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The Effect of Surface Inhomogeneities on the Propagation of Elastic Waves

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Abstract We investigate the effect of the surface inhomogeneities (defects) on the propagation of the elastic waves in a semi-infinite isotropic solid body (half-space). A perturbation-theoretical scheme is devised for small surface defects (in comparison with the relevant elastic disturbances propagating in the body), and the elastic waves equations are solved in the first-order approximation. It is shown that surface defects generate both scattered waves localized (and propagating only) on the surface (two-dimensional waves) and scattered waves reflected back in the body. Directional effects, wave slowness and attenuation by diffusive scattering, or possible resonance effects are discussed.

Keywords Surface roughness · Elastic waves scattering · Surface localized waves

Mathematics Subject Classification 74B05 · 74E05 · 74J20 · 74J15 · 35L05 · 86A15

1 Introduction

There is a great deal of interest in the role played by the surface defects (inhomogeneities) in a large variety of physical phenomena, ranging from mechanical properties of the elastic bodies [1, 2], to hydrodynamical flow of microfluids [3], dispersive properties of surface plasmon-polariton in nanoplasmonics [4], terahertz-waves generation [5] or electronic microstructures [6, 7]. Giant corrugations have been found on the graphite surface by scanning tunneling microscopy, due to the elastic deformations induced by atomic forces between tip and surface [8]. Periodic surface corrugation plays a central role in enhanced, or suppressed, optical transmission in the subwavelength regime [9], or in highly-directional optical emission [10]. An appreciable reduction in the thermal conductance has been assigned to the phonon scattering by surface defects [11]. Stick-slip instability responsible for earthquakes has been studied, as well as the associated radiation of seismic surface waves [12]. It has

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been recognized that elastic-waves propagation effects may play a central role in the surface defects associated with the cracks occurring in heterogeneous media, like aluminium alloys, ceramics or rock [13, 14]. The main difficulty in getting more definite results in this problem resides in modelling conveniently the surface inhomogeneities, such as to arrive at mathematically operational approaches [15, 16].

We introduce here a model of surface inhomogeneities, whose elastic characteristics are, in general, distinct from the ones of the underlying (isotropic) elastic half-space (semi-infinite solid). Such a model may account both for surface roughness and for surface coatings (in general, non-uniform). It is shown that the elastic waves propagating in the semi-infinite body (incident on and reflected specularly by the surface) generate a force localized on the surface, which is responsible for the scattered waves. This force arises mainly from both the presence of the surface layer and the more-or-less abrupt termination of the solid at its surface. The scattered waves are of two kinds: localized (and propagating only) on the surface (two-dimensional waves), and waves scattered back in the body. For an enhanced distribution of surface defects the waves scattered back in the body may get confined to the surface (damped surface waves). The method employed in the present paper is based on a perturbation-theoretical scheme, and the resulting coupled integral equations are solved in the first approximation with respect to the magnitude of the defects distribution. Multiple scattering is expected to occur in higher-order approximations. The perturbation method employed here differs from other perturbation methods. For instance, the perturbation treated in this paper is partially an intrinsic one, not a purely external one, as in the Born approximation. The introduction of a surface layer is equivalent to some extent with a double-scale treatment, so that, in this respect, there is a resemblance with a multi-scale method.

Forward and backward scattering of elastic waves have also been reported in corrugated waveguides [17]. Great insight has been obtained previously in the coupling of the surface (Rayleigh) waves to periodic corrugation (grating) [18–20], especially as regards the wave attenuation, slowing and leaking (outgoing increasing wave), corroborated with band gaps and stop bands, by using non-perturbational techniques. The reflection and refraction of elastic (acoustic) waves at a rough surface, or ducts with variable cross-sections, have been extensively studied, emphasizing the role of the boundary conditions [21–27]. Powerful numerical methods have been developed for such complex problems. A great deal of attention was given to the coupled modes propagating in elastic waveguides with rough surfaces, which highlighted a rich phenomenology [28–33]. The interplay between mode dispersion and surface roughness may lead to a well-defined selectivity in the transmission coefficient and anomalous backscattering enhancement. Most of such important results are obtained numerically. Similar results have been reported for sound and ultrasound waves propagating in fluids [34–38].

In addition to such results, we show here that the surface inhomogeneities may cause localized waves, propagating only on the surface, which may store a certain amount of energy, due to the localization effects. Attenuation of crustal waves across the Alpine range has been reported, which might be associated with the localization of energy in the surface defects region [39]. The method presented here can be extended to electromagnetic waves, or fluid waves, propagating in a semi-infinite body with surface defects. It was employed recently to analyze the elastic waves produced by localized forces in semi-infinite solids [40].

2 Elastic Body with Surface Inhomogeneities

We consider an isotropic elastic body extended boundlessly along the directions $\mathbf{r} = (x, y)$ and limited along the z-direction by a free surface $z = h(\mathbf{r})$, where $h(\mathbf{r}) > 0$ is a function to be further specified (roughness function). The body, which may also be termed a semi-infinite solid (elastic half-space) with a non-planar surface, occupies the region $z < h(\mathbf{r})$. It is convenient to write the well-known equation for free elastic waves in an isotropic body [41] as

$$\frac{1}{v_t^2}\ddot{\mathbf{u}} - \Delta\mathbf{u} = m \cdot \mathrm{grad} \cdot \mathrm{div}\, \mathbf{u}, \tag{1}$$

where $\mathbf{u}(\mathbf{r}, z, t)$ is the displacement field, t denotes the time, v_t is the velocity of the transverse waves, $m = v_l^2/v_t^2 - 1 > 1/3$ (actually 1) [41] and v_l is the velocity of the longitudinal waves. Indeed, Eq. (1) gives the free transverse waves ($\mathrm{div}\, \mathbf{u} = 0$) propagating with velocity v_t and the free longitudinal waves ($\mathrm{curl}\, \mathbf{u} = 0$) propagating with velocity v_l.

For a semi-infinite body with a surface described by equation $z = h(\mathbf{r})$ and extending in the region $z < h(\mathbf{r})$ the displacement field can be written as

$$\mathbf{u} = (\mathbf{v}, w)\theta\big[h(\mathbf{r}) - z\big], \tag{2}$$

where $\mathbf{v}$ lies in the (x, y)-plane, w is directed along the z-axis and θ is the step function ($\theta(z) = 0$ for $z < 0$, $\theta(z) = 1$ for $z > 0$). The magnitude of the surface inhomogeneities (deviation from a plane) is given by the function $h(\mathbf{r})$, which we assume to be very small in comparison with the relevant wavelengths along the z-directions of the elastic disturbances propagating in the body. Consequently, we may use the first-order approximation

$$\mathbf{u} = (\mathbf{v}, w)\big[\theta(-z) + h(\mathbf{r})\delta(z)\big] \tag{3}$$

for Eq. (2), where $\delta(z)$ is the Dirac function. This is the usual approximation employed in the perturbation-theoretical approaches [42–44]. The specific conditions of validity for this approximation will be discussed on the final results.

We write such a displacement field as

$$\mathbf{u} = \mathbf{u}_0 + \delta\mathbf{u}_0, \tag{4}$$

where

$$\mathbf{u}_0 = (\mathbf{v}_0, w_0)\theta(-z), \qquad \delta\mathbf{u}_0 = (\mathbf{v}_0, w_0)|_{z=0} h\delta(z), \tag{5}$$

and assume that $\mathbf{u}_0$ satisfies the wave equation (1)

$$\frac{1}{v_t^2}\ddot{\mathbf{u}}_0 - \Delta\mathbf{u}_0 = m \cdot \mathrm{grad} \cdot \mathrm{div}\, \mathbf{u}_0 \tag{6}$$

with specific boundary conditions at $z = 0$. This equation describes incident and (specularly) reflected waves propagating in a semi-infinite solid with a plane surface $z = 0$. We can see that $\delta\mathbf{u}_0$ generates a source-term localized on the surface (a force), which can produce scattered waves. We denote the displacement field associated with these scattered waves by $\mathbf{u}_1$; it satisfies the wave equation

$$\frac{1}{v_t^2}\ddot{\mathbf{u}}_1 - \Delta\mathbf{u}_1 = m \cdot \mathrm{grad} \cdot \mathrm{div}\, \mathbf{u}_1 + \frac{\mathbf{f}}{v_t^2}, \tag{7}$$

where the force is given by

$$\frac{\mathbf{f}}{v_t^2} = \frac{1}{v_t^2}\delta\ddot{\mathbf{u}}_0 - \Delta\delta\mathbf{u}_0 - m \cdot \mathrm{grad} \cdot \mathrm{div}\, \delta\mathbf{u}_0. \tag{8}$$

Equations (7) and (8) represent merely a different way of re-writing the wave equation for a semi-infinite solid with surface defects. For waves localized on the surface the solution of Eq. (7) is $\mathbf{u}_1 = \delta\mathbf{u}_0$. Another solutions are given by the waves scattered back in the body by the surface defects, i.e. waves generated in Eq. (7) by the source term $\mathbf{f}$ (a particular solution of Eq. (7)). We generalize this model of surface defects by assuming that the roughness is "inhomogeneous", i.e., it is a homogeneous elastic medium with different elastic characteristics than the plane-surface half-space bulk (for instance, different density and elastic constants). Therefore, we introduce distinct velocities $\bar{v}_{t,l}$ and denote all the changed parameters with an overbar (for instance, $\bar{m} = \bar{v}_l^2/\bar{v}_t^2 - 1$). The force is given in this case by

$$\frac{\bar{\mathbf{f}}}{v_t^2} = \frac{1}{\bar{v}_t^2}\delta\ddot{\mathbf{u}}_0 - \Delta\delta\mathbf{u}_0 - \bar{m}\cdot\mathrm{grad}\cdot\mathrm{div}\,\delta\mathbf{u}_0, \tag{9}$$

The results are expressed conveniently by using the relative differences $\eta_{t,l} = 1 - v_{t,l}^2/\bar{v}_{t,l}^2$. The displacement field $\mathbf{u}_1$ given by Eq. (7) can be written as $\mathbf{u}_1 = (\mathbf{v}, w)\theta(-z)$.

We may say that, in the presence of a displacement field $\mathbf{u}_0$, the surface inhomogeneities generate a force $\bar{\mathbf{f}}$, localized on the surface and of the same order of magnitude as the function h ($\delta u_0 \sim h\delta(z)$). This force is the difference between the inertial force $\delta\ddot{\mathbf{u}}_0/\bar{v}_t^2$ and the elastic force $\Delta\delta\mathbf{u}_0 + \bar{m}\cdot\mathrm{grad}\cdot\mathrm{div}\,\delta\mathbf{u}_0$; it represents the distinct way the surface follows the elastic motion in comparison with the bulk. Equation (6) gives the free incident and reflected waves propagating in a half-space with a plane surface, while Eq. (7) gives the scattered waves produced by the roughness of the surface, as a consequence of the source term $\bar{\mathbf{f}}/v_t^2$.

It is worth noting that such a model of inhomogeneous surface may correspond either to a surface whose physical properties have been changed, or to a solid which is homogeneous everywhere, including its rough surface. Indeed, in the latter case, it is precisely the spatial variations of the rough surface which affect its elastic properties, viewed as a homogeneous medium, and render it, in fact, a rough surface which is inhomogeneous with respect to the bulk.

The above perturbation-theoretical scheme can also be written in a different way, by recasting equation (1) into an equation involving the velocity v_l of the longitudinal waves and the parameter $n = 1 - v_t^2/v_l^2 = m/(1+m)$. Then, Eqs. (6)–(8) become

$$\begin{aligned}
&\frac{1}{v_l^2}\ddot{\mathbf{u}}_0 - \Delta\mathbf{u}_0 = n(-\Delta\mathbf{u}_0 + \mathrm{grad}\cdot\mathrm{div}\,\mathbf{u}_0),\\
&\frac{1}{v_l^2}\ddot{\mathbf{u}}_1 - \Delta\mathbf{u}_1 = n(-\Delta\mathbf{u}_1 + \mathrm{grad}\cdot\mathrm{div}\,\mathbf{u}_1) + \frac{\bar{\mathbf{f}}}{v_l^2},
\end{aligned} \tag{10}$$

where

$$\frac{\bar{\mathbf{f}}}{v_l^2} = \frac{1}{\bar{v}_l^2}\delta\ddot{\mathbf{u}}_0 - (1-\bar{n})\Delta\delta\mathbf{u}_0 - \bar{n}\cdot\mathrm{grad}\cdot\mathrm{div}\,\delta\mathbf{u}_0. \tag{11}$$

We solve Eq. (7) and the second equation (10) for the scattered transverse and, respectively, longitudinal waves by using the Green function method.

3 Plane Surface

As it is well known, the elementary solutions of Eq. (6), or the first equation (10), (homogeneous elastic waves equation) for a half-space with a plane surface are transverse and longitudinal plane waves of the form

$$\mathbf{u}_0 \sim \left(e^{\pm i\kappa_0 z}, e^{\pm i\kappa_0' z}\right)e^{-i\omega t+i\mathbf{k}_0\mathbf{r}}, \tag{12}$$

where both incident ($+\kappa_0, +\kappa_0'$) and reflected ($-\kappa_0, -\kappa_0'$) waves are included, ω is the frequency and $\mathbf{k}_0$ is the in-plane wavevector. For $\operatorname{div}\mathbf{u}_0 = 0$ we get the transverse waves, propagating with the velocity v_t ($\omega = v_t K_0$, where $\mathbf{K}_0 = (\mathbf{k}_0, \kappa_0)$), with the z-component of the wavevector $\kappa_0 = \sqrt{\omega^2/v_t^2 - k_0^2}$. For $\operatorname{curl}\mathbf{u}_0 = 0$ we get the longitudinal waves (through $\operatorname{curl}\cdot\operatorname{curl}\mathbf{u}_0 = -\Delta\mathbf{u}_0 + \operatorname{grad}\cdot\operatorname{div}\mathbf{u}_0 = 0$), propagating with the velocity v_l and the z-component of the wavevector $\kappa_0' = \sqrt{\omega^2/v_l^2 - k_0^2}$ ($\omega = v_l K_0'$ and $\mathbf{K}_0' = (\mathbf{k}_0, \kappa_0')$). The transverse waves have two polarizations, one in the propagating plane (the $(\mathbf{k}_0, \kappa_0)$-plane), which we call here the p-wave (parallel wave), another perpendicular to the propagating plane, which we call here the s-wave (from the German "senkrecht", which means "perpendicular"). Linear combinations of the plane waves given by Eq. (12) are subject to conditions imposed on the surface (e.g., free or fixed surface). The p- and s-notation is used in electromagnetism. In seismology the longitudinal waves, denoted here by the suffix l, are usually called primary waves and denoted by P, while the transverse s-waves discussed here are called shear horizontal waves and denoted by SH. The transverse p-waves discussed here have not a simple polarization with respect to the surface. It is worth noting that the results of the perturbation scheme applied here to the integral equations acquire their most simple and convenient form for longitudinal waves and p- and s-transverse waves as used here.

We derive here these free waves propagating in a half-space with a plane surface by a different method, which will be used subsequently in deriving the solutions for the scattered waves (Eq. (7) and the second equation (10)). In order to simplify the notations we omit here the subscript 0.

The solution of Eq. (6) is written as

$$\mathbf{u} = \left[\mathbf{v}(z), w(z)\right]\theta(-z)e^{-i\omega t+i\mathbf{k}\mathbf{r}}. \tag{13}$$

Introducing this $\mathbf{u}$ in Eq. (6) and leaving aside the exponential factor $e^{-i\omega t+i\mathbf{k}\mathbf{r}}$ we get

$$\frac{\partial^2\mathbf{u}}{\partial z^2} + \kappa^2\mathbf{u} = \mathbf{S}, \tag{14}$$

where $\kappa^2 = \omega^2/v_t^2 - k^2$ and the source $\mathbf{S}$ has the components

$$\begin{aligned}
\mathbf{S}_{(x,y)} &= -im\mathbf{k}\left(i\mathbf{k}\mathbf{v} + \frac{\partial w}{\partial z}\right)\theta(-z) \\
&\quad + \left(\left.\frac{\partial\mathbf{v}}{\partial z}\right|_{z=0} + imkw|_{z=0}\right)\delta(z) + \mathbf{v}|_{z=0}\delta'(z), \\
S_z &= -m\left[i\mathbf{k}\frac{\partial\mathbf{v}}{\partial z} + \frac{\partial^2 w}{\partial z^2}\right]\theta(-z) + im\mathbf{k}\mathbf{v}|_{z=0}\delta(z) \\
&\quad + (1+m)\left[\left.\frac{\partial w}{\partial z}\right|_{z=0}\delta(z) + w|_{z=0}\delta'(z)\right].
\end{aligned} \tag{15}$$

We can see that the source $\mathbf{S}$, which collects all the contributions from $m\cdot\operatorname{grad}\mathbf{u}$ and the derivatives of $\theta(-z)$ in $\Delta\mathbf{u}$, acts as an "external force" in Eq. (14). As it is well known, the particular solution of Eq. (14) is given by

$$\mathbf{u}(z)=\int dz' G\left(z-z'\right)\mathbf{S}\left(z'\right), \tag{16}$$

where

$$G(z)=\frac{1}{2i\kappa}e^{i\kappa|z|} \tag{17}$$

is the Green function for Eq. (14) (Green function of the one-dimensional Helmholtz equation). Making use of the notations $v_1=\mathbf{vk}/k$ and $v_2=\mathbf{vk}_\perp/k$, where $\mathbf{k}_\perp$ is a vector perpendicular to $\mathbf{k}$ and of the same magnitude k, Eqs. (15)–(17) lead to

$$v_2=-\frac{i}{2\kappa}\left.\frac{\partial v_2}{\partial z}\right|_{z=0}e^{-i\kappa z}-\frac{1}{2}v_2|_{z=0}e^{-i\kappa z} \tag{18}$$

and

$$\begin{aligned}
v_1={}&-\frac{imk^2}{2\kappa}\int^0 dz' v_1\left(z'\right)e^{i\kappa|z-z'|}-\frac{mk}{2\kappa}\frac{\partial}{\partial z}\int^0 dz' w\left(z'\right)e^{i\kappa|z-z'|}\\
&-\frac{i}{2\kappa}\left.\frac{\partial v_1}{\partial z}\right|_{z=0}e^{-i\kappa z}-\frac{1}{2}v_1|_{z=0}e^{-i\kappa z},\\
(1+m)w={}&-\frac{mk}{2\kappa}\frac{\partial}{\partial z}\int^0 dz' v_1\left(z'\right)e^{i\kappa|z-z'|}+\frac{im\kappa}{2}\int^0 dz' w\left(z'\right)e^{i\kappa|z-z'|}\\
&-\frac{i}{2\kappa}\left.\frac{\partial w}{\partial z}\right|_{z=0}e^{-i\kappa z}-\frac{1}{2}w|_{z=0}e^{-i\kappa z}.
\end{aligned} \tag{19}$$

Equation (18) corresponds to the s-wave. It is easy to see that the particular solution given by Eq. (18) is identically vanishing. Therefore, we are left with the free s-waves given by Eq. (12), as expected ($\sim e^{\pm i\kappa z}e^{-i\omega t+i\mathbf{kr}}$).

Let us take the second derivative of Eq. (19) with respect to z and use the identity

$$\frac{\partial^2}{\partial z^2}\int dz' f\left(z'\right)e^{i\kappa|z-z'|}=-\kappa^2\int dz' f\left(z'\right)e^{i\kappa|z-z'|}+2i\kappa f(z) \tag{20}$$

for any arbitrary function $f(z)$. We get

$$\begin{aligned}
&\frac{\partial^2 v_1}{\partial z^2}+\kappa^2 v_1=-imk\left(ikv_1+\frac{\partial w}{\partial z}\right),\\
&\frac{\partial^2 w}{\partial z^2}+\kappa^2 w=-m\frac{\partial}{\partial z}\left(ikv_1+\frac{\partial w}{\partial z}\right).
\end{aligned} \tag{21}$$

We can see that for $\operatorname{div}(v_1,w)=0$, i.e., for $ikv_1+\partial w/\partial z=0$, we get the free p-waves ($\kappa=\sqrt{\omega^2/v_t^2-k^2}$), according to Eq. (12) ($\sim e^{\pm i\kappa z}e^{-i\omega t+i\mathbf{kr}}$). Similarly, for $\operatorname{curl}\mathbf{u}=0$, i.e., for $ikw-\partial v_1/\partial z=0$, Eq. (21) become

$$(1+m)\frac{\partial^2(v_1,w)}{\partial z^2}+\left(\kappa^2-mk^2\right)(v_1,w)=0, \tag{22}$$

or, making use of $m = v_l^2/v_t^2 - 1$,

$$\frac{\partial^2 (v_1, w)}{\partial z^2} + \kappa'^2 (v_1, w) = 0, \tag{23}$$

where $\kappa' = \sqrt{\omega^2/v_l^2 - k^2}$, i.e., free longitudinal waves $\sim e^{\pm i\kappa' z} e^{-i\omega t + i\mathbf{kr}}$.

The longitudinal waves can also be obtained by noting that the coupled equations (19) imply the relationship

$$\frac{\partial v_1}{\partial z} - ikw = Ce^{-i\kappa z}, \tag{24}$$

where

$$C = -\frac{1}{2}\left(\frac{\partial v_1}{\partial z} - ikw\right)\bigg|_{z=0} + \frac{1}{2}\left(i\kappa v_1 - \frac{k}{\kappa}\frac{\partial w}{\partial z}\right)\bigg|_{z=0}. \tag{25}$$

We use this relationship in one of Eq. (21), and get

$$\frac{\partial^2 v_1}{\partial z^2} + \kappa'^2 v_1 = -\frac{im\kappa}{1+m} Ce^{-i\kappa z}. \tag{26}$$

The particular solution of this equation is vanishing identically, and we are left with free longitudinal waves. Indeed, Eq. (24) with $C = 0$ corresponds to $\mathrm{curl}(v_1, w) = 0$.

The p-waves are obtained in a similar way, by starting with the first equation (10). Using $\mathbf{u}$ given by an equation similar with Eq. (13) we get

$$\begin{aligned} (1-n)v_2 = {} & \frac{in(\kappa'^2 + k^2)}{2\kappa'} \int^0 dz' v_2(z') e^{i\kappa'|z-z'|} \\ & - \frac{i}{2\kappa'} \frac{\partial v_2}{\partial z}\bigg|_{z=0} e^{-i\kappa' z} - \frac{1}{2} v_2|_{z=0} e^{-i\kappa' z} \end{aligned} \tag{27}$$

and

$$\begin{aligned} (1-n)v_1 = {} & \frac{in\kappa'}{2} \int^0 dz' v_1(z') e^{i\kappa'|z-z'|} - \frac{nk}{2\kappa'} \frac{\partial}{\partial z} \int^0 dz' w(z') e^{i\kappa'|z-z'|} \\ & - \frac{i}{2\kappa'} \frac{\partial v_1}{\partial z}\bigg|_{z=0} e^{-i\kappa' z} - \frac{1}{2} v_1|_{z=0} e^{-i\kappa' z}, \\ w = {} & -\frac{nk}{2\kappa'} \frac{\partial}{\partial z} \int^0 dz' v_1(z') e^{i\kappa'|z-z'|} + \frac{ink^2}{2\kappa'} \int^0 dz' w(z') e^{i\kappa|z-z'|} \\ & - \frac{i}{2\kappa'} \frac{\partial w}{\partial z}\bigg|_{z=0} e^{-i\kappa' z} - \frac{1}{2} w|_{z=0} e^{-i\kappa' z}. \end{aligned} \tag{28}$$

It is easy to see, by taking the second derivative with respect to z, that Eq. (27) gives the free s-waves. Similarly, by taking the second derivative with respect to z, Eq. (28) become

$$\begin{aligned} & \frac{\partial^2 v_1}{\partial z^2} + \frac{\kappa'^2}{1-n} v_1 = -ink\frac{\partial w}{\partial z}, \\ & \frac{\partial^2 w}{\partial z^2} + (1-n)\kappa^2 w = -ink\frac{\partial v_1}{\partial z} \end{aligned} \tag{29}$$

(where we have used the identity $\kappa'^2 + nk^2 = (1-n)\kappa^2$). On the other hand, from Eq. (28), we get easily the relationship

$$\frac{\partial v_1}{\partial z} + i\frac{\kappa^2}{k} w = \frac{C'}{1-n} e^{-i\kappa' z}, \tag{30}$$

where

$$C' = -\frac{1}{2}\left(\frac{\partial v_1}{\partial z} + \frac{i\kappa'^2}{k} w\right)\Big|_{z=0} + \frac{1}{2}\left(i\kappa' v_1 + \frac{\kappa'}{k}\frac{\partial w}{\partial z}\right)\Big|_{z=0}. \tag{31}$$

Making use of this relationship in Eq. (29) we get

$$\frac{\partial^2 w}{\partial z^2} + \kappa^2 w = -\frac{ink}{1-n} C' e^{-i\kappa' z} \tag{32}$$

and a similar equation for v_1. It is easy to see that the particular solution of Eq. (32) is identically vanishing, so we are left with the free p-waves. Indeed, Eq. (30) with $C' = 0$ corresponds to $\mathrm{div}(v_1, w) = 0$.

4 Scattered Waves

We consider now a bulk incident transverse wave and reflected transverse and longitudinal waves given by

$$\mathbf{u}_0 = \left(\mathbf{u}_0^{(1)} e^{i\kappa_0 z} + \mathbf{u}_0^{(2)} e^{-i\kappa_0 z} + \mathbf{u}_0^{(3)} e^{-i\kappa_0' z}\right) e^{-i\omega t + i\mathbf{k}_0\mathbf{r}} \tag{33}$$

(for $z < 0$), where the amplitudes $\mathbf{u}_0^{(1,2,3)}$ satisfy the corresponding conditions of transverse and, respectively, longitudinal waves. For instance, in the representation $\mathbf{u}_0 = (\mathbf{v}_0, w_0)$ we have $\mathbf{k}_0\mathbf{v}_0^{(1,2)} \pm \kappa_0 w_0^{(1,2)} = 0$ (including $w_0^{(1,2)} = 0$ for the s-waves) and $\kappa_0 \mathbf{v}_0^{(3)}\mathbf{k}_0/k_0 + k_0 w_0^{(3)} = 0$. In addition, the wave given by Eq. (33) must satisfy the conditions at the surface. For instance, for a fixed surface we have $\mathbf{u}_0|_{z=0} = 0$, while for a free surface, we impose the condition $\sigma_{iz} = 0$, where σ_{ij} is the stress tensor ($i = x, y, z$). All these conditions fix the amplitudes $\mathbf{u}_0^{(1,2,3)}$, up to the incidence angle and the amplitude of the incident wave, in terms of the reflection coefficients and reflection angles, ultimately in terms of the wave velocities $v_{t,l}$ [41]. For an incident s-wave we have only a reflected s-wave ($\mathbf{u}_0^{(3)} = 0$), while for an incident p-wave we have both p- and longitudinal waves. A similar situation occurs for an incident longitudinal wave, with κ_0 and κ_0' interchanged in Eq. (33). The displacement $\delta\mathbf{u}_0$ given by Eq. (5) implies $\mathbf{u}_0$ for $z = 0$, so that we may represent this localized contribution of the $\mathbf{u}_0$-wave as

$$\mathbf{u}_0|_{z=0} = (\mathbf{v}_0, w_0) e^{-i\omega t + i\mathbf{k}_0\mathbf{r}}, \tag{34}$$

where $\mathbf{v}_0$, w_0 include contributions corresponding to various polarizations.

First, we are interested in solving Eq. (7) for the scattered waves, with the force $\bar{\mathbf{f}}/v_t^2$ generated by the free waves $\mathbf{u}_0$, as given by Eq. (9). We consider a Fourier component of the form

$$h(\mathbf{r}) = h e^{i\mathbf{q}\mathbf{r}} \tag{35}$$

for the roughness function, where h is an amplitude (depending on $\mathbf{q}$) and $\mathbf{q}$ denotes a characteristic wavevector (in final results the contribution $\mathbf{q} \to -\mathbf{q}$ must be included). The localized displacement $\delta\mathbf{u}_0$ given by Eq. (5) can be written as

$$\delta\mathbf{u}_0 = h(\mathbf{v}_0, w_0)e^{-i\omega t+i\mathbf{kr}}\delta(z), \tag{36}$$

where $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$. Making use of this displacement $\delta\mathbf{u}_0$, the force $\bar{\mathbf{f}}/v_t^2$ given by Eq. (9) can be computed straightforwardly. Leaving aside the exponential factor $e^{-i\omega t+i\mathbf{kr}}$, it is given by

$$\begin{aligned} \frac{\bar{\mathbf{f}}_{(x,y)}}{v_t^2} &= -h\big[\bar{\kappa}^2\mathbf{v}_0\delta(z) + \mathbf{v}_0\delta''(z) - \bar{m}\mathbf{k}(\mathbf{k}\mathbf{v}_0)\delta(z) + i\bar{m}\mathbf{k}w_0\delta'(z)\big], \\ \frac{\bar{f}_z}{v_t^2} &= -h\big[\bar{\kappa}^2 w_0\delta(z) + w_0\delta''(z) + i\bar{m}\mathbf{k}\mathbf{v}_0\delta'(z) + \bar{m}w_0\delta''(z)\big], \end{aligned} \tag{37}$$

where

$$\bar{\kappa} = \sqrt{\omega^2/\bar{v}_t^2 - k^2} \tag{38}$$

and

$$\kappa = \sqrt{\omega^2/v_t^2 - k^2} = \sqrt{\kappa_0^2 - 2\mathbf{k}_0\mathbf{q} - q^2}. \tag{39}$$

We add the contributions arising from this force (via the Green function of Eq. (14)) to the *rhs* of Eqs. (18) and (19) and solve these equations by the procedure described in the previous section. For instance, Eq. (18) becomes

$$\begin{aligned} v_2 = &-\frac{i}{2\kappa}\frac{\partial v_2}{\partial z}\bigg|_{z=0} e^{-i\kappa z} - \frac{1}{2}v_2|_{z=0}e^{-i\kappa z} \\ &- \frac{ih}{2\kappa}\big(\bar{\kappa}^2 - \kappa^2\big)v_{02}e^{-i\kappa z} + hv_{02}\delta(z). \end{aligned} \tag{40}$$

The displacement v_2 given above includes the localized wave

$$v_{2l} = hv_{02}\delta(z)e^{-i\omega t+i\mathbf{kr}}, \tag{41}$$

which is a scattered wave propagating only on the surface (two-dimensional wave). The remaining contribution to Eq. (40) (terms without $\delta(z)$) represents scattered waves reflected back in the body. We denote this contribution by v_{2r}. Taking the second derivative with respect to z in Eq. (40) and using the self-consistency condition imposed by this equation on the displacement on the surface, we get immediately the solution

$$v_{2r} = -\frac{ih}{4\kappa}\big(\bar{\kappa}^2 - \kappa^2\big)v_{02}e^{-i\omega t+i\mathbf{kr}-i\kappa z}. \tag{42}$$

This is an s-wave, scattered back in the body by the surface roughness. We can see that it is the distinct elastic parameters of the surface roughness that ensure this scattering (through $\bar{\kappa}^2 - \kappa^2 = -\omega^2\eta_t/v_t^2 \neq 0$). The occurrence of the wavevector $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$ in Eq. (42) is indicative of the selective reflection phenomenon, associated with corrugated surfaces, and in general, of directional effects.

In likewise manner we get the equations for v_1 and w with the force terms given by Eq. (37). We get the amplitudes for localized waves

$$v_{1l} = hv_{01}\delta(z), \qquad w_l = h\frac{1+\bar{m}}{1+m}w_0\delta(z). \tag{43}$$

Equations (21) and (24) remain the same, but the constant C given by Eq. (25) (entering the relationship (24)) becomes now

$$C=-\frac{1}{2}\left(\frac{\partial v_1}{\partial z}-ikw\right)\Big|_{z=0}+\frac{1}{2}\left(i\kappa v_1-\frac{k}{\kappa}\frac{\partial w}{\partial z}\right)\Big|_{z=0} - \frac{h}{2\kappa}\left(\bar{\kappa}^2-\kappa^2\right)(\kappa v_{01}+kw_0). \tag{44}$$

Following the same procedure as described in the previous section we get the scattered waves

$$v_{1r}=-ih\frac{v_t^2}{4\omega^2}\left(\bar{\kappa}^2-\kappa^2\right)(\kappa v_{01}+kw_0)e^{-i\omega t+i\mathbf{kr}-i\kappa z} = \frac{i}{4}h\eta_t(\kappa v_{01}+kw_0)e^{-i\omega t+i\mathbf{kr}-i\kappa z} \tag{45}$$

and $w_r=kv_{1r}/\kappa$. We can see that this represent a p-wave ($\mathrm{div}(v_{1r},w_r)=0$, i.e., $kv_{1r}-\kappa w_r=0$).

We turn now to the second equation (10) with the force given by

$$\begin{aligned}\frac{\bar{\mathbf{f}}_{(x,y)}}{v_l^2}&=-h\big[(1-\bar{n})\bar{\kappa}^2\mathbf{v}_0\delta(z)+(1-\bar{n})\mathbf{v}_0\delta''(z)\\&\quad-\bar{n}\mathbf{k}(\mathbf{kv}_0)\delta(z)+i\bar{n}\mathbf{k}w_0\delta'(z)\big],\\ \frac{\bar{f}_z}{v_l^2}&=-h\big[(1-\bar{n})\bar{\kappa}^2w_0\delta(z)+(1-\bar{n})w_0\delta''(z)\\&\quad+i\bar{n}\mathbf{kv}_0\delta'(z)+\bar{n}w_0\delta''(z)\big].\end{aligned} \tag{46}$$

By using the procedure described in the previous section we get a localized displacement

$$\mathbf{v}_l=h\frac{1-\bar{n}}{1-n}\mathbf{v}_0\delta(z),\ w_l=hw_0\delta(z). \tag{47}$$

We can see, by comparing Eqs. (41), (43) and (47) that the inhomogeneous roughness affects the localized waves in different ways. For the scattered waves reflected back in the body, Eqs. (29) and (30) from the previous section remain unchanged, but the constant C' given by Eq. (31) (entering the relationship (30)) becomes

$$C'=-\frac{1}{2}\left(\frac{\partial v_1}{\partial z}+\frac{i\kappa'^2}{k}w\right)\Big|_{z=0}+\frac{1}{2}\left(i\kappa' v_1+\frac{\kappa'}{k}\frac{\partial w}{\partial z}\right)\Big|_{z=0} - \frac{h}{2k}\left(\bar{\kappa'}^2-\kappa'^2\right)\left(kv_{01}-\kappa' w_0\right). \tag{48}$$

We get straightforwardly the reflected waves

$$v_{1r}=-ih\frac{v_l^2k}{4\omega^2\kappa'}\left(\bar{\kappa'}^2-\kappa'^2\right)\left(kv_{01}-\kappa' w_0\right)e^{-i\omega t+i\mathbf{kr}-i\kappa' z} = \frac{i}{4}h\eta_l\left(kv_{01}-\kappa' w_0\right)e^{-i\omega t+i\mathbf{kr}-i\kappa' z} \tag{49}$$

and $w_r = -\kappa' v_{1r}/k$. We can see that this scattered wave is a longitudinal wave ($\mathrm{curl}(v_{1r}, w_r) = 0$, i.e., $-\kappa' v_{1r} = k w_r$).

According to Eqs. (42), (45) and (49), within the present model of surface roughness we get waves scattered back in the body only for a rough surface with elastic characteristics different from those of the body (inhomogeneous roughness, $\eta_{t,l} \neq 0$). For a homogeneous roughness, i.e., for $\eta_{t,l} = 0$, we get only scattered waves localized on the surface, given by

$$\mathbf{u}_l = \delta \mathbf{u}_0 = h(\mathbf{r})(\mathbf{v}_0, w_0) e^{-i\omega t + i\mathbf{k}_0 \mathbf{r}} \delta(z), \tag{50}$$

as expected.

5 Discussion

The localized waves have the general form of the incoming wave $e^{-i\omega t+i\mathbf{k}_0\mathbf{r}}$ modulated by the roughness function $h(\mathbf{r})$. If $\mathbf{q}$ is a characteristic wavevector of this roughness function and $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$, the velocity of the localized waves is given by $v_s = \omega/k = v_{t,l} k_0/k \sin\theta$, where θ is the incidence angle of the incoming (transverse or longitudinal) wave. The directional effects are clearly seen from the presence of $k == \sqrt{k_0^2 + 2\mathbf{k}_0\mathbf{q} + q^2}$ in the denominator of this relation. It is worth noting that for $\mathbf{q} = \pm \mathbf{k}_0$, i.e. for a surface distribution of defects modulated with the same wavelength as the original $\mathbf{u}_0$-wave, there appear scattered waves with half the wavelength of the original $\mathbf{u}_0$-waves (wavevector $2\mathbf{k}_0$) and the whole surface suffers a vibration (independent of the coordinate $\mathbf{r}$), a characteristic resonance phenomenon ($\mathbf{k} = 0$). The waves corresponding to the wavevector $2\mathbf{k}_0$ have a velocity $\omega/2k_0$, which is twice as small as the original velocity on the surface. This is indicative of the slowness phenomenon, associated with rough surfaces.

The $\mathbf{q} = \pm \mathbf{k}_0$ resonance phenomenon is exhibited also by the waves scattered back in the body. Another resonance phenomenon may appear for $\pm 2\mathbf{k}_0\mathbf{q} + q^2 = 0$, which is the well-known Laue-Bragg condition for the X-rays diffraction in crystalline bodies [29, 30, 45]. In this case, $k = k_0$, $\kappa = \kappa_0$ and $\kappa' = \kappa'_0$, and we can see that the scattered transverse (longitudinal) waves are generated only by the transverse (longitudinal) part in the original $\mathbf{u}_0$-waves, as expected, due to the presence of the factors $\kappa v_{01} + k w_0$ and $k v_{01} - \kappa' w_0$ in Eqs. (45) and, respectively, (49). For $\mathbf{k}_0$ and $\mathbf{q}$ antiparallel the scattered wave propagates in opposite direction with respect to the incident wave.

The results given above hold also for purely imaginary values of the wavevectors κ or κ', when the scattered waves become confined to the surface (surface waves), a situation which may occur especially for high values of the magnitude q of the characteristic wavevectors $\mathbf{q}$ ($q \gg k_0$). According to Eqs. (42), (45) and (49), the scattered waves are now damped ($\sim e^{|\kappa| z}$, $\sim e^{|\kappa'| z}$) and their amplitudes are proportional to the roughness function $h(\mathbf{r})$. It is worth noting that these surface waves are generated by the surface roughness.

As it is well known, the energy of the incident wave is transferred to the reflected waves. In the present case, it is transferred both to the reflected waves as well as to the scattered waves, including the waves localized on the surface and the waves scattered back in the body. According to Eqs. (42), (45) and (49) the energy density of the scattered waves reflected back in the body is proportional to $(h/\bar{\lambda})^2$, where $\bar{\lambda}$ is a characteristic "wavelength" of these waves (projection of the wavelength λ on the surface, or on the direction perpendicular to the surface, or combinations of these). It follows that the validity criterion for our perturbation-theoretical scheme is $h \ll \bar{\lambda}$. In the limit of small roughness ($h \to 0$), the energy of the scattering waves (their amplitude) is vanishing. It is worth estimating the energy of the waves localized on the surface. For simplicity, we consider a homogeneous roughness, with the localized waves given by

$$(\mathbf{v}_l, w_l) = h(\mathbf{v}_0, w_0)\delta(z)e^{-i\omega t+i\mathbf{kr}} \tag{51}$$

(according to Eq. (50)) and choose the wavevector $\mathbf{k}$ directed along the x-axis. The validity condition for these waves is obtained by assuming that the distribution of the surface defects extends over a distance of the order of $h_m = \max h(\mathbf{r})$ and use the representation $\delta(z) \simeq 1/h_m$ for the δ-function. Then, the perturbation calculations are valid for $\bar{h} \ll h_m$, where $\bar{h}$ is the average (mean value) of the function $h(\mathbf{r})$. This means that the surface distribution of defects has only a few spikes. As it is well known, the (elastic) energy density (per unit mass) can be expressed as

$$\mathcal{E}/\rho = v_t^2\left(u_{ij}^2 - u_{ii}^2\right) + \frac{1}{2}v_l^2 u_{ii}^2, \tag{52}$$

where $u_{ij} = (1/2)(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ is the strain tensor. In our case, we use for computing this strain tensor the displacement given by Eq. (51). The strain tensor includes factors proportional to $\delta(z)$ and $\delta'(z)$, and the energy density includes factors proportional to $\delta^2(z)$ and $\delta'^2(z)$. The leading contribution come from $\delta'^2(z)$-terms:

$$\mathcal{E}/\rho = \frac{h^2}{2}\left(v_t^2\mathbf{v}_0^2 + v_l^2 w_0^2\right)\delta'^2(z), \tag{53}$$

giving a surface energy (per unit mass)$\sim h_m\mathcal{E}/\rho$. Making use of the representation $\delta'^2(z) \simeq 1/h_m^4$, this surface energy is proportional to h^2/h_m^3, while the corresponding energy of the incident wave goes like h_m/λ^2; the ratio of the two quantities is of the order of $h^2\lambda^2/h_m^4$. We can see that this ratio may acquire large values, even for $h \ll h_m$ (perturbation criterion satisfied), for $\lambda \gg h_m$. Therefore, the surface waves may store an appreciable amount of energy, as a result of their localization. This phenomenon is related to the discontinuities experienced by the strain tensor along the direction perpendicular to the surface.

6 Particular Cases and Concluding Remarks

From Eqs. (42), (45) and (49) we can get the reflection coefficients, related to the energy, of the waves scattered back in the body. Their general characteristic is the directionality effects. The derivation of these coefficients is complicated in the general case, where we should fix the amplitudes of the original $\mathbf{u}_0$-waves according to the nature of these waves and the boundary conditions. Another complication arises from the fact that we should "renormalize" the amplitudes of the reflected original $\mathbf{u}_0$-waves such as to include (accommodate) the scattered waves in the boundary conditions (a procedure specific to theoretical-perturbation calculations). We limit ourselves here to give the reflection coefficients for a few particular cases.

First, one of the simplest case is an original s-wave, described by

$$\mathbf{u}_0 = 2(0, u_0, 0)\cos\kappa_0 z \cdot e^{-i\omega t+i\mathbf{k}_0\mathbf{r}}, \tag{54}$$

where $\mathbf{k}_0$ is directed along the x-axis. Making use of Eq. (52), the energy density (per unit mass) of the incident wave in Eq. (54) is $\mathcal{E}_0/\rho = \omega^2 u_0^2$. We must compute the projections $v_{01,2}$ of the amplitude of this wave on $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$ and $\mathbf{k}_\perp$. Introducing the angle α between $\mathbf{q}$

and $\mathbf{k}_0$, we get $v_{01} = 2u_0 q \sin\alpha/k$ and $v_{02} = 2u_0(k_0 + q\cos\alpha)/k$ (and, of course, $w_0 = 0$). We can see, from Eqs. (42), (45) and (49), that an incident s-wave produce both s- and p-scattered transverse waves as well as a scattered longitudinal wave, due to the surface inhomogeneities. Making use of these equations we compute easily the amplitudes of these waves and get the reflection coefficients

$$R_s = \eta_t \frac{h\omega^2}{4v_t^2 \kappa k}(k_0 + q\cos\alpha), \qquad R_p = \eta_t \frac{h\omega q}{4v_t k}\sin\alpha, \qquad R_l = \eta_l \frac{h\omega q}{4v_l k}\sin\alpha. \tag{55}$$

The energy density carried on by these waves is given by $\mathcal{E}_{s,p,l}/\mathcal{E}_0 = R_{s,p,l}^2$. We stress upon the complicated direction-dependence (angle α) of these reflection coefficients, included both in κ and k. The formulae given by Eq. (55) become more simple for normal incidence ($\mathbf{k}_0 = 0$).

For normal incidence there is another simple case concerning longitudinal waves described by

$$\mathbf{u}_0 = 2(0, 0, u_0)\cos\kappa_0' z \cdot e^{-i\omega t}, \tag{56}$$

where $\kappa_0' = \omega/v_l$. The energy density per unit mass of this incident wave is $E_0/\rho = \omega^2 u_0^2$. According to Eqs. (42), (45) and (49), the scattered waves in this case are a p-wave and a longitudinal wave. Their reflection coefficients are much more simple now,

$$R_p = \eta_t \frac{h\omega q}{4v_t\kappa}, \qquad R_l = \eta_l \frac{h\omega\kappa'}{4v_l q}. \tag{57}$$

The squares of these coefficients give the fraction of energy carried on by these waves.

It is worth stressing that all the above formulae are valid only for $\kappa, k, q \neq 0$ (non-vanishing denominators).

We can see from the above particular cases, as well as from the general equations (42), (45) and (49), that the total amount of energy carried on diffusively by the waves scattered by the surface roughness implies sums of the form $\sum_{\mathbf{q}} |h(\mathbf{q})|^2 f(\mathbf{q})$, where $h(\mathbf{q})$ is the Fourier transform of the roughness function $h(\mathbf{r})$ and $f(\mathbf{q})$ are specific functions corresponding to the waves' nature (factors implying k, κ, κ', etc). Qualitatively, in order to maximize this energy, it is necessary, apart from particular cases of gratings (one, or a few wavevectors $\mathbf{q}$), to include as many Fourier components as possible, i.e. the surface should be as rough as possible in order to have a good attenuation, a reasonably expected result.

In conclusion, we may say that we have introduced a model of inhomogeneous surface distribution of defects for a semi-infinite isotropic elastic body and solved the wave equations for the elastic waves scattered by this surface roughness in the first-order approximation with respect to the magnitude of the defects distribution. The scattered waves are of two kinds: waves localized (and propagating only) on the surface, given by Eqs. (43) and (47), and scattered waves reflected back in the body by the surface inhomogeneities, both transverse, as given by Eqs. (42) and (45), and longitudinal, as given by Eq. (49). The latter may become confined to the surface (damped surface waves) for an enhanced roughness (large wavevectors q). The reflected waves are absent for a homogeneous roughness ($\eta_{t,l} = 0$), where only the localized waves survive.

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