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ISOLATING LAYER IN FOUNDATION-STRUCTURE DYNAMIC BEHAVIOR

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Abstract. We investigate the effect of inserting an isolating layer between a built structure and its base. Such a device is often used in order to reduce the damaging effects of the earthquakes. In this paper the structure and the base are viewed as coupled oscillators, with a base excitation due to the seismic ground motion. We give results for the realistic case of a structure-layer eigenfrequency much lower than the ground-base eigenfrequency. We show that the resonance frequency of the rigid structure may be appreciably shifted by such a device, although an additional, low, resonance frequency may appear. In addition, the oscillation amplitude of the structure is appreciably reduced in comparison with the rigid structure-base coupling.

Key words: structural behaviour, coupled oscillators, isolating layer, resonance.

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1. INTRODUCTION

The main concern in the mitigation of the seismic risk is to diminish the damaging effects of the earthquakes, including those regarding the buildings safety. This can be achieved through the implementation of earthquake-resistant design and rehabilitation of older structures, some built without building norms. The effects of the earthquakes on buildings are generally evaluated by using representative models, involving elements of Structural Engineering; such models include usually an analysis of the effects of the seismic excitation, with different shape wavefronts [1–3]. In this paper a simple model of dynamic behavior of structures is employed, suitable, for example for those equipped with base-isolated systems. Usually, such structures are viewed as localized harmonic oscillators, with one or several degrees of freedom and corresponding eigenfrequencies (characteristic frequencies). It is assumed that the seismic motion acts as an external force upon such oscillators, and the resonant regime is highlighted [4–6]. We assume an isolating layer, which decouples, to some extent, the structure from its base. Both the structure and its base are represented by two harmonic oscillators. The necessary conditions for such a representation are discussed in a previous paper [7]. The effects of the seismic motion upon the localized structure are included by considering a harmonic oscillator coupled to a homogeneous elastic medium. The resonance regime of the structure and the changes in its inertia and oscillator's frequency were highlighted previously [8]. The buildings can be fitted with "base isolation" systems, to separate the building from its foundations, through the use of special devices (*e.g.*, bearings, springs, runners) [9, 10]. When the seismic shock starts to act, the resulting movement will not impose a supplementary stress on the structure of the building. The procedure of the base isolation, as an effective measure of protecting urban areas exposed to high seismic risk, has gain a great interest, and a fast, continuous development is recorded, for highly developed cities, with highly-rising buildings, or economically important structures, located in near-fault areas [11–18]. An additional effect in base isolation technique, for this simplified system, is the shift of the fundamental period of the superstructure towards higher values. By such a device the eigenfrequency of a rigid base-structure may be removed from its resonance regime with the ground oscillations.

2. RIGID STRUCTURE

In structural engineering there is an increasing interest in isolating-foundation structures. Such an isolated structure consists of a flexible, soft layer interposed between the structure and its foundation (base). By such a device the eigenfrequency of a rigid base-structure may be removed from its resonance regime with the ground oscillations. At the same time, far from resonance, the stress suffered by the structure is appreciably diminished. An isolated base-structure can be viewed as two coupled oscillators, excited by the ground motion.

Let us consider the Lagrangian of two coupled linear oscillators

$$L = \frac{1}{2}m_s\dot{x}_s^2 + \frac{1}{2}m_b\dot{x}_b^2 - \frac{1}{2}k(x_s - x_b)^2 - \frac{1}{2}k_g\left(x_b - x_g\right)^2 \tag{1}$$

where $m_{s,b}$ are the masses of the structure and the base, k is the elastic coupling constant between structure and the base and k_g is the elastic coupling constant between the base and the ground. For $x_{b,g}$=0 the structure has the frequency given by $\omega_s^2 = k/m_s$; for $x_{s,g}$=0 the base has the frequency given by $\omega_b^2 = \frac{k+k_g}{m_s} = \omega_g^2 + \mu\omega_s^2$, where $\omega_g^2 = k_g/m_b$ and $\mu = m_s/m_b$.

Let us consider a rigid base-structure, *i.e.* $x_s = x_b = x_r$; then, the Lagrangian becomes

$$L = \frac{1}{2}(m_s + m_b)\dot{x}_r^2 - \frac{1}{2}k_g\left(x_r - x_g\right)^2, \tag{2}$$

which indicates a frequency given by

$$\omega_r^2 = \frac{k_g}{m_s+m_b} = \frac{\omega_g^2}{1+\mu} \tag{3}$$

We show below how this frequency is modified by a base which may move with respect to the structure. If ω_r happens to be in the resonant regime with the ground, the shift in frequency caused by the base-structure coupling may withdraw the building from this resonance regime.

The equation of motion of the rigid structure is

$$\ddot{x}_r + \omega_r^2 x_r = \omega_r^2 x_g \tag{4}$$

Let us assume that the ground excitation is $x_g = x_g^0 \cos\omega t$; then, the solution of the equation with the initial conditions $x_r(t=0) = \dot{x}_r(t=0) = 0$ is

$$x_r = \frac{\omega_r^2 x_g^0}{\omega^2 - \omega_r^2}(\cos\omega_r t - \cos\omega t) \tag{5}$$

We can see that for $\omega \gg \omega_r$ the amplitude of the rigid-structure oscillations is much smaller than the excitation amplitude, $(\omega_r/\omega)^2 x_g^0 \ll x_g^0$, while for low frequencies ($\omega \ll \omega_r$) the amplitude is comparable to the excitation amplitude x_g^0. These are far-from-resonance regimes. On approaching the resonance, we put $\omega = \omega_r + \varepsilon$, $\varepsilon \to 0$, and get

$$x_r \simeq \frac{1}{2}\omega_r x_g^0 t \sin\omega_r t \tag{6}$$

i.e. the oscillations increase indefinitely with increasing time; they are damped by friction. If we introduce the damping coefficient γ (see below), the resonant solution is

$$x_r \simeq \frac{\omega_r x_g^0}{2\gamma}(1 - e^{-\gamma t})\sin\omega_r t; \tag{7}$$

since, usually, $\gamma/\omega_r \ll 1$, the oscillation amplitude is very large, and usually the material fails.

3. COUPLED OSCILLATORS

The equations of motion corresponding to the lagrangian given by equation (1) are

$$\begin{gathered} m_s \ddot{x}_s + k(x_s - x_b) = 0\,, \\ m_b \ddot{x}_b + k(x_b - x_s) + k_g(x_b - x_g) = 0 \end{gathered} \tag{8}$$

or

$$\begin{gathered} \ddot{x}_s + \omega_s^2 x_s - \omega_s^2 x_b = 0\,, \\ -\mu\omega_s^2 x_s + \ddot{x}_b + (\omega_g^2 + \mu\omega_s^2)x_b = \omega_g^2 x_g \end{gathered} \tag{9}$$

The free solution of this system of equations is proportional to $e^{-i\omega t}$, where the frequency ω is given by

$$\begin{aligned}(\omega^2-\omega_s^2)x_s+\omega_s^2 x_b&=0\\ \mu\omega_s^2 x_s+\left(\omega^2-\omega_g^2-\mu\omega_s^2\right)x_b&=0.\end{aligned} \tag{10}$$

The determinant of this system of equations is

$$\Delta=\omega^4-\left(\omega_g^2+\omega_s^2+\mu\omega_s^2\right)\omega^2+\omega_s^2\omega_g^2=0. \tag{11}$$

We solve this equation by assuming $\omega_s^2 \ll \omega_g^2$, in accordance with the realistic situations; we get

$$\begin{aligned}\omega_1^2&=\omega_g^2+\mu\omega_s^2+\mu\omega_s^4/\omega_g^2+\cdots\\ \omega_2^2&=\omega_s^2-\mu\omega_s^4/\omega_g^2+\cdots\end{aligned} \tag{12}$$

and

$$\Delta=(\omega^2-\omega_1^2)(\omega^2-\omega_2^2). \tag{13}$$

We can see that the eigenfrequency of the rigid structure $\omega_r=\omega_g/\sqrt{1+\mu}$ (equation (3)) is changed into $\omega_1\simeq\omega_g$. Since, usually, $\mu\gg 1$, this frequency shift can be appreciable. At the same time, the frequency ω_s of the structure is only slightly modified, though, this time, it may represent a resonance frequency.

The free solutions of the coupled system of equations (9) are of the form

$$x_{s,b}=x_{s,b}^{(1)}e^{-i\omega_1 t}+x_{s,b}^{(2)}e^{-i\omega_2 t}+c.c. \tag{14}$$

where $x_{s,b}^{(1,2)}$ are solutions of the system

$$\begin{aligned}\left(\omega_{1,2}^2-\omega_s^2\right)x_s^{(1,2)}+\omega_s^2 x_b^{(1,2)}&=0\\ \mu\omega_s^2 x_s^{(1,2)}+\left(\omega_{1,2}^2-\omega_g^2-\mu\omega_s^2\right)x_b^{(1,2)}&=0.\end{aligned} \tag{15}$$

By solving this system of equations, we get

$$\begin{aligned}x_s^{(1)}&=-\frac{\omega_s^2}{\omega_g^2}\left[1+(1-\mu)\frac{\omega_s^2}{\omega_g^2}\right]A\,,\ \ x_b^{(1)}=A\\ x_b^{(2)}&=\mu\frac{\omega_s^2}{\omega_g^2}B\,,\qquad B=x_s^{(2)}\end{aligned} \tag{16}$$

where A and B are two constants which are determined from the initial conditions. Therefore, we get the free solution

$$x_s = -\frac{\omega_s^2}{\omega_g^2}\left[1 + (1-\mu)\frac{\omega_s^2}{\omega_g^2}\right]Ae^{-i\omega_1 t} + Be^{-i\omega_2 t} + c.c.$$
$$x_b = Ae^{-i\omega_1 t} + \mu\frac{\omega_s^2}{\omega_g^2}Be^{-i\omega_2 t} + c.c. \quad (17)$$

it is worth noting that the constants *A* and *B* are complex numbers.

A particular solution can be obtained easily from equations (9); by assuming an external excitation $x_g = x_g^0 \cos\omega t$; we get the particular solution

$$x_s^0 = \frac{\omega_s^2 \omega_g^2}{(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)} x_g^0$$
$$x_b^0 = -\frac{(\omega^2 - \omega_s^2)\omega_g^2}{(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)} x_g^0 \quad (18)$$

The general solution is the sum of the free solution (equations (17)) and the particular solution. Passing from the imaginary exponentials to trigonometric functions, we get the general solution

$$x_s = -\frac{\omega_s^2}{\omega_g^2}\left[1 + (1-\mu)\frac{\omega_s^2}{\omega_g^2}\right](A_1\cos\omega_1 t + A_2\sin\omega_1 t) +$$
$$+(B_1\cos\omega_2 t + B_2\sin\omega_2 t) + \frac{\omega_s^2\omega_g^2}{(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)} x_g^0\cos\omega t,$$
$$x_b = (A_1\cos\omega_1 t + A_2\sin\omega_1 t) + \mu\frac{\omega_s^2}{\omega_g^2}(B_1\cos\omega_2 t + B_2\sin\omega_2 t) -$$
$$-\frac{(\omega^2 - \omega_s^2)\omega_g^2}{(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)} x_g^0\cos\omega t. \quad (19)$$

The constants $A_{1,2}$ and $B_{1,2}$ are determined from the initial conditions $x_{s,b}(t=0) = 0$, $\dot{x}_{s,b}(t=0) = 0$. We limit ourselves to the leading contributions in powers of ω_s^2/ω_g^2, and get finally

$$x_s = \frac{\left(\omega^2 - \omega_g^2\right)\omega_s^2}{(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)}(\cos\omega_2 t - \cos\omega_1 t)x_g^0 +$$
$$+\frac{\omega_s^2\omega_g^2}{(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)} x_g^0\cos\omega t, \quad (20)$$
$$x_b = \frac{\left(\omega^2 - \omega_g^2\right)\omega_g^2}{(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)} x_g^0\cos\omega_1 t - \frac{(\omega^2 - \omega_s^2)\omega_g^2}{(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)} x_g^0\cos\omega t$$

First, we note that the resonance is shifted to $\omega = \omega_1 \simeq \omega_g$, from $\omega = \omega_r = \omega_g/\sqrt{1+\mu}$, and a new resonance frequency $\omega = \omega_2 \simeq \omega_s$ appears. Secondly, outside the resonance region, the oscillation amplitude of the structure is proportional to ω_s^2, which, in comparison with the rigid structure (equation (5)), is appreciably reduced. We may neglect the motion of the structure (x_s) in comparison with the motion of the base (x_b), due to the factor $\omega_s^2/\omega_g^2 \ll 1$. In these conditions we may limit ourselves to the study of one linear oscillator with frequency $\omega_1 \simeq \omega_g$ (the base).

4. DAMPING

For reference purposes we give here results for an oscillator with damping. Usually, the equation of motion of such an oscillator is

$$\ddot{x} + \omega_0^2 x + 2\gamma\dot{x} = \omega_0^2 x_g \tag{21}$$

where ω_0 is the frequency, m is the mass of the oscillator and $\gamma(\ll \omega_0)$ is the damping coefficient (this is so-called Kelvin-Voigt model). The solution (with zero initial conditions) is

$$\begin{gathered} x = \frac{(\omega^2-\omega_0^2)\omega_0^2}{(\omega^2-\omega_0^2)^2+4\gamma^2\omega^2} x_g^0 (e^{-\gamma t}\cos\omega_0 t - \cos\omega t) - \\ -\frac{\omega_0\gamma x_g^0}{(\omega^2-\omega_0^2)^2+4\gamma^2\omega^2}[(\omega^2+\omega_0^2)e^{-\gamma t}\sin\omega_0 t - 2\omega\omega_0\sin\omega t]; \end{gathered} \tag{22}$$

for long times the free solution is vanishing (it is damped out), and we are left with the forced oscillation

$$x = -\omega_0^2 x_g^0 \frac{(\omega^2-\omega_0^2)\cos\omega t - 2\omega\gamma\sin\omega t}{(\omega^2-\omega_0^2)^2+4\gamma^2\omega^2}. \tag{23}$$

The energy balance can be estimated by multiplying equation (21) by $\dot{x}$; we get

$$\frac{d}{dt}\left(\frac{1}{2}\dot{x}^2 + \frac{1}{2}\omega_0^2 x^2\right) + 2\gamma\dot{x}^2 = \omega_0^2 x_g^0 \dot{x}\cos\omega t \tag{24}$$

where

$$E = \frac{1}{2}\dot{x}^2 + \frac{1}{2}\omega_0^2 x^2 \tag{25}$$

is the energy of the oscillator (per unit mass),

$$E_d = 2\gamma \dot{x}^2 \tag{26}$$

is the dissipated energy per unit mass and unit time, and

$$W = \omega_0^2 x_g^0 \dot{x} \cos\omega t \tag{27}$$

is the mechanical work done by the external force per unit mass and unit time. By writing the solution (23) as

$$x = A\cos\omega t + B\sin\omega t \tag{28}$$

we can see that the oscillator absorbs an amount of energy and absorbs and returns periodically another amount. Similarly, the dissipated energy and the work oscillate periodically around some mean values. Far from resonance the coefficients *A* and *B* are

$$A = -\frac{\omega_0^2 x_g^0}{\omega^2 - \omega_0^2} \;, \quad B = \frac{2\omega\omega_0^2 \gamma x_g^0}{\left(\omega^2 - \omega_0^2\right)^2} \tag{29}$$

and we can check that the mean values are

$$\bar{E} = \frac{1}{4}(\omega^2 + \omega_0^2) A^2 \tag{30}$$

and

$$\bar{W} = \bar{E}_d = \frac{\omega^2 \omega_0^4 \gamma x_g^{02}}{\left(\omega^2 - \omega_0^2\right)^2} \tag{31}$$

i.e. the work compensates the dissipation. At resonance $A = 0$ and $B = \omega_0 x_g^0 / 2\gamma$, and we get and $\bar{E} = \omega_0^2 B^2 / 4$ and

$$\bar{W} = \bar{E}_d = \frac{\omega_0^4 \gamma x_g^{02}}{4\gamma} \tag{32}$$

we can see that the energy absorbed and dissipated at resonance is very large.

5. DISCUSSION AND CONCLUSIONS

As it is well known, damaging effect appears at resonance, when one or more seismic frequencies equal one or more eigenfrequencies of the buildings. Usually, the seismic periods are mainly in the range 0.1–10 s, and, unfortunately, the lower part of this range is shared by buildings' eigenperiods. The usual designs attempt to modify the buildings, *e.g.* by implementing isolating layers between the base and the structure, such that the buildings' eigenfrequencies be removed from the range of the seismic frequencies. As a key element in earthquake engineering, means of achieving to some extent a building-foundation decoupling are pursued, such that the response of the building to vibrations be not (too) damaging.

In a simplified model, a rigid base-structure behaves as a harmonic oscillator with one eigenfrequency. Usually, this eigenfrequency is a high one. If a soft, elastic layer is inserted between the base and the structure, the base and the structure behave as two coupled oscillators. This coupled oscillation system has now two eigenfrequencies: one is an upright shifted rigid base-structure high frequency, and another is a low frequency, appearing by the structure-layer coupling. The occurrence of this low frequency increases the seismic risk of building damage. At the same time, if we are far from resonance, the low frequency diminishes appreciably the mechanical effort on the structure, which means lower construction costs. However, the deformability processes are located at the isolating layer, which may rise additional problems.

Therefore, as regards a possible seismic base isolation of the buildings by designing special foundations, we may say that the answer is not definite. The concern is to definitely remove the building frequency from the range of the seismic shock, an aim which is not always achieved.

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