EARTHQUAKE PARAMETERS AND SEISMIC HAZARD. EXAMPLES OF SPECIFIC CALCULATIONS

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Abstract. We present examples of specific calculations for determining earthquake parameters and estimating the seismic hazard. First, we examine an earthquake occurred on May 25, 2021, in Vrancea, Romania. We show how to calculate the parameters of this earthquake, and the parameters of the corresponding seismic source. These parameters include the tensor of the seismic moment, the earthquake magnitude and energy, the focal volume, slip, duration and the orientation of the seismic fault. Similar results are given for a Vrancea earthquake of November 3, 2022. Next, we give an example of estimating the seismic hazard, which includes the peak values of soil displacement, velocity and acceleration. These calculations are based on a previously presented theoretical procedure. Also, we give an estimation of the critical dimensions of a building on Earth's surface, in order to be resistant to an earthquake with given magnitude.

Key words: P and S seismic waves; seismic mainshock; earthquakes' parameters; seismic hazard.

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1. INTRODUCTION

There exist at least two main problems in seismology which enjoy a large interest. The first problem consists in determining earthquake parameters, like earthquake energy, magnitude, seismic moment tensor, etc, from measurements on Earth's surface. This problem helps in understanding the seismic motion. The second problem requires to know the maximal soil displacement, velocity and acceleration (the so-called peak values), produced by a hypothetical seism. This problem, known as the seismic hazard problem, is of great practical importance. We provide below two examples of specific calculations for both these problems. The answer is made possible by knowing the solution of the elastic wave equation with seismic source (this is known as the seismological problem) [1, 2]. Also, we present a formula for estimating the critical dimensions of a building on Earth's surface, in order to be resistant to an earthquake with given magnitude.

The seismic sources are shear faults, localized in space and time, characterized by a tensor of seismic moment. They produce seismic waves, which have the shape of scissor-like spherical-shell waves, known as the P and S (primary) seismic waves. From the measurement of the displacement produced by these waves on Earth's surface we can derive the parameters of the earthquakes, like energy, magnitude, moment tensor, focal volume, slip, duration and fault orientation (this is known as the inverse seismological problem) [3–5]. Once arrived at Earth's surface, the seismic waves produce secondary wave sources, which generate the seismic mainshock. The mainshock has the shape of a propagating wall with a long tail (there are two mainshocks, for the two primary waves). It produces the main soil displacement (velocity and acceleration), which provide the answer to the seismic hazard problem [6]. We can see that the two problems - determination of earthquake parameters and the seismic hazard - are inter-related. We give below examples of specific calculations for these problems.

2. EARTHQUAKE PARAMETERS

2.1. EARTHQUAKE OF MAY 25, 2021

On 25 May 2021 an earthquake occurred in Vrancea, Romania, with epicentre latitude $\theta_E = 45.52°$, epicentre longitude $\varphi_E = 26.51°$ and depth $H = -128$ km, as reported by the National Institute for Earth's Physics (NIEP, http://www.infp.ro). The NIEP announced a local magnitude $M_L = 4.7$ for this earthquake. According to the NIEP procedure, the moment magnitude is $M_w = 4.3$ ($M_w = 0.74M_L + 0.8$, for $M_L < 4.7$ and $\mid H \mid > 60$ km). We report herein upon the parameters of this earthquake, as calculated from the theory given in Refs. [3–5], by using the data recorded at the Magurele station (latitude $\theta_0 = 44.35°$, longitudine $\varphi_0 = 26.03°$).

The seismic moment tensor is currently determined from teleseismic data [7–9], regional long-period waveforms [10, 11], simultaneous inversion of body and surface waves [12], or intermediate-period surface waves [13]. Synthetic seismograms with fitting parameters (like, for instance, location coordinates) are compared in these procedures with the recorded data. Waves measured at different locations (or times) may lead to overdetermined systems of equations, such that the solutions must be "compatibilized". In Refs. [3–5] an analytical procedure is given for determining the seismic moment tensor, energy and magnitude of the earthquake, as well as the volume of the seismic focus, the slip in the focal region, the duration of the seismic activity in focus and the orientation of the fault. These parameters are determined from the P and S primary seismic waves measured at Earth's surface. The theory is based on the solution of the elastic wave equation with tensorial seismic force in a homogeneous and isotropic body, the Kostrov vectorial representation of the shearing faults and energy conservation.

In the North-South (1), West-East (2), Vertical (3) reference frame, with the origin at the observation point, the focus coordinates are $x_1 = -R_0(\theta_E - \theta_0) =$

-130 km, $x_2 = R_0 \cos\theta_E \cdot (\varphi_E - \varphi_0) = 37.4$ km, $x_3 = H = -128$ km, where $R_0 = 6370$ km is the mean radius of the Earth. The distance from the focus to the observation point is $R = \sqrt{x_1^2 + x_2^2 + x_3^2} = 186$ km. The unit vector from the observation point to the focus is

$$\boldsymbol{n} = (x_1/R, x_2/R, x_3/R) = (-0.7, 0.2, -0.69) \,. \tag{1}$$

In reading the longitudinal displacement $\boldsymbol{v}_l$ (P wave) from seismograms we should observe two rules. First, there is the so-called "sign rule". The longitudinal displacement should be either along $\boldsymbol{n}$, or along the opposite direction $-\boldsymbol{n}$. The two options correspond to one or other of the two sides of the scissor-like shape of the P wave. Let us assume that the longitudinal displacement $\boldsymbol{v}_l$ is oriented along the vector $\boldsymbol{n}$, from the observer to the focus. For an observer in Bucharest and a Vrancea focus the first componenet of $\boldsymbol{v}_l$ is oriented to the North, so it should has the minus sign (the axis is North to South); usually, from an observer in Bucharest and a Vrancea focus the second component of the longitudinal displacement is oriented to the East, so the second component of $\boldsymbol{v}_l$ should carry the sign plus; and, finally, the vertical component of $\boldsymbol{v}_l$ is oriented downwards, so it carries the minus sign. Therefore, the sign rule for the components of $\boldsymbol{v}_l$ is $(-,+,-)$. If the longitudinal displacement is oriented in the opposite direction of $\boldsymbol{n}$, the sign rule is, of course, $(+,-,+)$. In general, this rule gives a different result for other observers. For example, for an observer placed at Cernavoda, the second component of $\boldsymbol{v}_l$ is oriented to the West, so it has the minus sign, and the displacement has the signs $(-,-,-)$, or $(+,+,+)$. This rule should be compatible with the maximum values read on the same side of the (scissor-like) seismogram for all three components of the longitudinal displacement. If we use an average of the maximum values on the two sides, we attribute this average value to one side, such as to observe the sign rule. These signs do not affect the magnitude v_l of the longitudinal displacement $\boldsymbol{v}_l$, and we may take $\boldsymbol{v}_l = v_l\boldsymbol{n}$, as long as the angle made by $\boldsymbol{v}_l$ and the direction vector $\boldsymbol{n}$ is close to zero; also, we may take $\boldsymbol{v}_l = -v_l\boldsymbol{n}$,if this angle is close to $180°$.

Also, a similar rule should be observed for the transverse displacement $\boldsymbol{v}_t$ (S wave). We should always read the data on the same side of the seismograms, for all the components of $\boldsymbol{v}_{l,t}$. The transverse displacement $\boldsymbol{v}_t$ should be perpendicular to $\boldsymbol{v}_l$. Usually, this is not the case, and the approximation $\boldsymbol{v}_l = \pm v_l\boldsymbol{n}$ may increase the error. We note that the tensor of the seismic moment changes sign under the transformation $\boldsymbol{v}_{l,t} \rightarrow -\boldsymbol{v}_{l,t}$ (see below). By choosing $\boldsymbol{v}_l = v_l\boldsymbol{n}$, we fix the sign of the seismic moment (and the "force" vector $\boldsymbol{m}$, see below).

The second rule refers to units. Usually, the seismograms provided by the NIEP have m written as unit on the ordinate axis. We should apply a factor 10^2, arising from data processing, and another factor 10^2 to get the values in cm; this makes a total factor 10^4.

The recorded longitudinal displacement (P wave) for the earthquake analyzed here is

$$\boldsymbol{v}_l = (5.5, -6, 1.1) \times 10^{-3}\text{cm}, \tag{2}$$

with a magnitude $v_l = 8.2 \times 10^{-3}$ cm and a unit vector $\boldsymbol{g} = (0.67, -0.73, 0.13)$. The recorded transverse displacement (S wave) is

$$\boldsymbol{v}_t = (-1.7, -1.5, 0.06) \times 10^{-2}\text{cm}, \tag{3}$$

with a magnitude $v_t = 2.3 \times 10^{-2}$ cm and a unit vector $\boldsymbol{t} = (-0.74, -0.65, 0.03)$. The scalar product $\boldsymbol{gt} \simeq -0.02$, shows that the two vectors $\boldsymbol{v}_l$ and $\boldsymbol{v}_t$ are practically orthogonal ($\boldsymbol{v}_l\boldsymbol{v}_t \simeq 0$), as expected (with an error 2%); the angle between $\boldsymbol{v}_l$ and $\boldsymbol{v}_t$ is approximately $91°$. But the scalar product $\boldsymbol{ng}$, which is expected to be close to ±1, is $\boldsymbol{ng} \simeq -0.7$; this shows an appreciable error, of approximately 30%, in the input data. The seismogram of the P-wave for the direction N-S is shown in Fig. 1. A particularity of the recorded data is the small values of the longitudinal displacement in comparison with the the transverse displacement, and the very small values of the vertical components of both displacements. It is likely that the vertical component of the longitudinal displacement is larger. Also, the seismogram in Fig. 1 exhibits several oscillations (in comparison with scissor-like shape), due to seismograph oscillations and local-site effects [6].

It seems that the deviation from colinearity of the vectors $\boldsymbol{n}$ and $\boldsymbol{v}_l$, as well as the deviation from orthogonality of the vectors $\boldsymbol{v}_l$ and $\boldsymbol{v}_t$ are systematic in input data. They may arise from the small differences in elastic wave velocities in the crust and the upper mantle (for instance, the so-called Mohorovicic discontinuity), though errors in the coordinates of the focus cannot be excluded. In Refs. [14–16] procedures are given of accounting for the effect of such deviations within the approximation of a homogeneous and isotropic medium. Herein, we leave aside such corrections and limit ourselves to use a longitudinal displacement given by $\boldsymbol{v}_l = v_l\boldsymbol{n}$.

The deviation angle due to a superposed homogeneous and isotropic layer with thickness d, with a continuous decrease in velocity, is

$$\delta\theta \simeq \delta c \frac{r}{c_0 H} , \tag{4}$$

where $\delta c = c_0 - c$ is the average variation of velocity, c_0 is the velocity in the background medium, $c < c_0$ is the wave velocity in the layer, r is the epicentral distance and H is the depth of the focus; the formula is valid for $\delta c \ll c_0$ and $d \ll H$, where d is the thickness of the superposed layer. For the full derivation of the deviation angle we refer to Ref. [16].

We adopt the magnitude $v_l = 8.2 \times 10^{-3}$ cm of the longitudinal displacement and the components of the vector $\boldsymbol{v}_l$ given by $\boldsymbol{v}_l = v_l\boldsymbol{n}$ (this introduces an error in the orthogonality of the two vectors $\boldsymbol{v}_l$ and $\boldsymbol{v}_t$). The energy of the earthquake, the magnitude of the seismic moment, the magnitude of the earthquake, the duration of the seismic activity in the focus as well as the focal volume and the focal slip do not depend on the components of the vectors $\boldsymbol{v}_{l,t}$ (see below). The sign of the moment tensor, the vector "force" $\boldsymbol{m}$ and the fault vector $\boldsymbol{s}$ is changed by this assumption ($\boldsymbol{v}_l = v_l\boldsymbol{n}$).

According to theory [3–5], the reduced magnitude of the seismic moment is given by

$$M = (M_{ij}^2/2)^{1/2} = 4\pi\sqrt{2}\rho R^{3/2}\left(c_l v_l^2 + c_t v_t^2\right)^{1/2}\left(c_l^6 v_l^2 + c_t^6 v_t^2\right)^{1/4} \ , \tag{5}$$

the earthquake energy is $E = M/2$ and the magnitude of the seismic moment is $\overline{M} = \sqrt{2}M = (M_{ij}^2)^{1/2}$. By using the present data, we get $E = 1.9\times10^{21}$ erg and $\overline{M} = 5.3\times10^{21}$ erg ($\rho = 5\mathrm{g/cm}^3$ is Earth's mean density, $c_l = 7\mathrm{km/s}$ is the velocity of the longitudinal wave and $c_t = 3$ km/s is the velocity of the transverse wave; all the equations are written in units cm,g,s). Using the Hanks-Kanamori (Gutenberg-Richter) law

$$\lg E = 1.5M_w + 15.65 \tag{6}$$

(or $\lg\overline{M} = \frac{3}{2}M_w + 16.1$), we get the moment magnitude of the earthquake

$$M_w = \frac{1}{1.5}\left(\lg E - 15.65\right) \simeq 3.75 \ . \tag{7}$$

This magnitude is in good agreement with $M_w = 4.3$ given by the NIEP, with due allowance for the errors in the input data.

The focal volume is given by

$$V = \frac{M}{2\rho c_t^2} \simeq 3.8\times10^9\mathrm{cm}^3 \ , \tag{8}$$

whence we may infer the dimension of the focal region and the magnitude of the fault slip $l = V^{1/3} \simeq 15$ m. The duration of the seismic activity in the focal region is given by

$$T = (2R)^{1/2}\frac{\left(c_l v_l^2 + c_t v_t^2\right)^{1/2}}{\left(c_l^6 v_l^2 + c_t^6 v_t^2\right)^{1/4}} \simeq 2\times10^{-3}\mathrm{s} \ . \tag{9}$$

Usually, this parameter is underestimated. The rate of the focal slip is l/T.

According to theory [3–5], the seismic-moment tensor is given by

$$M_{ij} = \frac{M}{1-m_4^2}\left[m_i n_j + n_i m_j - m_4\left(m_i m_j + n_i n_j\right)\right] \ , \tag{10}$$

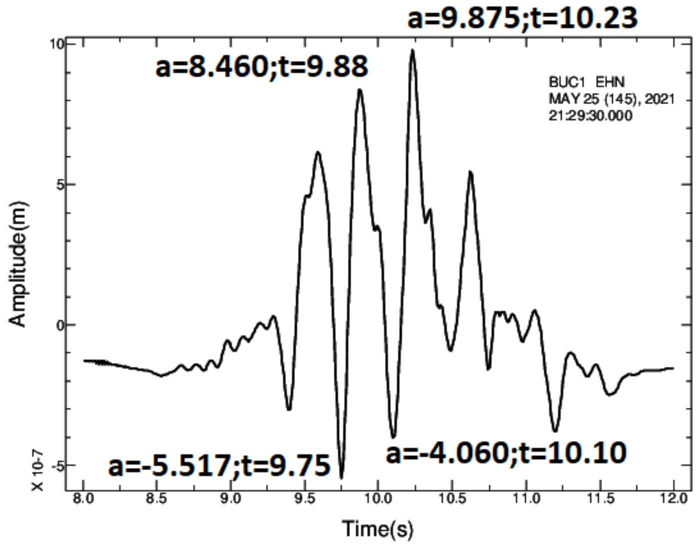

Fig. 1 – P-wave along the $N-S$ direction, earthquake of 26 May 2021 (amplitude multiplied by 10^4).

where

$$m_i = -\frac{c_l^3 v_{li} + c_t^3 v_{ti}}{\left(c_l^6 v_l^2 + c_t^6 v_t^2\right)^{1/2}} \ ,$$

$$m_4 = -\frac{c_l^3 (\boldsymbol{v}_l \boldsymbol{n})}{\left(c_l^6 v_l^2 + c_t^6 v_t^2\right)^{1/2}} \simeq -0.41 \ . \tag{11}$$

The components M_{ij} can be viewed as generalized force couples, while the vector $\mathbf{m}$ may be viewed as indicating the direction of a "force" acting in the focus; m_4 is a measure of the "force" acting along the observation radius (longitudinal "force"). We can check the traceless condition $M_{ii} = 0$ and the covariance condition $m_i^2 = 1$.

The focal strain is given by

$$u_{ij}^0 = \frac{M_{ij}}{2M} \ , \tag{12}$$

where $u_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i)$ are the strain components for the displacement vector $\mathbf{v}$ (the superscript 0 stands for the focus). Making use of the data of this earthquake we get

$$\boldsymbol{m} \simeq (0.38, 0.67, -0.51) \tag{13}$$

and

$$M_{ij} \simeq \begin{pmatrix} 22 & 26 & -6.5 \\ 26 & 7.6 & 12.6 \\ -6.5 & 12.6 & -15.7 \end{pmatrix} \times 10^{20}\text{erg}. \tag{14}$$

We note a deviation from the traceless condition ($M_{ii} \neq 0$), due to the errors in the input data.

The fault is characterized by the unit vector $\boldsymbol{s}$ perpendicular to it (the normal to the fault) and the slip unit vector $\boldsymbol{a}$ lying in the fault, with the symmetries $\boldsymbol{s},\boldsymbol{a} \rightarrow -\boldsymbol{s},-\boldsymbol{a}$ and $\boldsymbol{s} \rightarrow \boldsymbol{a}$. The vectors s and $\mathbf{a}$ are given by [3–5, 14, 15]

$$\begin{gathered}\mathbf{s} = \tfrac{\alpha}{\alpha^2-\beta^2}\mathbf{m} - \tfrac{\beta}{\alpha^2-\beta^2}\mathbf{n} \ , \\ \mathbf{a} = -\tfrac{\beta}{\alpha^2-\beta^2}\mathbf{m} + \tfrac{\alpha}{\alpha^2-\beta^2}\mathbf{n} \ , \end{gathered} \tag{15}$$

where

$$\begin{gathered}\alpha = \sqrt{\tfrac{1+\sqrt{1-m_4^2}}{2}} \simeq 0.98 \ , \\ \beta = sgn(m_4)\sqrt{\tfrac{1-\sqrt{1-m_4^2}}{2}} \simeq -0.21 \ . \end{gathered} \tag{16}$$

The two vectors are

$$\begin{gathered}\boldsymbol{s} = (0.31, 0.67, -0.36) \ , \\ \boldsymbol{a} = (0.98, 0.15, 0.58) \ . \end{gathered} \tag{17}$$

The fault slip is oriented mainly along the N-S and the vertical directions.

These vectors intersect the Earth's surface at the coordinates $\theta^{'} = \theta_0 + \theta$ and $\varphi^{'} = \varphi_0 + \varphi$, where θ and φ are given by

$$\begin{gathered}H\tfrac{s_1}{s_3} + x_1 = -R_0\theta \ , \\ H\tfrac{s_2}{s_3} + x_2 = R_0 \cos\theta^{'} \cdot \varphi \end{gathered} \tag{18}$$

for the vector $\boldsymbol{s}$ ($s_3 > 0$; a similar equation for the vector $\boldsymbol{a}$); we get $\theta^{'}_s \simeq 46.50°$, $\varphi^{'}_s \simeq 23.40°$ and $\theta^{'}_a \simeq 43.59°$, $\varphi^{'}_a \simeq 26.91°$. All the formulae used above can be found in Refs. [3]-[5] and [14, 15].

Finally, we note a quick and simple method of estimating the order of magnitude of earthquake parameters. We use a generic velocity c for the seismic waves and a generic vector $\mathbf{v}$ for the displacement in the far-field seismic waves. By using the covariance equation $m^2 = 1$ in equation (5) (or directly from equation (9)), we get immediately $cT \simeq \sqrt{2Rv}$, which provides an estimate of the duration T of the seismic activity in the focus in terms of the displacement measured at distance R. The focal volume can be estimated as $V \simeq \pi\,(2Rv)^{3/2} \simeq \pi(cT)^3$, as expected (dimension l of the focal region of the order cT; the rate of the focal slip is $l/T \simeq c$). The earthquake energy is $E \simeq \mu V \simeq M/2 \simeq \rho c^2 V$, where $\mu = \rho c^2$ is the Lame coefficient and M is the reduced magnitude ($\left(M_{ij}^2\right)^{1/2} = \sqrt{2}M$) of the seismic moment (and the magnitude of the vector $M_{ij}n_j$). The focal strain is of the order unity, as expected.

The magnitude of the earthquake is given immediately by equations (6) and (7). In addition, we can see the relationship $\lg v = M_w + 10.73 - \lg\left[R(8\pi\rho c^2)^{2/3}\right]$, or $\lg(Rv) = M_w + 1.6$ (for $\rho = 5\mathrm{g/cm}^3$, $c = 5$ km/s and v measured in cm). For instance, we may see that the displacement measured at Bucharest for a Vrancea earthquake of magnitude $M_w = 7$ is of the order $v \simeq 30$ cm. A local magnitude given by $M_l = M_w - 3 = \lg(Rv) - 4.6$ can be used.

2.2. EARTHQUAKE OF NOVEMBER 3, 2022

The epicentre coordinates for this earthquake are $\theta_E = 45.50°$ and $\varphi_E = 26.52°$; the depth of the focus is $H = -148.8$ km. The unit vector from the Bucharest station ($\theta_0 = 44.35°$, $\varphi_0 = 26.03°$) to the focus is

$$\boldsymbol{n} = (-0.64, 0.19, -0.74)\,. \tag{19}$$

The recorded displacement vectors are

$$\begin{gathered}\boldsymbol{v}_l = (1.92, -1.49, 2.83) \times 10^{-3}\mathrm{cm}\,,\; v_l = 3.77 \times 10^{-3}\mathrm{cm},\\ \boldsymbol{v}_t = (3.5, 8.37, 0.71) \times 10^{-2}\mathrm{cm}\,,\; v_t = 9.1 \times 10^{-2}\mathrm{cm}.\end{gathered} \tag{20}$$

The angle made by $\boldsymbol{v}_l$ and $\boldsymbol{n}$ is $\simeq 166°$; the angle $(\boldsymbol{v}_l, \boldsymbol{v}_t) \simeq 82.6°$. We choose $\boldsymbol{v}_l = v_l\boldsymbol{n}$.

By making use of the formulae given above, we get the energy released by the earthquake $E = 1.1 \times 10^{22}$ erg, the magnitude of the seismic moment $\overline{M} = 3.1 \times 10^{22}$ erg and the moment magnitude of the earthquake $M_w = 4.3$; the NIEP announced $M_w = 4.9$. The focal volume is $V = 2.2 \times 10^{10}$ cm^3, with dimension $l = V^{1/3} \simeq 28$ m; the duration of the seismic activity in the focus is $T = 6 \times 10^{-3}$ s. We give the seismic moment with changed sign (as if we take $\boldsymbol{v}_l = -v_l\boldsymbol{n}$)

$$M_{ij} \simeq \begin{pmatrix} 1.16 & 0.91 & 0.9 \\ 0.91 & -1.1 & 1.34 \\ 0.9 & 1.34 & 0.47 \end{pmatrix} \times 10^{22}\mathrm{erg}; \tag{21}$$

we note the deviation from $M_{ii} = 0$, due to the errors in the input data. Also, we give the fault and slip vectors

$$\begin{aligned}\boldsymbol{s} &= (-0.44\;\; -0.9\;\; -0.15)\,,\\ \boldsymbol{a} &= (0.6\;\; -0.28\;\, 0.73)\,;\end{aligned} \tag{22}$$

they intersect the Earth's surface at $\theta_a' = 44.4°$, $\varphi_a' = 25.8°$ and $\theta_s' = 41.5°$, $\varphi_s' = 37.3°$.

The data recorded at the Cernavoda station for this earthquake are

$$\begin{gathered}\theta_0 = 44.32^\circ\,, \qquad \varphi_0 = 28.06^\circ,\\ \boldsymbol{n} = (-0.57, -0.52, -0.64),\\ \boldsymbol{v}_l = (1.03, 1.74, 3.26)\times 10^{-3}\mathrm{cm}\,, \quad v_l = 3.8\times 10^{-3}\mathrm{cm},\\ \boldsymbol{v}_t = (-1.3, -1.6, 0.63)\times 10^{-2}\mathrm{cm}\,, \quad v_t = 2.1\times 10^{-2}\mathrm{cm}.\end{gathered} \tag{23}$$

The product $\boldsymbol{gn}$ is -0.94 (the cosine of the angle made by $\boldsymbol{v}_l$ and $\boldsymbol{n}$), which is acceptable; the angle made by $\boldsymbol{v}_l$ and $\boldsymbol{v}_t$ is 122.5°, which indicates a large error (in comparison with 90°). We limit ourselves to give the moment magnitude $M_w = 3.7$, which has a large deviation from 4.9.

3. SEISMIC HAZARD

A typical seismic hazard problem requires to estimate the maximal soil displacement, velocity and acceleration (peak values) for a hypothetical earthquake with given magnitude. For example, in the time interval $1986-2018$ there were 1004 Vrancea earthquakes with magnitude $3.5-4.5$, 55 with magnitude $4.6-5.5$, 7 with magnitude $5.6-6.5$ and 2 with magnitude $6.6-7.5$. The total number of earthquakes in this time interval is 4320 with magnitude greater than 3 [17]. It is of great interest to estimate the seismic hazard for such earthquakes, especially for those with great magnitude.

Current studies of seismic hazard include both (neo-) deterministic and probabilistic approaches. In the former, peak values are estimated for a given, usually big, earthquake; in the latter, probabilities are given for peak values [19]. Waveform modelling is used for regional structure models, employing finite-diference calculations. The effects of the seismic source and local sites are included. A review of seismic hazard studies for Vrancea earthquakes is given in Refs. [20, 21].

In Ref. [6] a method of estimating the seismic hazard is given, site effects including. The ground peak values of the displacement, velocity and acceleration are greater for the mainshock than those corresponding to the primary waves, except, possibly, for acceleration, when the site effects are included. Usually, the site effects bring only a small contribution, so we may leave them aside. Within this approximation the peak values correspond to the mainshock. They are valid over the limited range of validity of the mainshock, from $r = z_0/\sqrt{3}$ to $r = 2z_0$, where r is the epicentral distance and z_0 is the depth of the focus. For $r > 2z_0$ the peak values are given by the primary waves, which are much smaller; while for $r < z_0/\sqrt{3}$ we enter the epicentral zone.

Therefore, the peak values of the displacement (u), velocity (v) and acceleration

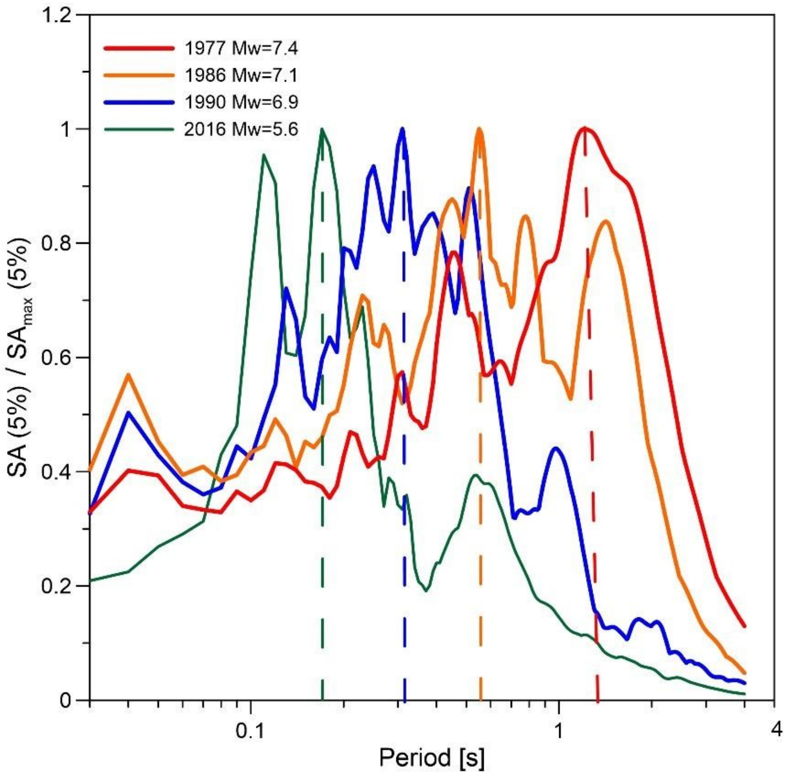

Fig. 2 – Normalized spectral response of acceleration for various magnitudes of Vrancea earthquakes (N-S component, [18]).

(a) are

$$u \simeq \frac{l^3 r^{1/2}}{2 l_0^{3/2} R} \ , \ v \simeq \frac{3 c l^3 r^{1/2}}{4 l_0^{5/2} R} \ ,$$
$$a \simeq \frac{15 c^2 l^3 r^{1/2}}{8 l_0^{7/2} R} \ , \tag{24}$$

where r is the epicentral distance, R is the focal distance, c is a mean velocity of the seismic waves, l is the size of the seismic focus and l_0 is the same size modified by dissipation. The focal distance is given by $R = \sqrt{r^2 + z_0^2}$, where z_0 is the depth of the focus. We use the mean value $c = 5$ km/s (see Ref. [6]).

By using the Hanks-Kanamori law ($\lg \overline{M} = \frac{3}{2} M_w + 16.1$) [22, 23] and the mean seismic moment ($\overline{M} = 2\pi\sqrt{2}\rho c^2 l^3$) [3, 14, 15] the focus size is given by

$$\lg l = \frac{1}{2} M_w + 1 \tag{25}$$

(for density $\rho = 5 \mathrm{g/cm^3}$), where M_w is the moment magnitude, l is measured in cm and $\overline{M}$ is measured in erg [24].

The parameter l_0 is determined from the Fourier spectrum of the primary seismic waves. A survey of a few tens of moderate Vrancea earthquakes (magnitude $M_w = 3.5 - 5$, average depth $\simeq 100$ km) indicates $l/l_0 \simeq 1/10$. We expect a slight dependence of the ratio l/l_0 on site and magnitude, though for large magnitudes we may have a stronger dependence. A typical spectral response of acceleration is shown in Fig. 2 for various magnitudes of Vrancea earthquakes. We can see the maximum period T_a from $0.1s$ (small earthquakes) to $1s$ (big earthquakes; the length l_0 is given by $l_0 = cT_d/2$ (displacement), $l_0 = 2cT_v/\pi$ (velocity) and $l_0 = 3cT_a/4$ (acceleration), where $T_{d,v,a}$ are the corresponding maximum periods). We note that for small magnitudes the length l decreases, such that we may take the ratio l/l_0 approximately the same for all magnitudes.

By using these numerical data we re-write equations (24) as

$$u \simeq \frac{10^{\frac{3M_w}{4}} r^{1/2}}{2R} \ , \ v \simeq \frac{15 \cdot 10^3 10^{\frac{M_w}{4}} r^{1/2}}{4R} \ , \qquad a \simeq \frac{15 \cdot 25 \cdot 10^6 10^{-\frac{M_w}{4}} r^{1/2}}{8R} \tag{26}$$

(in cm, cm/s and cm/s^2, respectively). We note that these quantities have a maximum value for $r = z_0$, so, in some cases, we may have higher values at larger distances. Another interesting feature is a decrease of acceleration with increasing magnitudes. This is due, on one hand, to the fact that big earthquakes lose more energy, and, on the other, by the poor approximation ($l/l_0 = const$) used herein. However, the main cause is the size l of the focal region, which increases with increasing magnitude; its inverse $1/l$ controls the sharpness of the distribution of the secondary-wave sources on the Earth's surface, which, for higher magnitude becomes more diffuse and, consequently, have a weaker effect. Numerical data are given in Table 1.

These estimations may look sometimes too large, especially the acceleration. We note that these estimations are affected by errors in the parameters c, l, l_0. The high-powers of l_0 in equations (24) may bring appreciable variations. For example, if l_0 increases by a factor 2, the acceleration decreases by a factor 12.

The acceleration felt by a building is different from the ground acceleration, because the building is an inhomogeneity with respect to the ground. We can estimate the building acceleration by dividing the ground displacement u to the time during which the displacement is present. This time is given by the propagation of the ground perturbation along the depth d of the foundation, for a compression wave; or along the smallest cross-sectional dimension D of the foundation. We may take the geometrical mean $\sqrt{dD}$ for this length, such that we have a building acceleration

$$a = \frac{c^2 u}{dD} \ , \tag{27}$$

Table 1

Peak values for displacement, velocity and accelerations for various magnitudes, focal depths and epicentral distances: (a) $z_0 = 200\,\mathrm{km}$, $r = 100\,\mathrm{km}$ ($R = 223\,\mathrm{km}$); (b) $z_0 = 200\,\mathrm{km}$, $r = 200\,\mathrm{km}$ ($R = 283\,\mathrm{km}$); (c) $z_0 = 100\,\mathrm{km}$, $r = 100\,\mathrm{km}$ ($R = 141\,\mathrm{km}$); (d) $z_0 = 100\,\mathrm{km}$, $r = 200\,\mathrm{km}$ ($R = 223\,\mathrm{km}$).

M_w (a)	4	5	6	7
$u(\mathrm{cm})$	0.07	0.38	2.17	12.2
$v(\mathrm{cm/s})$	5.15	9.16	16.3	28.9
$a(\mathrm{cm/s^2})$	644.5	362.4	203.8	114.6

M_w (b)	4	5	6	7
$u(\mathrm{cm})$	0.08	0.42	2.39	13.42
$v(\mathrm{cm/s})$	5.67	10.1	17.93	31.8
$a(\mathrm{cm/s^2})$	708.9	398.6	224.2	126.06

M_w (c)	4	5	6	7
$u(\mathrm{cm})$	0.11	0.6	3.43	19.3
$v(\mathrm{cm/s})$	8.14	14.5	25.7	45.7
$a(\mathrm{cm/s^2})$	1018	572.6	322	181.1

M_w (d)	4	5	6	7
$u(\mathrm{cm})$	0.09	0.54	3.1	17.2
$v(\mathrm{cm/s})$	7.26	12.9	22.9	40.7
$a(\mathrm{cm/s^2})$	908.7	510.9	287.3	161.6

where c is a mean wave velocity ($c = 5\,\mathrm{km/s}$). (Since this is a random process, the half-harmonic mean $dD/(d+D)$ is more suitable). The force acting upon the building is ρSHa, where ρ is the building density, S is the area of the foundation and H is the height of the building. This force should be smaller than the elastic force μS acting upon the building, where μ is the generic elasticity modulus of the building material (for example, concrete). Therefore, we must have

$$H < \frac{\mu dD}{\rho c^2 u} \ , \tag{28}$$

or by making use of equation (26),

$$H < \frac{2\mu dD}{\rho c^2}\frac{R}{r^{1/2}}10^{-3M_w/4} \ . \tag{29}$$

For a numerical estimation we take $\mu = 3\times10^{11}\mathrm{dyn/cm^2}$, $\rho = 2.4\,\mathrm{g/cm^3}$ (concrete), $d = 1\,\mathrm{m}$, $D = 10\,\mathrm{m}$, $c = 5\mathrm{km/s}$, $R = 223\,\mathrm{km}$, $r = 100\,\mathrm{km}$ ($z_0 = 200\,\mathrm{km}$) and $M_w = 7$; we get $H \lesssim 40\,\mathrm{m}$. Such an estimation should be taken with large errors. For example, since $c = 5\,\mathrm{km/s}$ is a mean velocity of the longitudinal velocity

$c_l = 7\,\mathrm{km/s}$ and the transverse velocity $c_t = 3\,\mathrm{km/s}$, we expect a 50% error at least. On the other hand, equation (29) exhibits all the expected features of a seismic-resistant building, which should have a large foundation, with a large depth; on the other hand, the building should be placed far away from the epicentre ($r \gg z_0$), and, of course, the earthquake shoud be small (small M_w).

4. CONCLUDING REMARKS

By making use of the solutions to the seismological and inverse seismological problems we provide examples of specific calculations for determining the earthquake parameters and estimating the seismic hazard. The earthquake parameters are determined for the Vrancea earthquakes of 25 May 2021 and 3 November 2022, with magnitudes 4.3 and 4.9, respectively. The seismic hazard calculations are carried out for various magnitudes, focal depths and epicentral distances. The numerical results are affected by errors arising from uncertainties in various input parameters. Critical dimensions of a building resistant to earthquakes are given.

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