Optik - International Journal for Light and Electron Optics 220 (2020) 165009

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Original research article

Penetration depth of an electric field in a semi-infinite classical plasma

M. Apostol

Department of Theoretical Physics, Institute for Physics and Nuclear Engineering, Institute of Atomic Physics, Magurele-Bucharest MG-6, POBox MG-35, Romania

ARTICLE INFO

Keywords:
Landau damping
Semi-infinite plasma
Electric field
Penetration depth

ABSTRACT

It is shown that the penetration of an oscillating electric field in a semi-infinite classical plasma obeys the standard exponential attenuation law e^{-x/λ_e} (besides oscillations), where x is the distance from the wall and λ_e is the extinction length (penetration depth, attenuation length). This solves the problem of various approximations and semi-empirical models of the surface sheath. The penetration depth λ_e is computed here explicitly; it is shown that it is of the order $\lambda_e \simeq [|\varepsilon|/(1-\varepsilon)]^{1/3} v_{\rm th}/\omega$, where ε is the dielectric function, ω is the frequency of the field and $v_{\rm th} = \sqrt{T/m}$ is the thermal velocity (T being the temperature and m the particle (electron) mass). The result is obtained by including explicitly the contribution of the surface term in the (linearized) Boltzmann (Vlasov) equation for the semi-infinite plasma. A key point in solving the problem is the observation that there exists a uniform (oscillating in time) component of the response electric field, besides the spatially decaying and oscillating component.

1. Introduction

The penetration depth of a uniform oscillating electric field in a semi-infinite classical plasma is a basic problem in the physics of plasmas. The importance of this problem resides, on one hand, in the difficulties of estimating exactly the magnitude of the penetration depth and, on the other hand, in using the radiofrequency fields for heating plasmas and other applications [1,2]. The problem was initiated by Landau in his classical paper Ref. [3], where an approximate solution was given. The difficulties are related to the manipulation of the Landau damping and, especially, by the limitation of the semi-infinite plasma to a half-space. These difficulties led to various semi-empirical models of the surface sheath, supported by physical arguments and combined with numerical simulations. This situation is overcome in the present paper by introducing explicitly the surface boundary condition (surface term) in the (linearized) Boltzmann (Vlasov) equation and by using a consistent treatment of the Landau damping. It is realized that the external electric field generates in plasma two types of (response) internal fields. One field is spatially uniform (oscillating in time). It originates in the uniform (oscillating) displacement of the mobile charges, which generates an electric current under the action of the uniform external field. Since the plasma is unmagnetized, this current is compensated by the time variation of this uniform oscillating internal field. The other component, spatially varying, is generated by the spatial variations induced by the presence of the surface; it is associated with a charge density. In these conditions we solve (exactly) the (linearized) Boltzmann (Vlasov) equation and get the response of the plasma, which is a non-uniform, oscillating (both in space and time) electric field, attenuated over a characteristic length. This is the extinction length (penetration depth), which is calculated explicitly herein. The attenuation law is the standard attenuation exponential. Moreover, the treatment presented here allows us to estimate the variation of the temperature, localized,

E-mail address: apoma@theory.nipne.ro.

https://doi.org/10.1016/j.ijleo.2020.165009
Received 10 February 2020; Received in revised form 21 May 2020; Accepted 27 May 2020

mainly, at the surface. In addition, the surface parameter (boundary condition) may be viewed either as an empirical parameter or a model parameter, which opens the possibility of discussing various models for the surface (*e.g.*, contacts of various nature, like metallic, dielectric contacts, etc., or coated surfaces, or other types of experimentally conditioned surfaces).

The plasma relevant for the problem discussed herein is the classical collisionless plasma. [4–8] For the benefit of the general reader we recall that a plasma is a set of positive and negative charges (ions and electrons), equal in number, in a background of neutrals (atoms). The typical example is an ionized gas. If the charges are very dilute, their mean separation distance is very large and their Coulomb interaction is much smaller than the electron temperature (which may be, for instance, 10^4 K). This weakly-coupled plasma is a classical plasma. At the same time, the screening Debye length in such a plasma is much longer than the mean separation distance between charges, and the charge collisions are infrequent. Moreover, for a high dilution the charge-neutrals collisions are infrequent (and mainly elastic). This is a collisionless plasma. Usually, the frequency of an external field is much higher than the collision frequency.

It is well known that there exists a mechanism of energy transfer between collective modes and individual particles in collisionless classical plasmas, governed by the Landau damping [3]. The origin of this mechanism is the causal character of the response of the plasmas to external excitations. The Landau damping received much interest, due to its application to heating plasmas by radio-frequency electric fields [9–13]. Also, the Landau damping enjoyed controversies along the years, as a consequence of the counter-intuitive character of an energy loss in collisioness plasmas [14–26]. Apart from theoretical and experimental investigations, numerical-analysis [9–12] and mathematical studies are devoted to the phenomenon, [27–30] which show both the complexity of the concept and difficulties related to its understanding at the fundamental level.

In semi-infinite plasmas the Landau damping appears as attenuated spatial oscillations (vibrations). The phenomenon, with its characteristic penetration depth, has a particular relevance for surface effects. Specifically, the Landau damping in semi-infinite plasmas implies an attenuated electric field, with spatial and temporal oscillations, besides a uniform component, as a response to a uniform oscillating external electric field, perpendicularly applied to the plasma surface. The calculation of the exact form of this response raises a few problems, due, on one hand, to the difficulties related to the Landau damping, and, on the other hand, as a consequence of the presence of the surface. The surface introduces an assymmetry in the problem, related to the limitation of the plasma to a half-space. The latter point is particularly interesting, because the response is discontinuous at the surface, and the usual Fourier or Laplace techniques may not include properly this discontinuity. In addition, the surface boundary conditions may bring further complications. These problems have been analyzed recently in Ref. [31]. In various approximations (see, for instance, Refs. [31–34]), including the original calculation in Ref. [3], the asymptotically attenuated field is presented as being proportional to $x^{2/3}e^{-\frac{3}{4}(\omega x/v_{\rm th})^{2/3}}$, where x is the distance from the wall, ω is the frequency of the field and $v_{\rm th}=\sqrt{T/m}$ is the thermal velocity, T being the temperature and m being the particle mass (electrons); sometimes, an exponential attenuation $\sim e^{-\omega_0 x/v_{\rm th}}$ is included, where ω_0 is the plasma frequency ($\omega_0=(4\pi {\rm n}{\rm q}^2/m)^{1/2}$, where n is the concentration of mobile charges, q and m are the particle charge and mass, respectively). A non-linear x-dependence ($\sim x^{2/3}$) is related to model assumptions made upon the surface and an asymptotic treatment of the Landau damping for the Boltzmann kinetic equation (see, for instance, Ref. [31]). We show here that, when the surface condition (surface term) is included explicitly, the attenuated field obeys the standard exponential attenuation law e^{-x/λ_e} (apart from factors oscillating in space), where λ_e is an extinction length (penetration depth, attenuation length) which is computed here explicitly; up to immaterial numerical factors, it is of the order $\lambda_e\simeq[|\varepsilon|/(1-\varepsilon)]^{1/3}v_{\rm th}/\omega$, where ε is the dielectric function (the dielectric function ε is the ratio of the external electric field to the total electric field).

We note that experimental studies indicate a damping very close to a purely exponential law [22,23]. The penetration depth estimated from the decrease of the extinction laws by a factor $1/e$ is of the same order of magnitude ($v_{\rm th}/\omega$) both for the 2/3-law and for the purely exponential law. In this context the relevance of our calculations presented here resides mainly in their theoretical (methodological) character. In addition, most of the numerical simulations deal with the plasma discharge regime, which is different from the stationary regime treated here; however, we include here an estimation of the transient regime with a small attenuation parameter.

2. Semi-infinite plasma

We consider a classical plasma at thermal equilibrium, consisting of mobile charges q with mass m and concentration n (electrons), moving in a rigid neutralizing background. We confine this plasma to a semi-infinite space (half-space) $x>0$, bounded by a plane surface $x=0$. The plasma is subject to a uniform oscillating external electric field $E_0e^{-i\omega t}$, where E_0 is directed along the x-direction (capacitively coupled plasma). The plasma is governed by the Maxwell distribution. The mean thermal velocity is sufficiently small to consider plasma unmagnetized. Since the field is directed along the x-direction we may integrate over the transverse velocities and use $F=n(\beta m/2\pi)^{1/2}e^{-\frac{1}{2}\beta {\rm m}{\rm v}^2}$ for the Maxwell distribution, where v is the velocity along the x-direction and $\beta=1/T$ is the reciprocal temperature. In the collisionless regime (where the collision frequency is much smaller than the frequency of the external perturbations [35]) the change $f(x,v)e^{-i\omega t}$ in the Maxwell distribution is governed by the (linearized) Boltzmann (Vlasov) equation [3]

$$-i\omega f+v\frac{\partial f}{\partial x}+\frac{q}{m}(E_0+E+E_1)\frac{\partial F}{\partial v}=0, \tag{1}$$

where E is a uniform internal electric field and E_1 is another internal electric field, which may vary in space; these fields are generated by internal charges and currents. Since the plasma is in the weak-coupling regime ($q^2n^{1/3}/T\ll 1$) and the external perturbation varies slowly in space and time, we may limit ourselves to the Boltzmann (Vlasov) equation where the external-force term is governed by the unperturbed Maxwell distribution. The uniform reaction field E occurs in an infinite space too, *i.e.* a space bounded by surfaces at infinity (it is a bulk reaction field), while the non-uniform field E_1 is due to the presence of the surface (it is a surface field). Also, we note that we seek a solution for the stationary regime generated by a monochromatic uniform external field, which is the most interesting experimental situation (in contrast with a transient regime). We seek the solution of Eq. (1) as $f(x, v) = f_0(v) + f_1(x, v)$, where

$$-i\omega f_0 + \frac{q}{m}(E_0 + E)\frac{\partial F}{\partial v} = 0 \tag{2}$$

and

$$-i\omega f_1 + v\frac{\partial f_1}{\partial x} + \frac{q}{m}E_1\frac{\partial F}{\partial v} = 0. \tag{3}$$

The uniform part f_0 of the solution does not generate charge density in plasma; it generates a current density. This can be seen immediately from Eq. (2), which implies $\int dv \cdot f_0 = 0$. Therefore f_0 should satisfy the equation

$$i\omega E = 4\pi q\int \mathrm{dv}\cdot\mathrm{vf}_0; \tag{4}$$

it is easy to see that this equation arises from the general equation $\partial\mathbf{E}/\partial t + 4\pi\mathbf{j} = 0$, where $\mathbf{j}$ is the current density [3]; this equation ensures the vanishing of the (internal) magnetic field, as expected. We emphasize that the uniform external field produces a uniform current, which, by this equation, leads to a uniform electric field E. In addition, the presence of the surface introduce spatial variations, and generates a non-uniform electric field E_1. The non-uniform part f_1 of the solution generates a charge density in plasma (and a current density; the continuity equation can be verified immediately by taking the integral with respect to v in Eq. (3)); consequently, f_1 satisfies the equation

$$\frac{\partial E_1}{\partial x} = 4\pi q\int \mathrm{dvf}_1. \tag{5}$$

The solution of Eqs. (2) and (4) is

$$f_0 = -\frac{\mathrm{iq}\omega E_0}{m(\omega^2 - \omega_0^2)}\frac{\partial F}{\partial v} \tag{6}$$

and

$$E = \frac{\omega_0^2}{\omega^2 - \omega_0^2}E_0, \quad E_t = E_0 + E = \frac{\omega^2}{\omega^2 - \omega_0^2}E_0, \tag{7}$$

where $\omega_0 = (4\pi \mathrm{nq}^2/m)^{1/2}$ is the plasma frequency; we recognize here the response of a boundless plasma to an electric field (restricted to $x > 0$), where $\varepsilon = 1 - \omega_0^2/\omega^2$ is the dielectric function and E_t is the total field in plasma ($P = \chi E_t$ is the polarization and $\chi = (\varepsilon - 1)/4\pi = -nq^2/m\omega^2$, $\chi = -\omega_0^2/4\pi\omega^2$ is the electric susceptibility).

In realistic conditions ($\omega \ll \omega_0$) the total uniform field E_t given by Eq. (7) is very small. It remains the non-uniform field E_1, which controls the penetration length. This field is generated by the asymmetry introduced in the problem by the presence of the surface. Indeed, the charges present in the region $x > 0$ (plasma) may screen, to some extent, an external perturbation, while such a screening is absent in the region $x < 0$ where charges (plasma) are absent.

In order to deal conveniently with the boundary condition at the surface we multiply Eq. (3) by the step function $\theta(x)$ ($\theta(x) = 1$ for $x > 0$, $\theta(x) = 0$ for $x < 0$) and restrict ourselves to the solution for $x > 0$; Eq. (3) becomes

$$-i\omega f_1 + v\frac{\partial f_1}{\partial x} + \frac{q}{m}E_1\frac{\partial F}{\partial v} = \mathrm{vf}_s\delta(x), \tag{8}$$

where $f_s = f_s(v) = f_1(x = 0, v)$; we can check directly this surface term by integrating Eq. (8) along a small distance perpendicular to the surface $x = 0$. The passing from Eq. (3) to Eq. (8) is done by $v\frac{\partial f_1}{\partial x}\theta(x) = v\frac{\partial(f_1\theta)}{\partial x} - \mathrm{vf}_s\delta(x)$. In the solution of Eq. (8) we restrict ourselves to $x > 0$. Similarly, Eq. (5) becomes

$$\frac{\partial E_1}{\partial x} - E_{1s}\delta(x) = 4\pi q\int \mathrm{dvf}_1, \tag{9}$$

where $E_{1s} = E_1(x = 0)$. The inclusion of the surface δ-terms in Eqs. (8) and (9) is the main point of this paper. (If the transverse motion would be allowed on the plane surface, these terms would lead to surface plasmons).

In Eqs. (8) and (9) we use the Fourier transforms with respect to the coordinate x (and restrict ourselves to $x > 0$); we get

$$f_1(k, v) = \frac{i}{\omega - \mathrm{vk} + i\gamma}\left[\mathrm{vf}_s(v) - \frac{q}{m}\frac{\partial F}{\partial v}E_1(k)\right] \tag{10}$$

and

$$E_1(k) = \frac{4\pi q \int \mathrm{dv}\frac{\mathrm{v}f_s(v)}{\omega - \mathrm{vk} + i0^+} - \mathrm{i}E_{1s}}{k + \frac{4\pi q^2}{m}\int \mathrm{dv}\frac{\partial F / \partial v}{\omega - \mathrm{vk} + i0^+}}, \tag{11}$$

where $\gamma \to 0^+$; in Eq. (11), derived from Eq. (9), we used Eq. (10). It is worth noting that in the Fourier transforms we replace ω by $\omega + i\gamma$, $\gamma \to 0^+$, in order to ensure the causal behaviour (*i.e.* zero response for time $t < 0$, which requires a pole in the lower ω-half-plane). This procedure gives a pole in the upper k-half-plane (this is the connection between the Landau damping and the spatial decay). At the same time, in the integrals with respect to v we may take the limit $\gamma \to 0^+$, which avoids the singularity $\omega = vk$; the insertion of the parameter γ produces the Landau damping. We denote the denominator in Eq. (11) by A; it can be estimated as

$$\begin{aligned} A= \; & k + \frac{4\pi q^2}{m}\int \mathrm{dv}\frac{\partial F/\partial v}{\omega - \mathrm{vk} + i0^+} = k + \frac{4\pi q^2}{m}P\int \mathrm{dv}\frac{\partial F/\partial v}{\omega - \mathrm{vk}} - i\frac{4\pi^2 q^2}{\mathrm{mk}}\frac{\partial F}{\partial v}|_{v=\omega/k} \\ & \simeq k(1 - \omega_0^2/\omega^2) - i\frac{4\pi^2 q^2}{\mathrm{mk}}\frac{\partial F}{\partial v}|_{v=\omega/k}; \end{aligned} \tag{12}$$

we can see that the zeros of A give the damped collective eigenmodes $\omega = \pm\omega_0 - i\Gamma$ (plasma frequency), where Γ is given by the imaginary part in Eq. (12)

($\Gamma \simeq -2\pi^2 q^2\omega_0/\mathrm{mk}^2)(\partial F/\partial v)|_{v=\omega_0/k}$); this is the Landau damping.

3. Penetrating electric field

In order to estimate the field $E_1(x)$ we need the zeros of A with respect to k in Eq. (11). It is convenient to introduce the variable $\xi = \sqrt{\beta m/2}\,\omega/k$. We can see easily that the zeros of A (the roots of the equation $A = 0$) are given by $\xi^2|\xi|e^{-\xi^2} = -i\alpha$, where $\alpha = |\varepsilon|/2\sqrt{\pi}(1-\varepsilon)$; we consider the case $\omega < \omega_0$ ($\varepsilon < 0$; the rather unrealistic case $\omega > \omega_0$ can be treated similarly, by using the equation $\xi^2|\xi|e^{-\xi^2} = i\alpha$). For small values of α (*e.g.*, $\omega \ll \omega_0$) we get two roots of the equation $A = 0$, given by $k_{1,2} \simeq \pm\frac{1}{2\alpha^{1/3}}\sqrt{\beta m}\,\omega(1+i)$; only k_1 (placed in the upper half-plane) contributes to the k-integration for $x > 0$. In estimating the integral in the numerator of Eq. (11) we may leave aside the contribution of the principal value, in comparison with the contribution of the δ-function. For k near k_1 the field $E_1(k)$ has the form

$$\begin{aligned} E_1(k) &\simeq \frac{B}{k - k_1 + \frac{i}{5}(k-k_1)^*}, \\ B &= \frac{8\sqrt{2}\pi q\alpha^{2/3}v_{\mathrm{th}}^2}{5\omega|\varepsilon|}(1+i)f_s(\alpha^{1/3}v_{\mathrm{th}}(1-i)) + \frac{2i}{5|\varepsilon|}E_{1s}. \end{aligned} \tag{13}$$

Making use of Eq. (13), the inverse Fourier transformation can be performed straightforwardly, leading to

$$E_1(x) = E_{1s}e^{(i-1)\omega x/2\alpha^{1/3}v_{\mathrm{th}}} \tag{14}$$

where

$$E_{1s} = -\frac{8\sqrt{2}\pi q\alpha^{2/3}v_{\mathrm{th}}^2}{(2+5|\varepsilon|)\omega}(1-i)f_s(\alpha^{1/3}v_{\mathrm{th}}(1-i)) \tag{15}$$

(or $E_{1s} = iB$). The final result is given by $E_1(t, x) = \mathrm{Re}\,[E_1(x)e^{-i\omega t}]$. We can see that an additional, non-uniform, electric field $E_1(x)$ appears as a result of the presence of the surface. According to Eq. (14), this field oscillates in space and is attenuated with an attenuation length (penetration depth, extinction length) $\lambda_e = 2^{2/3}(1/\pi)^{1/6}[|\varepsilon|/(1-\varepsilon)]^{1/3}v_{\mathrm{th}}/\omega$ (making use of the definition of α given above). It is worth noting that the penetration depth and the wavelength of the spatial oscillations have the same order of magnitude.

The result obtained here has a simple physical interpretation. In a semi-infinite plasma at equilibrium, slightly perturbed by a uniform oscillating external electric field, the perturbation generates plasma oscillations, damped by the thermal motion. The damping of the plasma oscillations (Landau damping) affects the response of the plasma (and the equilibrium distribution) over a distance from the surface of the order $\lambda_e \simeq 1/k_1 \simeq \alpha^{1/3}(v_{\mathrm{th}}/\omega)$, where the plasma contribution is included in $\alpha = |\varepsilon|/2\sqrt{\pi}(1-\varepsilon)$. The root k_1 of the equation $A = 0$ (where A is the denominator in Eq. (11)) is directly related to the Landau damping. A different approach of attenuation is discussed in Ref. [31]. According to that approach, we may assume that the perturbation $\sim e^{-i\omega t}$ is carried by charges with velocity v, *i.e.* it is $\sim e^{-i\omega x/v}$ at distance x from the surface. This local perturbation is convoluted with the Maxwell distribution $\sim e^{-v^2/2v_{\mathrm{th}}^2}$, leading to an attenuated perturbation of the form $\sim e^{-\frac{3}{4}(\omega x/v_{\mathrm{th}})^{2/3}}$. This mechanism of attenuation is termed the phase-mixing scale mechanism. In this approach, the plasma damping is not present, the main role being played by the external perturbation, modified by the thermal motion. The presence of (undamped) plasma oscillations is analyzed in Ref. [31], with the same result for the 2/3-attenuation law.

4. Discussion and conclusions

Making use of $E_1(k)$ given by Eqs. (11) and (13) we can calculate the change $f_1(x, v)$ in the distribution function (Eq. (10)). The poles contributions occurring in this calculation should be estimated separately for $v > 0$ and $v < 0$. In addition, we need to limit ourselves to slow spatial oscillations (associated with the pole at k_1); we get

$$f_1(x, v) \simeq -\frac{\mathrm{iq}}{m\omega}\mathrm{sgn}(v)\frac{\partial F}{\partial v}E_1(x) \tag{16}$$

(compare with Eqs. (6) and (7)). Within this approximation $f_s(v) = -(\mathrm{iq}/m\omega)\mathrm{sgn}(v)(\partial F/\partial v)E_{1s}$ and the polarization charge and current densities are zero (as expected in the limit of slow oscillations).

The amplitude of the field $E_1(x)$ depends on the parameter E_{1s}, which accounts for the boundary condition at $x = 0$. It is related to $f_s(v) = \frac{1}{2\pi}\int \mathrm{dkf_1}(k, v)$ by Eq. (15), where $f_1(k, v)$ is given by Eq. (10); it is easy to see that the integration of the first term in Eq. (10) gives f_s, while, making use of Eq. (13), the integration of the term which includes $E_1(k)$ with respect to k is zero.

Within the kinetic approach we may estimate the local change in temperature by $\delta T = 2T\overline{f}/\overline{F}$, where the overbar implies an integration over velocities (thermal average). We can see that only f_1 contributes to this integration. Making use of Eq. (16) we get $\delta T = 0$. However, if we keep the contribution of the fast spatial oscillations ($\omega \gg vk$), we get a surface change of temperature

$$\delta T \simeq \frac{2\mathrm{iT}}{n\omega}\int \mathrm{dv}\cdot \mathrm{vf}_s(v)\cdot\delta(x) + \cdots. \tag{17}$$

(*i.e.* $\mathrm{Re}(\delta \mathrm{T}e^{-i\omega t})$). The δ-type contribution in Eq. (17) corresponds to the surface sheath in plasma heating models [13,33].

Similar calculations of the penetration depth can be made for a plasma confined between two plane-parallel walls (or other geometries); the result depends on the boundary conditions incorporated in parameters like f_s[32]. The boundary parameter f_s may be viewed either as an empirical, or model parameter; we may take $f_0 + f_s = 0$ ($f(x = 0, v) = 0$) as a natural assumption, an equation which provides the parameter f_s. For $f_s = -f_0$ the field E_1 at the surface (maximum value) is of the order $E_1 \simeq E/|\varepsilon|$, where E is the internal uniform field given by Eq. (7). The surface change in temperature (Eq. (17)) can be written in this case as

$$\delta T = \frac{1}{2\pi}\left(\frac{E_0}{q/a^2}\right)T\cdot a\delta(x) \tag{18}$$

(for $\omega \ll \omega_0$), where a is the mean separation distance between the particles ($a = n^{-1/3}$); $q/a^2 \gg E_0$ is an electric field of the order of the microscopic (inter-particle) field.

It is worth discussing a numerical example. Recently, PIC (particle-in-cell) simulations have been performed for discharges in capacitively coupled collisionless plasmas [36]. Besides sheath phenomena, like field reversal and ion reflection, these simulations discuss also plasma waves and Landau damping. For an electron concentrations $n = 10^{10}\,\mathrm{cm}^{-3}$ the plasma frequency is $\omega_0 \simeq 5\times 10^9\,\mathrm{s}^{-1}$, such that the dielectric function ε acquires large negative values for frequencies of the order $\omega \simeq 10^7 s^{-1}$ ($\ll\omega_0$). In these conditions the extinction length given above is $\lambda_e \simeq v_{\mathrm{th}}/\omega$; for temperatures $T \simeq 10^4$ K the thermal velocity is $v_{\mathrm{th}} \simeq 6\times 10^7$ cm/s, such that the extinction length is of the order 6 cm. This result is of the same order of magnitude as the one obtained from numerical simulations.

Finally, we include here two related comments. First, we note that the kinetic equation has a slightly more general form

$$\frac{\partial\tilde{f}}{\partial t} + v\frac{\partial\tilde{f}}{\partial x} + \gamma\tilde{f} + \frac{q}{m}(E_0 e^{-i\omega t} + E + E_1)\frac{\partial F}{\partial v} = 0, \tag{19}$$

where $\gamma > 0$ is the collision-frequency parameter and the time-dependence of the external field is written explicitly. The solution can be written as $\tilde{f} = f_t + f$, where f_t satisfies the free equation

$$\frac{\partial f_t}{\partial t} + v\frac{\partial f_t}{\partial x} + \gamma f_t = 0 \tag{20}$$

and f is a particular solution of the inhomogeneous equation. In the inhomogeneous equation we may include γ as a small positive imaginary part of the frequency ω (*i.e.* we view ω as $\omega + i\gamma$), since $\gamma \ll \omega$ in the collisionless regime. The inhomogeneous equation becomes Eq. (1) and its treatment goes as above. However, the fields are determined by

$$i\omega E = 4\pi q\int \mathrm{dv}\cdot v\tilde{f}_0, \quad \frac{\partial E_1}{\partial x} = 4\pi q\int \mathrm{dv}\tilde{f}_1 \tag{21}$$

(Eqs. (4) and (5)). This enlarged framework allows the inclusion of the initial condition. Indeed, the solution of Eq. (20) is a superposition of damped plane waves $C(k, v)e^{-\mathrm{ivkt}+\mathrm{ikx}-\gamma t}$, where the amplitudes $C(k, v)$ are determined from the initial condition. We choose here the natural condition $\tilde{f}(t = 0, x, v) = 0$. It is easy to find

$$\tilde{f}_0 = -\frac{\mathrm{iq}\omega E_0 h(t)}{m[\omega^2 - \omega_0^2 h(t)]}\frac{\partial F}{\partial v}, \quad E = \frac{\omega_0^2 h(t)}{\omega^2 - \omega_0^2 h(t)}E_0 \tag{22}$$

and

$$\begin{aligned}\tilde{f}_1(k, v) &= \frac{i}{\omega - \mathrm{vk} + i\gamma}\left[\mathrm{vf}_s(v) - \frac{q}{m}\frac{\partial F}{\partial v}E_1(k)\right](1 - e^{i(\omega-\mathrm{vk})t-\gamma t}),\\ E_1(k) &= \frac{4\pi q\int \mathrm{dv}\frac{\mathrm{vf}_s(v)}{\omega - \mathrm{vk} + i0^+}(1 - e^{i(\omega-\mathrm{vk})t-\gamma t}) - \mathrm{iE}_{1s}}{k + \frac{4\pi q^2}{m}\int \mathrm{dv}\frac{\partial F/\partial v}{\omega - \mathrm{vk} + i0^+}(1 - e^{i(\omega-\mathrm{vk})t-\gamma t})},\end{aligned} \tag{23}$$

where $h(t) = 1 - e^{-\gamma t}$. The contribution of the time factor to these integrals ca be estimated straightforwardly. Both the damping parameter and the penetrating electric field are modified according to $\Gamma \longrightarrow \Gamma h(t)$ and $E_1 \longrightarrow E_1 h(t)$. Similarly, the distribution $\widetilde{f}_1$ acquires a factor $\simeq 1 - e^{i\omega t - \gamma t}$. We can see that all these factors ($h(t)$, $1 - e^{i\omega t - \gamma t}$) tend to unity in the limit of the long time. They govern the transient regime, until the perturbation becomes a stationary one. A rough estimate of the (electron) collision frequency is given by $\gamma \simeq nv_{\mathrm{th}}(q^2/T)^2$, which, by using the numerical data given above, leads to $\gamma \simeq 10^3\,\mathrm{s}^{-1}$ (it is increased for collisions with neutral atoms); according to this estimation, the duration of the transient regime is of the order 10^{-3} s.

The second comment concerns the penetration depth of an electric field in electron plasma of solids (metals). In this case the collision frequency is much higher (of the order $10^{12} - 10^{14}\,\mathrm{s}^{-1}$), such that the equilibrium is reached rapidly and the Boltzmann equation (1) can be applied for slowly oscillating fields. The only difference in comparison with the classical plasma is the much shorter duration of the transient regime and the occurrence of the Fermi velocity instead of the thermal velocity. The parameter γ in the Boltzmann equation is related, in fact, to the effect of the causality principle in estimating the singularities contribution. The calculations proceed as above, with the Fermi-Dirac distribution instead of the Maxwell distribution. We get a penetration depth of the order ω/v_F, where v_F is the Fermi velocity. We note that typical Fermi velocities are of the order $v_F \simeq 10^8$ cm/s, which is comparable with the order of magnitude thermal velocities of a classical plasma. The treatment becomes inapplicable when the oscillations of the external field interfere with the collisions, *i.e.* when the frequency ω is comparable with the collision frequency.

In conclusion, it is shown in this paper that the penetration of an oscillating electric field in a semi-infinite classical plasma obeys the standard exponential penetration law e^{-x/λ_e} (besides a uniform component), which may exhibit spatial oscillations, the extinction length λ_e (penetration depth, attenuation length) being of the order $\lambda_e \simeq [|\varepsilon|/(1-\varepsilon)]^{1/3} v_{\mathrm{th}}/\omega$; ($\varepsilon$ is the dielectric function, ω is the frequency of the field and $v_{\mathrm{th}} = \sqrt{T/m}$ is the thermal velocity). The surface term is included explicitly in these calculations.

Conflicts of interest

None declared.

Acknowledgements

The author is indebted to the members of the Laboratory of Theoretical Physics at Magurele-Bucharest for many fruitful discussions. This work has been supported by the Scientific Research Agency of the Romanian Government through Grants 04-ELI/2016 (Program 5/5.1/ELI-RO), PN 16 42 01 01/2016 and PN (ELI) 16 42 01 05/2016.

References

[1] M.A. Lieberman, A.J. Lichtenbergm, Principles of Plasma Discharges and Material Processing, Wiley, NY, 2005.
[2] D.M. Manos, D.L. Flamm, Plasma Etching, Academic Press, NY, 1989.
[3] L. Landau, On the vibrations of the electronic plasma, ZhETF 16 (1946) 574–586 (J. Phys. USSR 10 25–34 (1946))..
[4] F.F. Chen, Introduction to Plasma Physics, Plenum, NY, 1974.
[5] D.R. Nicholson, Introduction to Plasma Theory, Wiley, NY, 1983.
[6] P.A. Sturrock, Plasma Physics, Cambridge University Press, Cambridge, 1994.
[7] D.G. Swanson, Plasma Kinetic Theory, CRC Press, Taylor&Francis, NY, 2008.
[8] S. Ichimaru, Statistical Plasma Physics, vol. I: Basic Principles, vol. II: Condensed Plasmas, CRC Press, Taylor&Francis, NY, 2018.
[9] M.M. Turner, Collisionless electron heating in an inductively coupled discharge, Phys. Rev. Lett. 71 (1993) 1844–1847.
[10] M.M. Turner, Simulation of kinetic effects in inductive discharges, Plasma Sources Sci. Technol. 5 (1996) 159–165.
[11] R.H. Cohen, T.D. Rognlien, Electron kinetics in radio-frequency magnetic fields of inductive palsma sources, Plasma Sources Sci. Technol. 5 (1996) 442–452.
[12] G. Gozadinos, D. Vender, M.M. Turner, M.A. Liberman, Collisionless electron heating by capacitive radio-frequency plasma sheaths, Plasma Sources Sci. Technol. 10 (2001) 117–124.
[13] I.D. Kaganovich, V.I. Kolobov, L.D. Tsendin, Stochastic electron heating in bounded radio-frequency plasmas, Appl. Phys. Lett. 69 (1996) 3818–3820.
[14] D. Bohm, E.P. Gross, Theory of plasma oscillations. A. Origin of medium-like behaviour, Phys. Rev. 75 (1949) 1851–1864.
[15] D. Bohm, E.P. Gross, Theory of plasma oscillations. B. Excitation and damping of oscillations, Phys. Rev. 75 (1949) 1864–1876.
[16] R.W. Twiss, Propagation in electron-ion streams, Phys. Rev. 88 (1952) 1352–1407.
[17] N.G. van Kampen, On the theory of stationary waves in plasmas, Physica 21 (1955) 949–963.
[18] K.M. Case, Plasma oscillations, Ann. Phys. 7 (1959) 349–364.
[19] J. Dawson, On Landau damping, Phys. Fluids 4 (1961) 869–874.
[20] J. Ecker, J. Holling, Limits of collective description and their consequences for Landau damping, Phys. Fluids 6 (1963) 70–75.
[21] H. Weitzner, Plasma oscillations and Landau damping, Phys. Fluids 6 (1963) 1123–1127.
[22] J.H. Malmberg, C.B. Wharton, Collisionless damping of electrostatic plasma waves, Phys. Rev. Lett. 13 (1964) 184–186.
[23] J.H. Malmberg, C.B. Wharton, Dispersion of electron plasma waves, Phys. Rev. Lett. 17 (1966) 175–178.
[24] F.G.R. Crownfield Jr., Plasma oscillations and Landau damping, Phys. Fluids 20 (1977) 1483–1487.
[25] J.E. Allen, A.D.R. Phelps, Waves and microinstabilities in plasmas-linear effects, Rep. Prog. Phys. 40 (1977) 1305–1368.
[26] J. Weiland, A derivation of Landau damping from the Vlasov equation without contour integration, Eur. J. Phys. 2 (1981) 171–173.
[27] Y. Elskens, Irreversible behaviours in Vlasov equation and many-body Hamiltonian dynamics: Landau damping, chaos and granularity in the kinetic limit in Topics in Kinetic Theory, in: T. Passsot, C. Sulem, P.L. Sulem (Eds.), Fields Institute Communications, vol. 46, Amer. Math. Soc., Providence, 2015, pp. 89–108.
[28] C. Villani, Landau damping, Proc. Int. Congress of the Mathematicians, Hyderabad, India, 2010.
[29] R.J. Mason, Electric-field penetration into a plasma with a fractionally accommodating boundary, J. Math. Phys. 9 (1968) 868–874.
[30] D.F. Escande, F. Doveil, Y. Elskens, *N*-body description of Debye shielding and Landau damping, Plasma Phys. Control. Fusion 58 (2016) 014040.
[31] I.D. Kaganovich, O.V. Polomarov, C.E. Theodosiou, Revisiting anomalous RF field penetration into a warm plasma, IEEE Trans. Plasma Sci. 34 (2006) 696–717.
[32] V.M. Gokhfeld, M.I. Kaganov, G.Y. Lyubarskii, Anomalous penetration of longitudinal alternating electric field into a degenerate plasma with an arbitrary specularity parameter, ZhETF 92 (1987) 523–530 (Sov. Phys.-JETP 65 295-299 91987)).
[33] I.D. Kaganovich, Anomalous capacitive sheath with deep radio-frequency electric-field penetration, Phys. Rev. Lett. 89 (2002) 265006.
[34] S. Sharma, S.K. Mishra, P.K. Kaw, Observation of transient electric fields in particle-in-cell simulation of capacitively coupled discharges, Phys. Plasmas 21 (2014) 073511.
[35] L. Landau, E. Lifshitz, Course of Theoretical Physics, vol. 10, Physical Kinetics (E. Lifshitz and L. Pitaevskii L), Elsevier, Oxford, 1981.
[36] S. Sharma, M.M. Turner, Simulation study of wave phenomena from the sheath region in single frequency capacitively coupled plasma discharges; field reversals and ion reflection, Phys. Plasmas 20 (2013) 073507.