\mathbf{k})\delta(z), \qquad (14)$$

where

$$(\mathbf{g}(\mathbf{k}), g_z(\mathbf{k})) = \int d\mathbf{r}h(\mathbf{r})(\mathbf{u}(\mathbf{r},z),\, u_z(\mathbf{r},z)\big|_{z=0}\exp(-i\mathbf{kr}),$$
$$(\mathbf{s}(\mathbf{k}), s_z(\mathbf{k})) = \int d\mathbf{r}h^2(\mathbf{r})(\mathbf{u}(\mathbf{r},z),\, u_z(\mathbf{r},z))\big|_{z=0}\exp(-i\mathbf{kr}),$$
$$(\mathbf{s}'(\mathbf{k}), s'_z(\mathbf{k})) = \int d\mathbf{r}h^2(\mathbf{r})\left(\frac{\partial\mathbf{u}(\mathbf{r},z)}{\partial z},\, \frac{\partial u_z(\mathbf{r},z)}{\partial z}\right)\bigg|_{z=0}\exp(-i\mathbf{kr}) \qquad (15)$$

($\mathrm{sgn}(z) = +1$ for $z > 0$, $\mathrm{sgn}(z) = -1$ for $z < 0$). Occasionally, we may omit the argument $\mathbf{k}$ of these functions. We can see that higher-order terms in powers of $h(\mathbf{r})$ can be included in the expansion of the polarization given by Equation (11), leading to higher-order contributions to the electromagnetic potentials and fields.

Similarly, we get the expansion

$$E_1^{(0)} = 2\pi i\kappa\int_0^\infty dz' u_1\exp\left(i\kappa|z-z'|\right) - \frac{2\pi k}{\kappa}\frac{\partial}{\partial z}\int_0^\infty dz' u_z\exp\left(i\kappa|z-z'|\right),$$
$$E_2^{(0)} = \frac{2\pi i\lambda^2}{\kappa}\int_0^\infty dz' u_2\exp\left(i\kappa|z-z'|\right),$$
$$E_z^{(0)} = -\frac{2\pi k}{\kappa}\frac{\partial}{\partial z}\int_0^\infty dz' u_1\exp\left(i\kappa|z-z'|\right) + \frac{2\pi i k^2}{\kappa}\int_0^\infty dz' u_z\exp\left(i\kappa|z-z'|\right) - 4\pi u_z\theta(z), \qquad (16)$$

$$E_1^{(1)} = -2\pi i(\kappa g_1 - \mathrm{sgn}(z)kg_z)\exp(i\kappa|z|),$$
$$E_2^{(1)} = -\frac{2\pi i\lambda^2}{\kappa}g_2\exp(i\kappa|z|),$$
$$E_z^{(1)} = 2\pi ik\left(\mathrm{sgn}(z)\,g_1 - \frac{k}{\kappa}g_z\right)\exp(i\kappa|z|) + 4\pi g_z\delta(z) \qquad (17)$$

and

$$E_1^{(2)} = -\pi\kappa\left(is'_1 + \kappa\,\mathrm{sgn}(z)s_1\right)\exp(i\kappa|z|) + \pi k\left(i\,\mathrm{sgn}(z)s'_z + \kappa s_z\right)\exp(i\kappa|z|) - 2\pi iks_z\delta(z),$$
$$E_2^{(2)} = -\frac{\pi\lambda^2}{\kappa}\left(is'_2 + \kappa\,\mathrm{sgn}(z)s_2\right)\exp(i\kappa|z|),$$
$$E_z^{(2)} = \pi k\left(i\,\mathrm{sgn}(z)s'_1 + \kappa s_1\right)\exp(i\kappa|z|) - \frac{\pi k^2}{\kappa}\left(is'_z + \kappa\,\mathrm{sgn}(z)s_z\right)\exp(i\kappa|z|) - 2\pi\left(iks_1 - s'_z\right)\delta(z) - 2\pi s_z\delta'(z) \qquad (18)$$

for the electric field, where $g_{1,2}$ and $s_{1,2}$ are the projections of $\mathbf{g}$ and, respectively, $\mathbf{s}$ on the vectors $\mathbf{k}$ and, respectively, $\mathbf{k}_\perp$. Equations (10) become

$$ikE_1^{(0)} + \frac{\partial E_z^{(0)}}{\partial z} = -4\pi\left(iku_1 + \frac{\partial u_z}{\partial z}\right)\theta(z) - 4\pi u_z(z)\big|_{z=0}\delta(z), \qquad (19)$$
$$k\frac{\partial E_1^{(0)}}{\partial z} + i\kappa^2 E_z^{(0)} = -4\pi i\lambda^2 u_z\theta(z),$$

$$ikE_1^{(1)} + \frac{\partial E_z^{(1)}}{\partial z} = 4\pi ikg_1\delta(z) + 4\pi g_z\delta'(z),$$
$$k\frac{\partial E_1^{(1)}}{\partial z} + i\kappa^2 E_z^{(1)} = 4\pi i\lambda^2 g_z\delta(z) \qquad (20)$$

and

$$ikE_1^{(2)} + \frac{\partial E_z^{(2)}}{\partial z} = 2\pi iks'_1\delta(z) - 2\pi i(ks_1 + is'_z)\delta'(z) - 2\pi s_z\delta''(z),$$
$$k\frac{\partial E_1^{(2)}}{\partial z} + i\kappa^2 E_z^{(2)} = 2\pi i\lambda^2(s'_z\delta(z) - s_z\delta'(z)). \qquad (21)$$

3. Equations of motion

In polarizable homogeneous (non-magnetic) matter the displacement field $\mathbf{u}(\mathbf{R},t)$ is subjected to Newton's equation of motion

$$m\ddot{\mathbf{u}} = q(\mathbf{E} + \mathbf{E}_0) - m\omega_c^2\mathbf{u} - m\gamma\dot{\mathbf{u}}, \qquad (22)$$

where m is the mass of the mobile charges, $\mathbf{E}$ is the electric field of the polarization charges and currents (calculated in the previous section), $\mathbf{E}_0$ is an external electric field, ω_c is a characteristic frequency and γ is a damping coefficient. This is the well-known Lorentz–Drude (plasma) model of matter polarization [31–33]. Taking the temporal Fourier transform of Equation (22), with $\mathbf{E}_t = \mathbf{E} + \mathbf{E}_0$ the total electric field, we get the electric susceptibility $\chi(\omega) = P/E_t$ ($\mathbf{P} = nq\mathbf{u}$) and the dielectric function

$$\varepsilon(\omega) = 1 + 4\pi\chi(\omega) = \frac{\omega^2 - \omega_c^2 - \omega_p^2}{\omega^2 - \omega_c^2 + i\omega\gamma}, \qquad (23)$$

where $\omega_p = (4\pi nq^2/m)^{1/2}$ is the plasma frequency. As is well known, for $\omega_c = 0$ in Equation (23) we get the dielectric function of a conductor, while $\omega_c \neq 0$ corresponds to dielectrics. This is also known in solids as the Lydane–Sachs–Teller dielectric function [34], with the longitudinal frequency $\omega_L = (\omega_c^2 + \omega_p^2)^{1/2}$ and the transverse frequency $\omega_T = \omega_c$. In general, the damping coefficient γ is much smaller than these frequencies, so we limit ourselves to the ideal case $\gamma = 0$. It is worth noting the absence of the magnetic part of the Lorentz force in Equation (22), according to the non-relativistic motion of the slight displacement $\mathbf{u}$. It is easy to see that, apart from relativistic contributions, it would introduce non-linearities in Equation (22), which are beyond our assumption of a small displacement $\mathbf{u}$. Using spatial Fourier transforms, this approximation can be formulated as $\mathbf{K}\mathbf{u}(\mathbf{K}) \ll 1$, where $\mathbf{K} = (\mathbf{k}, \kappa)$ is the wavevector.

For temporal Fourier transforms Equation (22) can also be written as ($\gamma = 0$)

$$(\omega^2 - \omega_c^2)\mathbf{u} = -\frac{q}{m}(\mathbf{E} + \mathbf{E}_0). \qquad (24)$$

This equation holds for $z > h(\mathbf{r})$. In principle, we can introduce here the electric field $\mathbf{E}$ whose in-plane Fourier transforms are given by Equations (9) and solve the resulting integral equation for $\mathbf{u}$. Unfortunately, the *rhs* of Equation (24) includes the in-plane Fourier transforms, while its *lhs* includes the components in the direct space, and either the inverse Fourier transforms $\mathbf{E}(\mathbf{r}, z)$ are complicated, or the in-plane $\mathbf{k}$-Fourier transforms of $\mathbf{u}(\mathbf{r}, z)$ are inapplicable (because of the $\mathbf{r}$-dependence of the z-coordinate in the condition $z > h(\mathbf{r})$). The expansion given by Equations (11) and (16)–(18) simplifies appreciably Equation (24). We solve Equation (24) for the displacement field $\mathbf{u}$, using this expansion and assuming that the external field $\mathbf{E}_0$ is a plane wave. According to Equation (24), the field transmitted into the body is proportional to $\mathbf{u}$, while the reflected field can be computed by making use of the same Equations (16)–(18). The field scattered by the rough surface can be identified on these solutions as corresponding to terms containing the surface roughness function $h(\mathbf{r})$.

4. Plane surface: zeroth order approximation

We focus now on Equations (24) for a plane surface ($h(\mathbf{r}) = 0$), with the electric field $\mathbf{E} = \mathbf{E}^{(0)}$ given by Equations (16); the displacement field is denoted by $\mathbf{u}^{(0)}$. The incident plane wave is described by $\mathbf{E}_0 \exp(-i\omega t + i\mathbf{k}\mathbf{r} + i\kappa z)$ (with $\kappa = (\lambda^2 - k^2)^{1/2}$). We take the second derivative of Equation (24) for $u_2^{(0)}$ with respect to z and use the relation given by Equation (8). We get

$$\frac{\partial^2 u_2^{(0)}}{\partial z^2} + \kappa'^2 u_2^{(0)} = 0, \qquad (25)$$

where

$$\kappa'^2 = \kappa^2 - \frac{\lambda^2\omega_p^2}{\omega^2 - \omega_c^2} = \lambda^2\varepsilon - k^2, \qquad (26)$$

ε being the dielectric function given by Equation (23). Therefore, the solution is

$$u_2^{(0)} = A_2^{(0)} \exp(i\kappa' z), \qquad (27)$$

where $A_2^{(0)}$ is a constant amplitude. Since, by Equation (24), $\mathbf{E}_t = \mathbf{E} + \mathbf{E}_0 \sim \mathbf{u}$, we can see that the field propagates in the half-space with a modified wavevector κ', according to the Ewald–Oseen extinction theorem [35]. By Equation (26), we can check the well-known polaritonic dispersion relation $\varepsilon\omega^2 = c^2K'^2$, where $\mathbf{K}' = (\mathbf{k}, \kappa')$ is the wavevector of the field propagating in the body. From $\sin\theta_0 = k/K$, $\sin\theta_r = k/K'$, and using Equation (26), we check the refraction law $\sin\theta_r = \sin\theta_0/\varepsilon^{1/2}$, where θ_0 is the angle of incidence and θ_r is the angle of refraction. Introducing the solution given by Equation (27) in Equation (24) (and making use of the field given by Equations (16)) we get the amplitude $A_2^{(0)}$ given by

$$A_2^{(0)} \frac{\omega_p^2\lambda^2}{2\kappa(\kappa' - \kappa)} = \frac{q}{m} E_{02}, \qquad (28)$$

where E_{02} is the amplitude of the s-component of the external field. Making use of Equations (16) for $z < 0$ we get the reflected field

$$E_2^{(0)} = -2\pi nq A_2^{(0)} \frac{\lambda^2}{\kappa(\kappa' + \kappa)} \exp(-i\kappa z) = \frac{\kappa - \kappa'}{\kappa + \kappa'} E_{02} \exp(-i\kappa z), \quad z < 0. \qquad (29)$$

The (total) electric field inside the half-space is obtained from Equation (24) as

$$E_2^{(0)} = -\frac{m}{q}(\omega^2 - \omega_c^2)A_2^{(0)}\exp(i\kappa' z) = \frac{2\kappa}{\kappa + \kappa'}E_{02}\exp(i\kappa' z), \quad z > 0. \tag{30}$$

It is easy to see that Equation (29) defines the well-known Fresnel reflection coefficient [36]

$$R_s = \left|\frac{E_2^{(0)}}{E_{02}}\right|^2 = \left|\frac{\kappa - \kappa'}{\kappa + \kappa'}\right|^2 = \left|\frac{\cos\theta_0 - \varepsilon^{1/2}\cos\theta_r}{\cos\theta_0 + \varepsilon^{1/2}\cos\theta_r}\right|^2, \quad z < 0, \tag{31}$$

for the s-wave. Similarly, Equation (30) defines the transmission coefficient for the s-wave ($T_s = (\kappa'/\kappa)|E_2^{(0)}/E_{02}|^2$).

Equation (24) is solved in a similar way for $u_{1,z}^{(0)}$. It is convenient to form the combinations $iku_1^{(0)} + \partial u_z^{(0)}/\partial z$ and $k\partial u_1^{(0)}/\partial z + i\kappa^2 u_z^{(0)}$, and use the relations given by Equations (19). We find immediately that $u_{1,z}^{(0)}$ satisfy the same Equation (25), with solutions

$$u_1^{(0)} = A_1^{(0)}\exp(i\kappa' z), \quad u_z^{(0)} = -\frac{k}{\kappa'}A_1^{(0)}\exp(i\kappa' z), \tag{32}$$

where the amplitude $A_1^{(0)}$ is given by

$$A_1^{(0)}\omega_p^2\frac{\kappa\kappa' + k^2}{2\kappa'(\kappa' - \kappa)} = \frac{q}{m}E_{01}, \tag{33}$$

E_{01} being the amplitude of the p-component of the external field. Inserting the solution given by Equation (32) in Equations (16) for $z < 0$ we get the reflected field

$$E_1^{(0)} = -2\pi nqA_1^{(0)}\frac{\kappa\kappa' - k^2}{\kappa'(\kappa' + \kappa)}\exp(-i\kappa z) = -\frac{\kappa' - \kappa}{\kappa' + \kappa}\cdot\frac{\kappa\kappa' - k^2}{\kappa\kappa' + k^2}E_{01}\exp(-i\kappa z), \quad z < 0 \tag{34}$$

and $E_z^{(0)} = (k/\kappa)E_1^{(0)}$; hence, the Fresnel reflection coefficient

$$R_p = \left|\frac{\varepsilon^{1/2}\cos\theta_0 - \cos\theta_r}{\varepsilon^{1/2}\cos\theta_0 + \cos\theta_r}\right|^2 \tag{35}$$

for the p-wave [36]. The transmission coefficient is obtained from the (total) electric field which is proportional to $\mathbf{u}^{(0)}$ (Equation (24)). In both cases (s- and p-waves) we can check that the reflection and transmission coefficients add to unity, as expected.

It is worth noting that there appears a resonance in Equation (34) for $\kappa\kappa' + k^2 = 0$, provided κ and κ' are both purely imaginary. This resonance is given by

$$\omega^2 = \frac{2c^2k^2(\omega_L^2 + \omega_T^2)}{\omega_L^2 + 2c^2k^2 + [(\omega_L^2 - 2c^2k^2)^2 - 4c^2k^2\omega_T^2]^{1/2}}. \tag{36}$$

We can see that in the long-wavelength limit $k \to 0$ the frequency given by Equation (36) approaches the (surface) polaritonic frequency $\omega \sim ck(1 + \omega_T^2/\omega_L^2)^{1/2}$, while in the opposite limit $k \to \infty$ we get the surface plasmon frequency $\omega \simeq [(\omega_L^2 + \omega_T^2)/2]^{1/2}$. We may call this resonance the surface plasmon-polariton mode.

We can see that the explicit introduction of the polarization (displacement field $\mathbf{u}$, u_z) and use of its equation of motion (24) allows one to derive straightforwardly the reflected and refracted field (via integral Equations (16)) for a semi-infinite body with a plane surface. This field is the zeroth order approximation of our perturbation scheme for the surface roughness.

5. Localized modes

From Equation (11) we can see that there is a polarization

$$\mathbf{P}_l = \mathbf{P}^{(1)} + \mathbf{P}^{(2)} + = -nqh(\mathbf{r})(\mathbf{u}, u_z)\delta(z) + \frac{1}{2}nqh^2(\mathbf{r})(\mathbf{u}, u_z)\delta'(z)\ldots, \tag{37}$$

localized on the rough surface. Leaving aside the factor nq and taking the in-plane Fourier transform, the components of this polarization can be written as

$$P_{l1,2} = -\left(g_{1,2} + \frac{1}{2}s'_{1,2}\right)\delta(z), \; P_{lz} = -\left(g_z + \frac{1}{2}s'_z\right)\delta(z). \tag{38}$$

It is subjected to the in-plane equation of motion (24)

$$(\omega^2 - \omega_c^2)\mathbf{P}_l = -\frac{\omega_p^2}{4\pi}\mathbf{E}_l \tag{39}$$

under the action of the localized electric field

$$E_{l1} = -2\pi iks_z\delta(z), \; E_{l2} = 0, \quad E_{lz} = 4\pi g_z\delta(z) - 2\pi\left(iks_1 - s'_z\right)\delta(z), \tag{40}$$

as given by Equations (17) and (18) (the δ'-term in Equation (18) does not contribute). We leave aside the external field, whose contribution to Equation (39) is very small. Equation (39) becomes

$$(\omega^2 - \omega_c^2)\left(g_2 + \frac{1}{2}s'_2\right) = 0, \; (\omega^2 - \omega_c^2)g_1 = 0, \quad (\omega^2 - \omega_c^2)s'_1 = -i\omega_p^2 ks_z, \; (\omega^2 - \omega_c^2)g_z = \omega_p^2 g_z, \quad (\omega^2 - \omega_c^2)s'_z = -\omega_p^2(iks_1 - s'_z). \tag{41}$$

From the first two Equations (41) we get $\omega = \omega_c$, from the fourth Equation (41) we get $\omega^2 = \omega_c^2 + \omega_p^2$. In the remaining Equations (41) we may use $u_{1,z}^{(0)}$ in estimating $s_{1,z}$, $s'_{1,z}$. Making use of Equation (32), we get $s_z = -(k/\kappa')s_1$, $s'_1 = i\kappa' s_1$, $s'_z = -iks_1$ and $\omega^2 = \omega_c^2 + 2\omega_p^2$, $(\omega^2 - \omega_c^2)(\lambda^2 - k^2) - \omega_p^2(\lambda^2 + k^2) = 0$.

All these frequencies correspond to resonant two-dimensional modes which are confined to the rough surface and propagate only on it. These modes are indicative of the dynamics of the surface roughness, viewed as an independent structure. The excitation of these modes at resonance may confine all the energy within the surface roughness. Further on, we consider only the propagating fields in Equations (17) and (18) (i.e. without $\mathbf{E}_l$). We note that for a superficial layer different in structure from the bulk the parameters ω_c and ω_p in Equation (39) are different.

6. The scattered field: *s*-wave

Equation of motion (24) for the *s*-wave reads

$$(\omega^2-\omega_c^2)u_2=-\frac{q}{m}(E_2^{(0)}+E_2^{(1)}+E_2^{(2)}+\cdots+E_{02}), \tag{42}$$

where the fields $E_2^{(0,1,2)}$ are given by Equations (16)–(18). Taking the second derivative of this equation with respect to z and using Equation (8) we find hat u_2 satisfies the same Equation (25) as the zeroth-order approximation $u_2^{(0)}$. Therefore, its solution is $u_2=A_2\exp(\mathrm{i}\kappa' z)$, where the amplitude A_2 is determined from Equations (42) and (16)–(18). We assume that the external field in this equation is a plane wave with fixed ω and $\mathbf{k}$ (and $\kappa=\sqrt{\lambda^2-k^2}$). It is convenient to write explicitly the argument $\mathbf{k}_1$ of the amplitude A_2. Equation (42) becomes

$$A_2(\mathbf{k}_1)+\mathrm{i}(\kappa_1'-\kappa_1)g_2(\mathbf{k}_1)+\frac{1}{2}(\kappa_1'-\kappa_1)\left[\mathrm{i}s_2'(\mathbf{k}_1)+\kappa_1 s_2(\mathbf{k}_1)\right]=\frac{q}{m}\frac{2\kappa(\kappa'-\kappa)}{\lambda^2\omega_p^2}E_{02}\delta_{\mathbf{k}_1,\mathbf{k}}, \tag{43}$$

where E_{02} is the amplitude of the *s*-component of the external field. The displacement field is then written as

$$u_2(\mathbf{r},z)=\frac{1}{S}\sum_{\mathbf{k}_1}A_2(\mathbf{k}_1)\exp(\mathrm{i}\mathbf{k}_1\mathbf{r}+\kappa_1' z) \tag{44}$$

(as in the Rayleigh hypothesis) and, making use of Equations (15),

$$\begin{aligned} g_2(\mathbf{k}_1)&=\frac{1}{S}\sum_{\mathbf{q}}A_2(\mathbf{k}_1-\mathbf{q})h(\mathbf{q}),\\ s_2(\mathbf{k}_1)&=\frac{1}{S^2}\sum_{\mathbf{q},\mathbf{q}'}A_2(\mathbf{k}_1-\mathbf{q}-\mathbf{q}')h(\mathbf{q})h(\mathbf{q}'); \end{aligned} \tag{45}$$

in addition, $s_2'(\mathbf{k}_1)=\mathrm{i}\kappa_1' s_2(\mathbf{k}_1)$. Equation (43) can now be solved up to the second order of the perturbation theory. We get

$$\begin{aligned} A_2(\mathbf{k}_1)=\frac{q}{m}\frac{2\kappa(\kappa'-\kappa)}{\lambda^2\omega_p^2}E_{02}\Bigg\{&\delta_{\mathbf{k}_1,\mathbf{k}}-\frac{\mathrm{i}}{S}f(\mathbf{k}_1)h(\mathbf{k}_1-\mathbf{k})\\ &-\frac{1}{S^2}\sum_{\mathbf{q}}f(\mathbf{k}_1)\left[f(\mathbf{k}_1-\mathbf{q})-\frac{1}{2}f(\mathbf{k}_1)\right]\\ &\times h(\mathbf{q})h(\mathbf{k}_1-\mathbf{k}-\mathbf{q})\Bigg\}, \end{aligned} \tag{46}$$

where $f(\mathbf{k})=\kappa'(\mathbf{k})-\kappa(\mathbf{k})=(\lambda^2\varepsilon-k^2)^{1/2}-(\lambda^2-k^2)^{1/2}$ ($h(\mathbf{q}=0)=0$). The amplitude of the total field is given by $E_{t2}=(-m/q)(\omega^2-\omega_c^2)A_2$ and the transmission coefficient can be written as

$$\begin{aligned} T(\mathbf{k}_1)=T_s\Bigg\{&1-\frac{2}{S^2}\sum_{\mathbf{q}}\mathrm{Re}\left[f(\mathbf{k})\left(f(\mathbf{k}-\mathbf{q})-\frac{1}{2}f(\mathbf{k})\right)\right]\\ &\times|h(\mathbf{q})|^2\Bigg\}\delta_{\mathbf{k}_1,\mathbf{k}}+\frac{1}{S^2}T_s\frac{\kappa_1'}{\kappa'}\left|f(\mathbf{k}_1)h(\mathbf{k}_1-\mathbf{k})\right|^2, \end{aligned} \tag{47}$$

where T_s is the transmission coefficient of the *s*-wave for a half-space.

The reflected field can be calculated from Equations (16)–(18), making use of $A_2(\mathbf{k}_1)$ given by Equation (46). We get the amplitude of the reflected field

$$\begin{aligned} E_{r2}=\frac{\kappa-\kappa'}{\kappa+\kappa'}E_{02}\Bigg\{&\delta_{\mathbf{k}_1,\mathbf{k}}+\frac{2\mathrm{i}}{S}\frac{\kappa(\kappa'+\kappa)}{\kappa_1'+\kappa_1}h(\mathbf{k}_1-\mathbf{k})\\ &-\frac{2}{S^2}\sum_{\mathbf{q}}\frac{\kappa(\kappa'+\kappa)}{\kappa_1'+\kappa_1}[\kappa_1'-f(\mathbf{k}_1-\mathbf{q})]\\ &\times h(\mathbf{q})h(\mathbf{k}_1-\mathbf{k}-\mathbf{q})\Bigg\},\quad z<0 \end{aligned} \tag{48}$$

and the reflection coefficient

$$\begin{aligned} R(\mathbf{k}_1)=R_s\Bigg\{&1-\frac{4}{S^2}\sum_{\mathbf{q}}\mathrm{Re}[\kappa(\kappa'-f(\mathbf{k}-\mathbf{q}))]|h(\mathbf{q})|^2\Bigg\}\delta_{\mathbf{k}_1,\mathbf{k}}\\ &+\frac{4}{S^2}R_s\left|\frac{\kappa(\kappa'+\kappa)}{\kappa_1'+\kappa_1}h(\mathbf{k}_1-\mathbf{k})\right|^2, \end{aligned} \tag{49}$$

where R_s is the reflection coefficient of the *s*-wave for a half-space, as given by Equation (31). From the above equations we can see the conditions of validity of the perturbation scheme used here: the generic mean Fourier transform of the roughness function $h(\mathbf{q})/S$ must be much smaller than the relevant wavelengths, as corresponding to factors like κ, κ', etc.

We can see from Equations (46) and (48) that the rough surface gives a diffuse scattering (in-plane wavevector $\mathbf{k}_1\neq\mathbf{k}$), beside the main (specularly) reflected and refracted waves (in-plane wavevector $\mathbf{k}$), as expected. The reflection and transmission coefficients given above do not conserve the energy, i.e. they do not add to unity,

$$U = \sum_{\mathbf{k}_1} [R(\mathbf{k}_1) + T(\mathbf{k}_1)] \neq 1. \quad (50)$$

This particularity arises from the expansion of the function $\theta(z - h(\mathbf{r}))$ as given by Equation (11), which leads to a bulk (half-space), which conserves the energy ($R_s + T_s = 1$), and an additional localized, superficial layer of zero thickness, contributing its own scattered field. The perturbation-theoretical model should be corrected by normalizing the coefficients to $R(\mathbf{k}_1)/U$ and $T(\mathbf{k}_1)/U$ (up to the second order of the perturbation theory). However, this normalization is not necessary in the limit of small $\mathbf{q}$ (the region where the present approach is valid), where we can take approximately $f(\mathbf{k} \pm \mathbf{q}) \simeq f(\mathbf{k})$ and $\kappa_1 = \kappa(\mathbf{k} \pm \mathbf{q}) \simeq \kappa$ in Equations (47) and (49); then, it is easy to check the energy conservation

$$U = \sum_{\mathbf{k}_1} [R(\mathbf{k}_1) + T(\mathbf{k}_1)] \simeq R_s + T_s = 1; \quad (51)$$

we can see that the amount of radiation taken from the main reflected (refracted) peak is transferred to the diffuse reflected (refracted) peaks. Within this approximation the reflection and transmission coefficients can be written as

$$R(\mathbf{k}_1) \simeq R_s \left\{ \left[1 - \frac{4}{S^2} \sum_{\mathbf{q}} \left| \kappa h(\mathbf{q}) \right|^2 \right] \delta_{\mathbf{k}_1, \mathbf{k}} + \frac{4}{S^2} \left| \kappa h(\mathbf{k}_1 - \mathbf{k}) \right|^2 \right\}, \quad (52)$$

and

$$T(\mathbf{k}_1) \simeq T_s \left\{ \left| 1 - \frac{1}{S^2} \sum_{\mathbf{q}} \left| f(\mathbf{k}) h(\mathbf{q}) \right|^2 \right| \delta_{\mathbf{k}_1, \mathbf{k}} + \frac{1}{S^2} \left| f(\mathbf{k}_1) h(\mathbf{k}_1 - \mathbf{k}) \right|^2 \right\}. \quad (53)$$

Equations (47) and (49) predict also the occurrence of two secondary peaks (both reflected and refracted) for a regular surface grating $h(\mathbf{r}) = 2h \cos \mathbf{Qr}$, modulated with the wavelength $2\pi/Q$ ($h(\mathbf{q}) = hS(\delta_{\mathbf{q},\mathbf{Q}} + \delta_{\mathbf{q},-\mathbf{Q}})$, small Q), corresponding to the in-plane wavevectors $\mathbf{k}_1 = \pm\mathbf{Q}$. Their intensity is proportional to the square h^2 of the amplitude parameter h. Higher-order terms in the expansion of the polarization (Equation (11)) generate corresponding higher-order reflected and refracted peaks. The angular distribution of the diffuse scattering can give information about the correlation function $\left|h(\mathbf{q})\right|^2$ of the surface structure, according to Equations (52) and (53). We note that this angular distribution of the diffuse scattering is different for the reflection and transmission coefficients (κ in Equation (52) versus $f(\mathbf{k}_1)$ in Equation (53)).

7. The scattered field: *p*-wave

In the equation of motion (24) for the p-wave it is convenient to use the combinations $\mathrm{i}ku_1 + \partial u_z/\partial z$ and $k\partial u_1/\partial z + \mathrm{i}\kappa^2 u_z$ and the relations given by Equations (19)–(21). We get immediately the solution

$$\begin{aligned} u_1(\mathbf{r}, z) &= \frac{1}{S} \sum_{\mathbf{k}_1} A_1(\mathbf{k}_1) \exp(\mathrm{i}\mathbf{k}_1\mathbf{r} + \kappa_1' z), \\ u_z(\mathbf{r}, z) &= \frac{1}{S} \sum_{\mathbf{k}_1} (-k_1/\kappa_1') A_1(\mathbf{k}_1) \exp(\mathrm{i}\mathbf{k}_1\mathbf{r} + \kappa_1' z), \end{aligned} \quad (54)$$

where the amplitudes $A_1(\mathbf{k}_1)$ satisfy the equation

$$\begin{aligned} &A_1(\mathbf{k}_1) + \mathrm{i}(\kappa_1' - \kappa_1) g_1(\mathbf{k}_1) - \frac{1}{2} (\kappa_1' - \kappa_1)^2 s_1(\mathbf{k}_1) \\ &= \frac{q}{m} \frac{2\kappa'(\kappa' - \kappa)}{\omega_{\mathrm{p}}^2 (\kappa\kappa' + k^2)} E_{01} \delta_{\mathbf{k}_1, \mathbf{k}}. \end{aligned} \quad (55)$$

This equation is similar with Equation (43). It is solved up to the second-order of the perturbation theory:

$$\begin{aligned} A_2(\mathbf{k}_1) = \frac{q}{m} \frac{2\kappa'(\kappa' - \kappa)}{\omega_{\mathrm{p}}^2 (\kappa\kappa' + k^2)} E_{01} \bigg\{ &\delta_{\mathbf{k}_1, \mathbf{k}} - \frac{\mathrm{i}}{S} f(\mathbf{k}_1) h(\mathbf{k}_1 - \mathbf{k}) \\ &- \frac{1}{S^2} \sum_{\mathbf{q}} f(\mathbf{k}_1) \left[f(\mathbf{k}_1 - \mathbf{q}) - \frac{1}{2} f(\mathbf{k}_1) \right] \\ &\times h(\mathbf{q}) h(\mathbf{k}_1 - \mathbf{k} - \mathbf{q}) \bigg\}, \end{aligned} \quad (56)$$

The total field transmitted in the body ($z > 0$) is proportional to the displacement $\mathbf{u}$, according to the equation of motion. We get easily the transmission coefficient

$$\begin{aligned} T(\mathbf{k}_1) = T_{\mathrm{p}} \bigg\{ &1 - \frac{2}{S^2} \sum_{\mathbf{q}} \mathrm{Re} \left[f(\mathbf{k}) \left(f(\mathbf{k} - \mathbf{q}) - \frac{1}{2} f(\mathbf{k}) \right) \right] \\ &\times \left| h(\mathbf{q}) \right|^2 \bigg\} \delta_{\mathbf{k}_1, \mathbf{k}} \\ &+ \frac{1}{S^2} T_{\mathrm{p}} \frac{\kappa'}{\kappa_1'} \left| f(\mathbf{k}_1) h(\mathbf{k}_1 - \mathbf{k}) \right|^2, \end{aligned} \quad (57)$$

where T_{p} is the transmission coefficient of the p-wave for the half-plane. The reflected field is computed from Equations (16)–(18), by making use of Equations (54) and (56). We get the reflection coefficient

$$R(\mathbf{k}_1) = R_\mathrm{p}\left\{1 - \frac{4}{S^2}\sum_{\mathbf{q}}\mathrm{Re}[\kappa(\kappa' - f(\mathbf{k}-\mathbf{q}))]\left|h(\mathbf{q})\right|^2\right\}\delta_{\mathbf{k}_1,\mathbf{k}} + \frac{4}{S^2}R_\mathrm{p}\kappa\kappa_1\left|\frac{\kappa'(\kappa'+\kappa)(\kappa_1\kappa_1' - k_1^2)}{\kappa_1'(\kappa_1'+\kappa_1)(\kappa\kappa' - k^2)}h(\mathbf{k}_1 - \mathbf{k})\right|^2, \tag{58}$$

where R_p is the reflection coefficient of the *p*-wave for the half-plane, as given by Equation (35). In the limit $\mathbf{q} \to 0$, $T(\mathbf{k}_1)$ and $R(\mathbf{k}_1)$ given above take the form given by Equations (52) and (53) for the *s*-wave. The structure of the reflection and transmission coefficients obtained here is similar with the results obtained by various other theoretical approaches like the small-parameter perturbation theory, phase perturbation theory, self-energy perturbation theory, Kirchhoff approximation, etc. [25].

8. Concluding remarks

A perturbation-theoretical scheme was devised here, with the surface roughness as a perturbation parameter, for the reflection and refraction of the electromagnetic waves for a semi-infinite solid. The polarization degrees of motion has been introduced explicitly, within the Lorentz–Drude (plasma) model of polarizable, non-magnetic, homogeneous matter. The field scattered by the surface roughness has been calculated within the second order of the perturbation scheme. The scattered field contributes both to the main (specularly) reflected and refracted fields of the half-space (bulk) and the diffuse scattering arising from the rough surface. Secondary peaks are obtained for a regular grating. Strictly two-dimensional modes, resonant at certain frequencies, have been identified, confined to the surface and propagating only on the surface. The model can also be applied to superficial thin films deposited on the surface, coatings or other structures grown on the surface.

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References

[1] Cuevas, M.; Depine, R.A. *Optics Commun.* **2011,** *284,* 5242–5247.
[2] Namiot, V.A.; Schurova, L.Yu. *Phys. Lett. A* **2011,** *375,* 2759–2766.
[3] Shokri, A.A.; Ebrahiminejad, Z.H. *Physica E* **2011,** *43,* 1579–1584.
[4] Sedelnikova, O.V.; Bulusheva, L.G.; Okotrub, A.V. *J. Chem. Phys.* **2011,** *134,* 244707.
[5] Lezec, H.; Thio, T. *Opt. Express* **2004,** *12,* 3629–3651.
[6] Martin-Moreno, L.; Garcia-Vidal, F.J.; Lezec, H.J.; Degiron, A.; Ebbesen, T.W. *Phys. Rev. Lett.* **2003,** *90,* 167401, (1–4).
[7] Knotts, M.E.; Michel, T.R.; O'Donnell, K.A. *J. Opt. Soc. Am. A* **1993,** *10,* 928–941.
[8] Maradudin, A.A.; Mills, D.L. *Phys. Rev. B* **1975,** *11,* 1392–1415, and references therein.
[9] Maradudin, A.A. Ed *Light Scattering and Nanoscale Surface Roughness*; Springer: New York, 2007.
[10] See, for instance, Meier, A.; Arens, T.; Chandler-Wilde, S.N.; Kirsch, A. *J. Integral Equations Appl.* **2000**, *12*, 281–321.
[11] Wood, R.W. *Philos. Mag.* **1902,** *4,* 396–402.
[12] Strutt, J.W. (Lord Rayleigh) *The Theory of Sound*, 2nd ed.; MacMillan: London, 1896; Vol. 2, pp 89, 297–311.
[13] Rayleigh, Lord *Proc. R. Soc. London A* **1907,** *79,* 399–416.
[14] van den Berg, P.M.; Fokkema, J.T. *Radio Sci.* **1980,** *15,* 723–732.
[15] Paulick, T.C. *Phys. Rev. B* **1990,** *42,* 2801–2824.
[16] Mandelstam, L. *Ann. Phys.* **1913,** *346,* 609–624.
[17] Fano, U. *J. Opt. Soc. Am.* **1941,** *31,* 213–222.
[18] Rice, S.O. *Commun. Pure Appl. Math.* **1951,** *4,* 351–378.
[19] Toigo, F.; Marvin, A.; Celli, V.; Hill, N.R. *Phys. Rev. B* **1977,** *15,* 5618–5626.
[20] Brown, G.C.; Celli, V.; Haller, M.; Marvin, A. *Surf. Sci.* **1984,** *136,* 381–397.
[21] McGurn, A.R.; Maradudin, A.A.; Celli, V. *Phys. Rev. B* **1985,** *31,* 4866–4871.
[22] Brown, G.; Celli, V.; Haller, M.; Maradudin, A.A.; Marvin, A. *Phys. Rev B* **1985,** *31,* 4993–5005.
[23] McGurn, A.R.; Maradudin, A.A. *Waves Random Media* **1996,** *6,* 251–267.
[24] Johnson, J.T. *J. Opt. Soc. Am. A* **1999,** *16,* 2720–2736.
[25] Navarette-Alcala, A.G.; Chaikina, E.I.; Mendez, E.R.; Leskova, T.A.; Maradudin, A.A. *Waves Random Complex Media* **2009,** *19,* 600–636.
[26] Imperatore, P.; Iodice, A.; Ricio, D. *IEEE Trans. Antennas Propag.* **2012,** *60,* 1505–1520.
[27] Soubret, A.; Berginc, G.; Bourelli, C. *Phys. Rev. B* **2001,** *63,* 245411.
[28] See, for instance, Leskova, T.A.; Letnes, P.A.; Maradudin, A.A.; Nordam, T.; Simonsen, I. *Proc. SPIE* **2011**, *8172*, 817209 and references therein.
[29] Gradshteyn, I.S.; Ryzhik, I.M. *Table of Integrals, Series and Products*, 6th ed.; Jeffrey, A., Zwillinger, D., Eds.; Academic Press: San Diego, 2000; pp 714–715, 6.677.1,2.

[30] Ulaby, F.T.; Moore, R.K.; Fung, A.K. *Microwave Remote Sensing: Active and Passive*; Addison Wesley: London, 1981.
[31] Drude, P. *Ann. Phys.* **1900,** *306,* 566–613.
[32] Drude, P. *Ann. Phys.* **1900,** *308,* 369–402.
[33] Lorentz, H.A. *The Theory of Electrons*; Teubner: Leipzig, 1916.
[34] Kittel, C. *Introduction to Solid State Physics*; Wiley: NJ, 2005.
[35] Born, M.; Wolf, E. *Principles of Optics*; Pergamon: London, 1959.
[36] Landau, L.; Lifshitz, E. *Electrodynamics of Continuous Media*, Course of Theoretical Physics, Vol. 8; Elsevier: Oxford, 2004.