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Fast Atom Ionization in Strong Electromagnetic Radiation

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Abstract: The Goeppert–Mayer and Kramers–Henneberger transformations are examined for bound charges placed in electromagnetic radiation in the non-relativistic approximation. The consistent inclusion of the interaction with the radiation field provides the time evolution of the wavefunction with both structural interaction (which ensures the bound state) and electromagnetic interaction. It is shown that in a short time after switching on the high-intensity radiation the bound charges are set free. In these conditions, a statistical criterion is used to estimate the rate of atom ionization. The results correspond to a sudden application of the electromagnetic interaction, in contrast with the well-known ionization probability obtained by quasi-classical tunneling through classically unavailable non-stationary states, or other equivalent methods, where the interaction is introduced adiabatically. For low-intensity radiation the charges oscillate and emit higher-order harmonics, the charge configuration is re-arranged and the process is resumed. Tunneling ionization may appear in these circumstances. Extension of the approach to other applications involving radiation-induced charge emission from bound states is discussed, like ionization of molecules, atomic clusters or proton emission from atomic nuclei. Also, results for a static electric field are included.

Keywords: Dissociation of Bound States; Electromagnetic Radiation; Fragmentation; Ionization.

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The investigation of the laser-matter interaction has been focused since the beginning on the radiation-induced atom ionization [1, 2]. Originally, the transitions from atomic non-stationary states have been approached by time-dependent perturbation theory. Keldysh [3] noticed that the presence of the radiation implies a quasi-classical tunneling through states which are not allowed by the classical dynamics (including imaginary time tunneling [4]). Later, it was realised [5] that the radiation-dressed states play an important role in the ionization process through the Kramers–Henneberger transformation [6–8]. Similarly, the ionization rate in static electric field was computed in classical works [9–11], either by quantum transitions or by tunneling through the potential barrier generated by the field. These calculations assume an adiabatic introduction of the electromagnetic interaction, which allows the use of atomic states [12]. The dynamics of the electrons in the presence of the radiation field is neglected in these approaches, which results in well-known ionization probabilities proportional to $e^{-\mathrm{const}/E}$, where E is the (low) strength of the electric field [13–16] (a result valid also for static fields [4]). In current experiments envisaging atom ionization, especially in high-intensity electromagnetic radiation, the interaction occurs suddenly in the focal region of the laser beam. In this case, the atomic states are wiped out and time evolution of the wavefunction is needed. On a sudden application of the interaction, both in static and time-dependent electromagnetic field, the particle energy is not determined. The need of a time-evolving picture of radiation-induced atom ionization has often been emphasised [17–19]. We present here an investigation into the time-evolution of bound states of charges for a sudden application of high-intensity electromagnetic interaction.

First, we show that the consistent inclusion of the interaction with the radiation field by means of the unitary transformations of the Goeppert–Mayer and Kramers–Henneberger type offers the opportunity to follow the time evolution of the wavefunction with both structural interaction (which is responsible for the charge bound state) and electromagnetic interaction. Second, it is shown that in high-intensity radiation, after a short lapse of time since its application, the well-known radiation-dressed interaction resulting from the Kramers–Henneberger transformations vanishes, setting the charges free. Using a statistical criterion, we derive the ionization rate, which differs from the known results obtained by introducing the interaction adiabatically. The effect of low-intensity radiation is that of an adiabatic perturbation. During its slow action,

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the electrons oscillate among stationary states, leading to well-known multi-photon absorption and production of high-order electromagnetic harmonics. The process implies a periodic re-arrangement of the charge configuration. Tunneling ionization may appear in this case. We apply the results of this new context to the ionization of atoms, assuming, for convenience, (non-relativistic) single-electron states in atomic mean-field. The extension of the results to multiply-charged ions, ionization of large molecules, atomic clusters, or proton emission from atomic nuclei is discussed.

We assume a charge q with mass m in the potential $V(\mathbf{r})$ in the presence of an electric field $\mathbf{E}$, introduced at time $t=0$. In the non-relativistic approximation the dipole hamiltonian is

$$H_d = H_0 - q\mathbf{r}\mathbf{E},\ H_0 = \frac{1}{2m}p^2 + V(\mathbf{r}), \tag{1}$$

where $\mathbf{r}$ denotes the charge position and $\mathbf{p}$ is the charge momentum. We assume that the electric field $\mathbf{E}$ is a radiation field. In the non-relativistic approximation we may limit ourselves to its time dependence. Consequently, we assume a typical component of the electric field of the form $\mathbf{E}=\mathbf{E}_0\sin(\omega t+\alpha)$, where ω is the radiation frequency (linear polarization), t denotes the time and α is an initial phase. We consider the associated Schrodinger equation $i\hbar\partial\psi/\partial t = H_d\psi$ and introduce the unitary transformation

$$\psi = e^{iS_1}\phi,\ S_1 = \frac{1}{\hbar}\int_0^t \mathrm{d}t' q\mathbf{r}\mathbf{E} = -\frac{q}{\hbar\omega}\mathbf{r}\mathbf{E}_0[\cos(\omega t+\alpha)-\cos\alpha], \tag{2}$$

where we recognise the vector potential $\mathbf{A}=(c/\omega)\mathbf{E}_0[\cos(\omega t+\alpha)-\cos\alpha]$ ($\mathbf{E}=-(1/c)\partial\mathbf{A}/\partial t$), $\hbar$ being Planck's constant and c the speed of light in vacuum). We can write $S_1=-(q/\hbar c)\mathbf{r}\mathbf{A}$. The transformation given by (2) leads to $\mathbf{p}\to\tilde{\mathbf{p}}=\mathbf{p}-q\mathbf{A}/c$ and the standard non-relativistic hamiltonian

$$\tilde{H}_d = e^{-iS_1}H_d e^{iS_1} = \frac{1}{2m}\left(\mathbf{p}-\frac{q}{c}\mathbf{A}\right)^2 + V(\mathbf{r}), \tag{3}$$

with the associated Schrodinger equation $i\hbar\partial\phi/\partial t = \tilde{H}_d\phi$. The transformation given by (2) is the well-known known Goeppert–Mayer transformation [20].

Let us write

$$\tilde{H}_d = H_0 - \frac{q}{mc}\mathbf{A}\mathbf{p} + \frac{q^2}{2mc^2}A^2 \tag{4}$$

and continue with the unitary transformations

$$\phi = e^{iS_2}e^{iS_3}\chi,$$
$$S_2 = -\frac{q^2}{2\hbar mc^2}\int_0^t dt' A^2 = -\frac{q^2E_0^2}{8\hbar m\omega^3}[\sin 2(\omega t+\alpha) - 8\cos\alpha\cos(\omega t+\alpha) + 2\omega t(2+\cos 2\alpha) + 3\sin 2\alpha],$$
$$S_3 = \frac{q}{\hbar mc}\int_0^t \mathrm{d}t'\mathbf{p}\mathbf{A} = \frac{q}{\hbar m\omega^2}\mathbf{E}_0\mathbf{p}[\sin(\omega t+\alpha) - \omega t\cos\alpha - \sin\alpha]. \tag{5}$$

These transformations lead to the Schrodinger equation

$$i\hbar\frac{\partial\chi}{\partial t} = \left[\frac{1}{2m}p^2 + \tilde{V}(\mathbf{r})\right]\chi \tag{6}$$

with the radiation-dressed potential

$$\tilde{V}(\mathbf{r}) = e^{-\frac{q}{m\omega^2}[\sin(\omega t+\alpha)-\omega t\cos\alpha-\sin\alpha]\mathbf{E}_0 grad}V(\mathbf{r}). \tag{7}$$

We note that the interaction with the radiation is applied suddenly at $t=0$, where $\psi(t=0)=\phi(t=0)=\chi(t=0)$ and the electromagnetic interaction vanishes at $t=0$ in the standard non-relativistic hamiltonian given by (4) and in $\tilde{V}(\mathbf{r})$. This establishes the equivalence of the three hamiltonians [(1), (4) and (6)] and the consistent inclusion of the interaction with the radiation field (see, for instance, [21–23]). The unitary transformations given by $S_{2,3}$ are the well-known Kramers–Henneberger transformations, including the radiation-dressed potential $\tilde{V}(\mathbf{r})$. [5–8]. The wavefunction $\psi=\exp(iS_1)\exp(iS_2)\exp(iS_3)\chi$ given by the above formulae is known sometimes as the non-relativistic Volkov wavefunction [18]. Its expansion in a temporal Fourier series indicates the presence of multiple "photons" with frequencies $n\hbar\omega$, where $n=0, 1, 2, \ldots$ integer [15, 24, 25]. The Kramers–Henneberger transformation has been used recently for estimating the energy levels of the hydrogen atom and an electron in helium atom in laser fields, or including magnetic fields [26–28].

We emphasise that in the above calculations we assume that the electric field $\mathbf{E}=\mathbf{E}_0\sin(\omega t+\alpha)$ is applied suddenly at $t=0$, it being zero for $t<0$. We envisage ionization experiments which proceed by placing a collection of atoms, molecules, atomic nuclei, atomic clusters, etc in the focal region of an (optical-) laser beam, the laser pulse being fired upon them. Strictly speaking, if there is not energy loss, and, especially, for strong fields, we may estimate the time of setting up the radiation upon a particle as $\Delta t = a/c$, where a is the dimension of the particle. For atoms $\Delta t\simeq 3\times 10^{-19}$ s ($a=a_H\simeq 0.53$Å, where a_H is the Bohr radius). This is a very short time in comparison with other relevant times. The perturbation implied by such a sudden application of the interaction generates an energy uncertainty

$\Delta E \simeq \hbar/\Delta t$, which is $\Delta E \simeq 1$ keV for atoms. The atomic stationary states are wiped out by such a perturbation (irrespective of the interaction strength), at least the high-energy ones. In the subsequent duration of time, the assembly of bound charges may follow two distinct regimes, depending on the field strength. For weak fields, the perturbed charges accommodate themselves in the field in a long time, which amounts to view the interaction as being adiabatically introduced. During this time, the charges radiate, re-arrange themselves (the mean-field potential changes) and tunneling ionization may appear. This is the standard perturbation-theoretical approach to the interaction of the bound charges with the electromagnetic radiation. For strong fields, there is no time for charges to accomodate in the interaction field, a circumstance which precludes the application of usual treatments of the ionization by tunneling, perturbation theory, or other equivalent approaches. We may say that the transient regime generated by the sudden application of a high-intensity interaction dominates, in comparison with the (quasi-) stationary regime. The quantitative criterion which separates the two regimes described here is given below.

Also, we note that the phase α in the expression of the electric field accounts for the spatial position of the charge in the assembly of bound-state particles. This parameter may be treated as a statistical parameter. It is also worth noting that $\mathbf{E}(t=0) \neq 0$, while the vector potential $\mathbf{A} = (c/\omega)\,\mathbf{E}_0[\cos(\omega t+\alpha) - \cos\alpha]$ is zero for $t=0$ ($\mathbf{A}(t=0)=0$). Also, we note that we may start directly with the standard non-relativistic hamiltonian given by (3), instead of the dipole hamiltonian given by (1), because the particles are immersed in the radiation field. However, we must pay attention to including the initial-time condition, such that $\mathbf{A}(t=0)=0$. This is the meaning of the term "consistent" used for characterizing the present procedure of treating the electromagnetic interaction.

We proceed now to apply these results to the ionization of atoms. We assume that in the absence of the radiation field the potential $V(\mathbf{r})$ is the mean-field potential which generates bound atomic single-electron states. For convenience, we consider the center of mass of the bound-state assembly fixed at the origin. Let us assume that the electric field $\mathbf{E}$ is directed along the z-axis. Then, (7) gives

$$\begin{aligned}\tilde{V}(x,y,z) &= V + \zeta(t)V_1 + \frac{1}{2!}\zeta^2(t)V_2 + \ldots \\ &= V(x,y,z+\zeta(t)),\end{aligned} \tag{8}$$

where $V_1 = \partial V/\partial z$, $V_2 = \partial^2 V/\partial z^2$, ... and

$$\zeta(t) = \frac{qE_0}{m\omega^2}[\omega t\cos\alpha - \sin(\omega t+\alpha) + \sin\alpha]. \tag{9}$$

We can see that the potential $\tilde{V}$ at the position of the charge is the original potential V at coordinate $z+\zeta(t)$, as if the charge is displaced in the potential V by $\zeta(t)$ along the z-direction. It is convenient to introduce the parameter $\xi = |q|E_0/m\omega^2 a$, where a denotes a length of the order of the dimension of the atom (in its ground-state); we asssume $a \simeq a_H = \hbar^2/me^2$, where a_H is the Bohr radius, $q=-e$ being the electron charge.

For $\xi \ll 1$ (low-intensity radiation) the electron charge oscillates and emits higher-order harmonics, due to the oscillations brought about by $\zeta(t)$ in the z-coordinate. During this process the charge is reconfigured, the time of charge re-arrangement being given approximately by $t_a = \hbar/\Delta\varepsilon$, where $\Delta\varepsilon$ is the perturbation energy generated by the radiation field via the potential $\tilde{V}$. During the time t_a the mean-field potential V changes and its radiation-dressing process is resumed periodically. This is a well-known process [29–32]. It is worth noting that the re-arrangement time t_a is long for low radiation fields, such that we may consider the electromagnetic radiation as being adiabatically applied after each reconfiguration process. Standard tunneling ionization may appear in this case. The case $\xi = |q|E_0/m\omega^2 a \ll 1$ corresponds to the case $\gamma = \omega(2ml_0)^{1/2}/|q|E_0 \gg 1$ in [3], where I_0 is the ionization potential. Keldysh showed that the tunneling (which proceeds by imaginary time) is equivalent in this case with multiple-photon absorption, especially for high frequencies [3].

For $\xi \gg 1$ (high-intensity radiation) the displacement $\zeta(t)$ may get rapidly larger than the atom size, and the electrons are set free. The electrons are left with their kinetic energy only. Consequently, we may expect the peripheral electrons be ejected from the atom. The attractive potential of the resulting ion suffers the same radiation-dressing reduction process and it is surpassed by the kinetic energy of the ejected electrons (note that the positively-charged ion moves in opposite direction than the electron).

The parameter ξ can also be written as $\xi = \eta(\lambda/a)$, where $\eta = |q|A_0/mc^2$, $A_0 = cE_0/\omega$, λ being the radiation wavelength (divided by 2π). In order to preserve the non-relativistic approximation we should have $\eta \ll 1$. The conditions $\eta < 1$ and $\xi > 1$ are satisfied for radiation intensity I in the (approximate) range $10^{11} < I < 10^{18}$ (W/cm^2) for electrons in atoms ($a = 10^{-8}$ cm) and optical radiation ($\omega = 10^{15}$ s^{-1}, $\lambda \simeq 10^{-4}$ cm); these intensities correspond approximately to an electric field in the range $10^4 < E_0 < 10^8$ (esu) (we use the notation esu for the electric field unit statvolt/cm as well as for the charge unit statcoulumb, $m = 10^{-27}$ g for the electron mass, $q = -e = 4.8\times10^{-10}$ esu for the electron charge, $\hbar = 10^{-27}$ erg·s for Planck's constant and $c = 3\times10^{10}$ cm/s for the speed of light in vacuum). Even for moderate laser

intensities the parameter ξ is large ($\xi \leq 10^3$ for $I = 10^{18}$ W/cm^2). Under these circumstances we may use $\omega t \ll 1$ in (9), which gives the duration τ for setting the electron free as

$$|\zeta(\tau)|/a = \xi\,|\omega\tau\cos\alpha - \sin(\omega\tau+\alpha) + \sin\alpha| \simeq \frac{1}{2}\xi(\omega\tau)^2\,|\sin\alpha| = 1. \quad (10)$$

We can see that the displacement of the electrons $\zeta(t)$ at the initial moment ($\omega t \ll 1$) is in the opposite direction with respect to the electric field, as expected. Depending on the sign of $\sin\alpha$, this displacement is either positive or negative. We assume the phase α randomly distributed and take the mean value $\overline{|\sin\alpha|} = 2/\pi$; we use $\xi(\omega\tau)^2/\pi = 1$ as a statistical criterion for ionization.

Let us consider a heavy atom with the nucleus charge Ze. According to the Thomas–Fermi model for heavy atoms ($Z \gg 1$) the electrons (in the ground state) are concentrated mainly at a distance of the order $a_H/Z^{1/3}$ from the nucleus. Therefore, we may estimate the release time τ from $\xi(\omega\tau)^2/\pi = 1 - 1/Z^{1/3}$. For a heavy ion with one electron and the nuclear charge Ze the radius of the electron orbit is of the order a_H/Z and the time τ can be estimated from $\xi(\omega\tau)^2/\pi = 1 - 1/Z$. In general, for a heavy ion with charge ne, $n \ll Z$, the charge localization distance b can be estimated from $Ze^2/b \simeq \hbar^2(Z-n)^{2/3}/mb^2$, which gives $b \simeq a_H(Z-n)^{2/3}/Z$. In all these cases, for large Z, we may take approximately the condition $\xi(\omega\tau)^2/\pi \simeq 1$ for estimating the release time τ. Similarly, the same condition is valid for light atoms.

Under these circumstances, the statistical criterion of ionization makes no distinction between successive ionization acts or multiple ionization, the very short release time being the same for each electron. However, we must be aware that after each process of electron ejection the electronic core suffers a reconfiguration (re-arrangement) process and the potential $V(\mathbf{r})$ is modified. This is the well-known process of core "shake-up" (which may imply also a core excitation) [31–34]. As a consequence of this reconfiguration process, the condition of setting the electrons free, derived from (8), is not valid anymore. A new bound state is formed and a new transformation process begins for the modified potential $V(\mathbf{r})$. It is important to note that for electrons ejected from high-energy states the re-arrangement time is short, such that the fast ionization may continue. This would be a succesive (or multiple) ionization. But when the ionization process begins to affect the electrons lying deep in energy, the re-arrangement time becomes longer and we may consider that the electromagnetic interaction is applied adiabatically for them. In these conditions, the ionization rate for deep-lying electrons becomes much smaller. A very rough estimate of the maximum number of electrons n_c ejected by the fast-ionization process is provided by the Thomas–Fermi model of heavy atoms, where the highest-energy state has a spatial (quasi-) degeneracy of the order $Z^{2/3}$. We may take, as a rough estimate, $n_c \simeq Z^{2/3}$. We note that the elimination of these electrons reduces appreciably the energy (which is of the order $\hbar^2 Z^{2/3}/ma^2$), such that the re-arrangement time begins to increase appreciably.

According to the discussion above, the fast-ionization rate is

$$\frac{1}{\tau} \simeq \sqrt{\xi/\pi}\,\omega = \sqrt{|q|\,E_0/\pi ma}. \quad (11)$$

The decay law of atom population N is $N = N_0 e^{-t/\tau}$, where N_0 is the initial number of atoms. For high-intensity radiation the rate given by (11) is appreciably enhanced in comparison with the tunneling-ionization rate. We can see from (11) that the uncertainty in energy brought about by the ionization is $\Delta\varepsilon = \hbar/\tau \simeq \sqrt{(\hbar^2/ma^2)\,|q|\,E_0 a}$. The mechanical work $|q|\,E_0 a$ done by the field to extract the electron is of the same order of magnitude as the localization energy $\hbar^2/ma^2$ for atomic fields $E_0 \simeq 10^6$ esu (intensity $I \simeq 10^{14}$ W/cm^2), as expected. In this case $\xi = (\hbar^2/ma^2)^2/(\hbar\omega)^2 = 10^2$ and $1/\tau = [(\hbar^2/ma^2)/\pi(\hbar\omega)]\omega = (10/\pi)\omega$ (it is assumed that the laser pulse duration is longer than ω^{-1} and the time τ). We can see that the emission time τ is shorter in this case than the period $(2\pi/\omega)$ of the radiation. In this limit, the very fast ionization rate is 20 times higher than the radiation frequency. For $E > E_0$ the process of interaction with the radiation is dominated by the fast-rate ionization. For lower fields the ionization process slows down appreciably as the mechanical work $|q|\,E_0 a$ becomes smaller than ε_b (>0), where $-\varepsilon_b$ is the binding energy of the electron (ionization potential).

We can get a critical value E_{0c} of the electric field for the transition from the low- to high-intensity regime from $\xi = 1$. This condition gives $E_{0c} \simeq 2\times 10^4$ esu for electrons, corresponding to a radiation intensity $I \simeq 10^{11}$ W/cm^2. We can see from (11) that the ionization rate increases with increasing field strength. In the intermediate-intensity range corresponding to $\xi \simeq 1$ the interplay between highly-excited states and ionization may generate a transient regime of atomic stabilization, with a lower ionization rate [35]. In this region the ionization rate $1/\tau$ can be estimated by solving the equation $|\zeta(\tau)|/a = 1$ for τ. It is easy to see that the solution $\tau(E)$ exhibits oscillations with respect to the field strength E_0.

It is worth estimating the spatial distribution of the ejected electrons. From (9) and (10) the average momentum given to an electron by the electric field during time τ is

$$\mathbf{p}_e = m\dot{\xi}\mathbf{e}_z = -\frac{1}{2}\sqrt{\pi m|q|E_0 a}\sin\alpha\cdot\mathbf{e}_z, \tag{12}$$

where $\mathbf{e}_z$ is the unit vector along the z-direction. In the absence of the radiation the momentum $p=\sqrt{2m\varepsilon}$ corresponding to the highest energy $\varepsilon=\hbar^2Z^{2/3}/ma^2$ in heavy atoms makes an angle β with the direction $\mathbf{e}_z$, being uniformly distributed in space. From the total momentum $\mathbf{P}=\mathbf{p}+\mathbf{p}_e$ we get

$$\cos\theta = \frac{\cos\beta - B}{1-2B\cos\beta+B^2}, \tag{13}$$

where θ is the angle made by $\mathbf{P}$ with the direction of the electric field ($\mathbf{e}_z$) and $B=\sqrt{\pi|q|E_0 a/8\varepsilon}\sin\alpha$. For our range of electric fields the parameter B is smaller than unity, such that we may use a series expansion in (13) in powers of B. After averaging over phase α we get

$$\cos\theta = \cos\beta\left[1+(4\cos^2\beta-1)\frac{\pi|q|E_0 a}{8\varepsilon}\right]. \tag{14}$$

We can see that the electric field brings only a small (anisotropic) contribution to the uniform distribution of the ejected electrons. Although high, the field E_0 is not as high as to dominate the kinetic energy of the electrons in heavy atoms.

We note that the above calculations are done for linearly polarised radiation. It is easy to see that similar calculations can be done for a general polarization.

An interesting question arises here related to very high-intensity radiation in the so-called relativistic regime, where $\eta\gg 1$. As long as the bound state of the charge subsists, the motion is, practically, non-relativistic. This means that the electromagnetic momentum $\mathbf{p}$ is sufficiently large to reduce to a large extent the contribution $q\mathbf{A}/c$, such that the velocity is small. The above non-relativistic formalism may be applied. However, this situation lasts a very short time (since $\xi\gg 1$), the charge being rapidly injected in the high-intensity radiation, where it is accelerated up to relativistic velocities [36–38].

We include here similar calculations for a static uniform electric field $\mathbf{E}$, which may be viewed as being derived from a vector potential $\mathbf{A}=-c\mathbf{E}t$. The wavefunction is

$$\psi = e^{-\frac{iq^2E^2t^3}{6\hbar m}}e^{\frac{iqt}{\hbar}\mathbf{Er}}e^{-\frac{iqt^2}{2\hbar m}\mathbf{Ep}}\chi, \tag{15}$$

where χ satisfies the Schrodinger equation

$$i\hbar\frac{\partial\chi}{\partial t} = \frac{1}{2m}p^2\chi + \tilde{V}(\mathbf{r})\chi \tag{16}$$

with the transformed potential given by

$$\tilde{V}(\mathbf{r}) = e^{\frac{qt^2}{2m}\mathbf{E}grad}V(\mathbf{r}). \tag{17}$$

It is easy to see that the ionization rate is $1/\tau=(|q|E/2ma)^{1/2}$. It coincides practically with the ionization rate in oscillating fields (11).

Finally, we note that the displacement occurring in the radiation-dressed interaction (8) is given, in general, by

$$\boldsymbol{\zeta}(t) = \frac{q}{m}\int_0^t \mathrm{d}t_1\int_0^{t_1}\mathrm{d}t_2\mathbf{E}(t_2). \tag{18}$$

By using this equation, we can estimate the ionization rate for a general time-dependence of the electric field in the laser pulse. For example, for a very short δ-like pulse $\mathbf{E}=T\mathbf{E}_0\delta(t-t_0)$ occurring at t_0 with a width T we get a ionization rate $1/\tau\simeq qTE_0/ma$. This equation differs from the $\sqrt{E_0}$-dependence given above. Since strong fields are obtained usually with very short pulses, the tendency towards the linear dependence on the field indicated here may be more suitable for analyzing experimental data in this case [39].

In conclusion, we may say that, by means of suitable unitary transformations of the Goeppert–Mayer and Kramers–Henneberger type, the interaction of bound charges with the electromagnetic radiation can be consistently taken into account for a sudden application of the electromagnetic interaction. It is shown that the radiation dressing of the structural interaction (which is responsible of the charge bound state) favours, in this case, the dissociation of the bound state (ionization) in high-intensity radiation. The atom ionization rate is estimated by using a statistical criterion. The new picture of atomic ionization described in this paper implies an explicit time evolution of the charge wavefunction with both structural and radiation interaction. In a short time τ after turning-on the high-intensity radiation the bound charges are set, practically, free.

The approach presented here can be extended to the ionization of molecules [40, 41] or atomic clusters [42], or to proton emission from atomic nuclei [43], or even ion emission from molecules (atomic clusters), as well as nuclear alpha decay. For electrons in atoms the conditions of ionization and non-relativistic approximation imply a radiation intensity in the range $10^{11}<I<10^{18}$ (W/cm^2), for atomic dimensions of the order $a=10^{-8}$ cm and optical radiation with frequency $\omega=10^{15}$ s^{-1}, as shown above. Similar conditions lead to $10^{7}<I<10^{23}$ (W/cm^2) for proton emission from atomic nuclei (nuclear dimension 10^{-13} cm) and $10^{17}<I/A^2<10^{23}$ (W/cm^2) for ion emission from molecules (dimension 10^{-18} cm), where A is the mass number of the ion.

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