COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in **Physical Review B.** *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on ''Single-particle Green functions in exactly solvable models of Bose and Fermi liquids''

L. C. Cune* and M. Apostol
Department of Theoretical Physics, Institute of Atomic Physics, Magurele-Bucharest MG-6, P.O. Box MG-35, Romania
(Received 7 December 1998)

It is shown that the construction introduced recently by Setlur and Chang [Phys. Rev. B **57**, 15 144 (1998)] for generalized Fermi sea-displacement operators contains undefined elements, which may lead to divergencies, and, in fact, these operators are not bosonic operators, in contrast to what these authors claim. [S0163-1829(99)05335-7]

In a recent paper,[1] Setlur and Chang engaged themselves in delineating a general theory, aimed at solving exactly, or, at least, ''exceedingly plausibly,''[1] the interaction problem in both Fermi and Bose systems, irrespective of the interaction strength (or its sign), and in any spatial dimensions. Claiming that they draw largely from the work of Castro-Neto and Fradkin,[2] and that they generalize the concepts of Haldane,[3] Setlur and Chang attempt at reworking almost the whole body of the many-body theories in a special manner, based on the central concept of bosonization.[4] In particular, the single-particle propagator is claimed to be computed ''exactly for all wavelengths and energies,'' including ''short-wavelength behavior,''[1] an assertion which is not proven. The main point of their approach, that of constructing bosonic operators for Fermi systems, is shown here to be groundless.

For Bose systems, Setlur and Chang[1] introduce condensate-displacement operators which satisfy Bose commutation relations. Similarly, sea-displacement operators are postulated for Fermi systems, satisfying Bose commutation relations, and it is assumed that products of Fermi operators have the same functional dependence on these operators as for the case of the Bose systems. Making use of the analogy with the Bose systems, the following relations are proposed for Fermi systems:

$$c^{+}_{k+q/2}c_{k-q/2}=\left(\frac{N}{\langle N\rangle}\right)^{1/2}[\Lambda_k(q)a_k(-q)+a^{+}_k(q)\Lambda_k(-q)]$$
$$+T_1(k,q)\sum_{q_1} a^{+}_{k+q/2-q_1/2}(q_1)a_{k-q_1/2}(q_1-q)$$
$$-T_2(k,q)\sum_{q_1} a^{+}_{k-q/2+q_1/2}(q_1)a_{k+q_1/2}(q_1-q), \tag{1}$$

where c_k are Fermi operators (spin label is irrelevant here and, therefore, it is omitted), $a_k(q)$ are sea-displacement operators,

$$[a_k(q),a^{+}_{k'}(q')]=\delta_{kk'}\delta_{qq'},\quad [a_k(q),a_{k'}(q')]=0, \tag{2}$$

$$a_k(0)=0, \tag{3}$$

and the coefficients T_1, T_2, and Λ are given by

$$T_1(k,q)=\sqrt{1-\overline{n}_{k+q/2}}\sqrt{1-\overline{n}_{k-q/2}},$$
$$T_2(k,q)=\sqrt{\overline{n}_{k+q/2}\overline{n}_{k-q/2}}, \tag{4}$$
$$\Lambda_k(q)=\sqrt{\overline{n}_{k+q/2}(1-\overline{n}_{k-q/2})};$$

n_k in the above formulas represents the Fermi occupation number (occupation number operator), $\overline{n_k}$ is its expectation value on the ground state, N stands for the operator of the total number of particles, and $\langle N\rangle$ denotes the average number of particles. As one can see, Eqs. (1) and (2) provide a bosonic representation of the particle-density operators for Fermi systems. It is claimed that the occupation number itself has a bosonic representation in this theory, given by

$$n_k=n^{\beta}(k)\frac{N}{\langle N\rangle}+\sum_q a^{+}_{k-q/2}(q)a_{k-q/2}(q)$$
$$-\sum_q a^{+}_{k+q/2}(q)a_{k+q/2}(q), \tag{5}$$

where

$$n^{\beta}(k)=\frac{1}{\exp[\beta(\epsilon_k-\mu)]+1} \tag{6}$$

is the Fermi distribution.

The ''exact bosonic'' character of the Fermi sea-displacement operators would be embodied in the ansatz[1]

$$a_k(q)=\frac{1}{\sqrt{n_{k-q/2}}}c^{+}_{k-q/2}M(k,q)c_{k+q/2}, \tag{7}$$

where the operator $M(k,q)$ has to be determined in such a way as to ensure the Bose commutation relations required by Eq. (2). In the limit of the random-phase approximation (RPA), Eq. (7) is written as

$$a_k(q)=\frac{1}{\sqrt{n_{k-q/2}}}c^+_{k-q/2}\left(\frac{n^\beta(k-q/2)}{\langle N\rangle}\right)^{1/2}e^{i\theta(k,q)}c_{k+q/2}, \tag{8}$$

where the phase $\theta(k,q)$ is a functional of the number operator. However, nowhere do the authors give the phase $\theta(k,q)$, and as such the central point of their work remains unspecified.

Equations (7) and (8) raise several difficulties. First, we note that the Fermi number operator n_k has the idempotency property $n_k^2=n_k$, and, therefore, $\sqrt{n_{k-q/2}}$ in Eqs. (7) and (8) might be taken simply as being equal with $n_{k-q/2}$. If a formal proof would still be required, we note here that, indeed, a general operatorial function $f(A)$ may formally be represented by the associated Taylor series

$$f(A)=\sum_{m=0}\frac{f^{(m)}(a)}{m!}(A-a)^m, \tag{9}$$

for instance; and for $A=n_k$ and $a=1$ one obtains from Eq. (9)

$$f(n_k)=f(1)+(1-n_k)[f(0)-f(1)], \tag{10}$$

which, for the particular case $f=\sqrt{x}$, leads to $\sqrt{n_k}=n_k$. Moreover, the factor $1/\sqrt{n_{k-q/2}}$ in Eqs. (7) and (8) implies, as it is written, a division by zero, since the fermion occupation number may have a vanishing eigenvalue, and, consequently, this factor may lead to divergencies. Therefore, a certain sense must be attached to this writing, as, for instance, replacing $n_{k-q/2}$ in this factor by $n_{k-q/2}+\varepsilon I$, where I is the identity operator, and taking the limit $\varepsilon\to 0$ at the end of the calculations; doing so, one obtains

$$\frac{1}{\sqrt{n_{k-q/2}}}\to\frac{1}{n_{k-q/2}+\varepsilon I}=\frac{1}{\varepsilon}\left(I-\frac{1}{\varepsilon+1}n_{k-q/2}\right) \tag{11}$$

and

$$\frac{1}{\sqrt{n_{k-q/2}}}c^+_{k-q/2}\to\frac{1}{\varepsilon}\left(I-\frac{1}{\varepsilon+1}n_{k-q/2}\right)c^+_{k-q/2} = \frac{1}{\varepsilon+1}c^+_{k-q/2}\to c^+_{k-q/2}, \tag{12}$$

as expected, i.e., the factor $1/\sqrt{n_{k-q/2}}$ in Eqs. (7) and (8) would be ineffective in this case. Of course, the same result is obtained working with the function $f=1/\sqrt{x+\varepsilon}$ and using the expansion (9). Equation (8) then becomes

$$a_k(q)=c^+_{k-q/2}\left(\frac{n^\beta(k-q/2)}{\langle N\rangle}\right)^{1/2}e^{i\theta(k,q)}c_{k+q/2}, \tag{13}$$

and using the fact that $\theta(k,q)$ is a functional of the number operator, as assumed by the authors, one obtains straightforwardly

$$[a_k(q),a_k^+(q)]=\frac{n^\beta(k-q/2)}{\langle N\rangle}(n_{k-q/2}-n_{k+q/2}). \tag{14}$$

Obviously, this is not a bosonic commutation relation as required in Eq. (2). Of course, future publications, which would ''bend the rules''[1] in order to ''capture what one is looking for,''[1] may try to clarify such points. These authors might try to suggest that Eq. (14) would become bosonlike commutations relations when averaged over the Fermi sea; if so, we point out that this would be at variance with their own claim that the new Fermi-sea displacement operators are ''no longer restricted to be close to the Fermi surface.''[1] Moreover, the sea-displacement operators defined by Eq. (13) are only consistent with Eq. (5) for

$$\frac{n_k}{\langle N\rangle}\left[\sum_{k_1}n_{k_1}n^\beta(k_1)-Nn^\beta(k)\right]=0, \tag{15}$$

which requires $n^\beta(k_1)=n^\beta(k)=\text{const}$, i.e., the absence of the Fermi surface. We cannot refrain ourselves from emphasizing the apparent ''consistency'' of such a conclusion: indeed, if the fermions are described entirely and exactly in terms of bosons, there would be no Fermi surface at all, since, indeed, bosons have no Fermi surface.

The above considerations are not restricted to the RPA limit. Indeed, making use of Eq. (12), the general ansatz expressed in Eq. (7) becomes

$$a_k(q)=c^+_{k-q/2}M(k,q)c_{k+q/2}; \tag{16}$$

let $|v\rangle=|1,1,1,...,1,0_{k-q/2},1,...,1,0,0,0,...,0,1_{k'\neq k+q/2},0,0,0,...\rangle$ be a state vector in the space of the occupation numbers, i.e., an empty fermion state at $k-q/2$ below the Fermi surface, and an occupied fermion state at $k'\neq k+q/2$ above the Fermi surface; then one obtains $\langle v|[a_k(q),a_k^+(q)]|v\rangle=0$, which certainly is at variance with the bosonic character of the sea-displacement operators. This shows again that the $a_k(q)$ operators as defined by Eq. (7) are not bosonic operators.

In conclusion, one may say that the bosonic construction proposed by Setlur and Chang for Fermi sea-displacement operators,[1] besides containing undefined elements (which may lead to divergencies), does not represent bosonic operators, contrary to the claim made by these authors.

*Electronic address: cune@theory.nipne.ro

[1] G. S. Setlur and Y. C. Chang, Phys. Rev. B **57**, 15 144(1998).
[2] A. H. Castro-Neto and E. Fradkin, Phys. Rev. Lett. **72**, 1393 (1994); Phys. Rev. B **49**, 10 877 (1994); **51**, 4084 (1995).
[3] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981); Helv. Phys. Acta **65**, 152 (1992); *Perspectives in Many-Particle Physics*, Proceedings of the International School of Physics ''Enrico Fermi,'' Course CXXI, Varenna, 1992, edited by R. A. Broglia, J. R. Schreiffer, and P. F. Bortignon (North-Holland, New York, 1994).
[4] A relevant series of references are included in Ref. 1 above.