

# ON THE MOTION OF AN ELECTRIC CHARGE IN HIGH-INTENSITY ELECTROMAGNETIC RADIATION

CRISTINA MIRON<sup>1</sup>, MARIAN APOSTOL<sup>2,a</sup>

<sup>1</sup>Department of Physics,  
University of Bucharest,

Măgurele-Bucharest, Romania

<sup>2</sup>Department of Theoretical Physics,  
Institute of Atomic Physics,

Magurele-Bucharest MG-6, POBox MG-35, Romania

*E-mail*<sup>a</sup>: apoma@theory.nipne.ro

*Received June 23, 2017*

*Abstract.* The motion of an electrically charged particle in high-intensity electromagnetic radiation is revisited; the problem is relevant for the charge acceleration in ultra-intense laser radiation in vacuum. It is shown that for many practical purposes a charged particle accelerated by a traveling high-intensity electromagnetic plane wave can be treated as a (quasi-) classical relativistic particle. The effects of the damping force are estimated by a direct method; it is shown that in realistic cases these effects are small. A “stopping” point (“turning”, “reflection” point) is identified for charges propagating initially (launched) against the electromagnetic beam; this circumstance has been considered recently by numerical simulations. Particular aspects of a relativistic quantum-mechanical charge accelerated by a high-intensity electromagnetic wave are discussed, which indicate that a (quasi-) classical treatment is appropriate for very strong fields in many respects. Also, it is shown that charges in a standing electromagnetic wave behave as non-relativistic and quantum-mechanical charges, as in an optical lattice. The increase in the particle-antiparticle energy gap in high electromagnetic fields is highlighted, which points out difficulties in achieving pair production assisted (induced) by high-intensity electromagnetic radiation in vacuum.

*Key words:* charges accelerated in high-intensity electromagnetic fields; damping force; Volkov states; charges in standing electromagnetic fields; optical lattice; pair creation from vacuum.

*PACS:* 41.75.Jv; 52.38.-r; 41.85.Ar; 41.85.Ja.

## 1. INTRODUCTION

The recent advances in the technique of focused pulses of optical laser beams offer the opportunity of reaching very high intensities of electromagnetic fields [1–8]. New and interesting phenomena can be made available for experimental study in the laboratory by using this technique [9, 10]. Particle acceleration in high-intensity laser fields has been achieved up to energies of GeV’s order [11–13]; vacuum polarization, vacuum breakdown, photon-photon scattering, electron-positron pair creation or non-linear quantum electrodynamics effects are envisaged [14–18]; multiple

Romanian Journal of Physics **62**, 117 (2017)

Compton scattering, generation of higher harmonics, atomic and even nuclear effects are currently produced and investigated in gaseous or solid plasmas subject to high-intensity laser fields [19–22]. In all these phenomena, the basic element is the electric charge acceleration in electromagnetic fields, and, generally, the motion of an electric charge in strong electromagnetic fields. We discuss in this Note some particular points of the motion of an electric charge in high-intensity electromagnetic radiation in vacuum, with a theoretical character, which, usually, are insufficiently emphasized, in spite of their relevance.

## 2. EQUATIONS OF MOTION

Let us consider the motion of a charge  $q$  with mass  $m$  in an electromagnetic field generated by the vector potential  $A = A_z = A_0 \cos \frac{\omega}{c}(ct - x)$ , propagating along the  $x$ -direction;  $\omega$  denotes the radiation frequency and  $c$  is the speed of light in vacuum. With usual notations the equation of motions are

$$mc \frac{du^i}{ds} = \frac{q}{c} F^{ik} u_k, \quad (1)$$

where  $u^i$  is the four-velocity. The non-vanishing components of the field intensity (electric field  $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$ , magnetic field  $\mathbf{H} = \vec{\nabla} \times \mathbf{A}$ ) are  $F^{03} = F^{13} = -F^{31} = -F^{32} = -E$ , where  $E = E_z = \frac{\omega}{c} A_0 \sin \frac{\omega}{c}(ct - x)$  (and  $H_y = -E$ ). Consequently, the equations of motion are re-written as

$$mc \frac{du^0}{ds} = \frac{qE}{c} u^3, \quad mc \frac{du^1}{ds} = \frac{qE}{c} u^3, \quad (2)$$

$$mc \frac{du^3}{ds} = \frac{qE}{c} (u^0 - u^1)$$

( $u^2 = 0$ ), where  $s = k_i x^i = ct - x$  is the world-line length and  $k_i = (1, -1, 0, 0)$  is the four-wavevector; we get immediately  $u^0 - u^1 = \text{const} = 1$ , for a charge initially at rest at the origin (we recall  $u^0 = \gamma$ ,  $\mathbf{u} = \gamma\mathbf{v}/c$ , where  $\gamma = (1 - v^2/c^2)^{-1/2}$ ); it follows

$$u^1 = \frac{q^2 A^2}{2m^2 c^4}, \quad u^0 = \gamma = 1 + \frac{q^2 A^2}{2m^2 c^4}, \quad (3)$$

$$u^3 = -\frac{q}{mc^2} A$$

(we can check  $u_i u^i = 1$ ). The velocities are given by

$$v_x = c \frac{q^2 A^2 / 2m^2 c^4}{1 + q^2 A^2 / 2m^2 c^4}, \quad v_z = -c \frac{qA/mc^2}{1 + q^2 A^2 / 2m^2 c^4} \quad (4)$$

and the energy is

$$\mathcal{E} = mc^2 \gamma = mc^2 + \frac{q^2 A^2}{2mc^2}. \quad (5)$$

We can see that for high fields the charge reaches the ultrarelativistic limit along the direction of propagation of the radiation field ( $v_x \rightarrow c$ ,  $v_z \rightarrow 0$ ). From  $u^i = dx^i/ds$  we get the coordinates

$$\begin{aligned} x &= \frac{q^2 A_0^2 / 4m^2 c^3}{1 + q^2 A_0^2 / 4m^2 c^4} \left[ t + \frac{1}{2\omega} \sin 2\frac{\omega}{c}(ct - x) \right], \\ z &= -\frac{qA_0}{mc\omega} \sin \frac{\omega}{c}(ct - x). \end{aligned} \quad (6)$$

### 3. HAMILTON-JACOBI EQUATION

The same results can be obtained from the Hamilton-Jacobi equation

$$g^{ij} \left( \frac{\partial S}{\partial x^i} + \frac{q}{c} A_i \right) \left( \frac{\partial S}{\partial x^j} + \frac{q}{c} A_j \right) = m^2 c^2, \quad (7)$$

where the metrics is  $g^{ij} = (1, -1, -1, -1)$ ,  $\mathcal{E} = -\partial S/\partial t$  is the energy,  $\mathbf{P} = \mathbf{p} + \frac{q}{c}\mathbf{A} = \partial S/\partial \mathbf{R}$  is the generalized momentum and  $S$  is the mechanical action ( $\mathbf{R}$  is the position vector). We seek a solution of the form  $S = -f_i x^i + F(s)$ , where  $f^i$  is the four-momentum of a free particle,  $f_i f^i = m^2 c^2$  (the initial momentum, the interaction is introduced adiabatically). Since  $k_i k^i = 0$  and  $k^i A_i = 0$  (transversality condition, with zero scalar potential), we get

$$F' = -\frac{q}{c\alpha} f_i A^i + \frac{q^2}{2\alpha c^2} A_i A^i, \quad (8)$$

where  $\alpha = k_i f^i$ . The solution is well known [23]. Since  $\alpha = f^0 - f^1$  and  $f^i = (f^0, f^1, \boldsymbol{\kappa})$ ,  $f_i f^i = (f^0)^2 - (f^1)^2 - \kappa^2 = m^2 c^2$ , we get

$$f^0 = \frac{1}{2}\alpha + \frac{m^2 c^2 + \kappa^2}{2\alpha}, \quad f^1 = -\frac{1}{2}\alpha + \frac{m^2 c^2 + \kappa^2}{2\alpha} \quad (9)$$

and

$$S = -\frac{1}{2}\alpha(ct + x) - \frac{m^2 c^2 + \kappa^2}{2\alpha} s + \boldsymbol{\kappa} \mathbf{r} + \frac{q}{c\alpha} \int^s ds' \boldsymbol{\kappa} \mathbf{A} - \frac{q^2}{2c^2 \alpha} \int^s ds' A^2, \quad (10)$$

where  $\boldsymbol{\kappa}$  is the transverse momentum and  $\mathbf{r} = (y, z)$  is the transverse position vector. The derivatives of  $S$  with respect to the momentum  $\boldsymbol{\kappa}$  and the parameter  $\alpha$  are constants of motion; they may be set equal to zero. We get

$$\begin{aligned} y &= \frac{\kappa_y}{\alpha} s - \frac{q}{c\alpha} \int^s ds' A_y, \\ z &= \frac{\kappa_z}{\alpha} s - \frac{q}{c\alpha} \int^s ds' A_z, \end{aligned} \quad (11)$$

$$x = \frac{1}{2} \left( \frac{m^2 c^2 + \kappa^2}{\alpha^2} - 1 \right) s - \frac{q}{c\alpha^2} \int^s ds' \boldsymbol{\kappa} \mathbf{A} + \frac{q^2}{2c^2 \alpha^2} \int^s ds' A^2$$

and

$$\begin{aligned} p_y &= \kappa_y - \frac{q}{c} A_y, \\ p_z &= \kappa_z - \frac{q}{c} A_z, \end{aligned} \quad (12)$$

$$p_x = -\frac{1}{2}\alpha + \frac{m^2 c^2 + \kappa^2}{2\alpha} - \frac{q}{c\alpha} \boldsymbol{\kappa} \cdot \mathbf{A} + \frac{q^2}{2c^2\alpha} A^2;$$

the energy is given by

$$\mathcal{E} = c(\alpha + p_x); \quad (13)$$

we note the linear dependence on the momentum. For a charge initially at rest at  $x = y = z = 0$  at the initial moment of time  $t = 0$  ( $\boldsymbol{\kappa} = 0$ ,  $f^1 = 0$ ), and a vector potential  $A = A_z = A_0 \cos(\omega t - kx) = A_0 \cos \frac{\omega}{c}(ct - x) = A_0 \cos \frac{\omega}{c}s$  (linear polarization) we get  $\alpha = mc$  ( $\alpha^2 = m^2 c^2$ ) and

$$z = -\frac{qA_0}{mc^2} \lambda \sin(\omega t - kx), \quad y = 0,$$

$$x = \frac{q^2 A_0^2 / 4m^2 c^4}{1 + q^2 A_0^2 / 4m^2 c^4} \left[ ct + \frac{\lambda}{2} \sin 2(\omega t - kx) \right], \quad (14)$$

$$p_x = \frac{q^2 A_0^2}{2mc^3} \cos^2(\omega t - kx), \quad p_z = -\frac{qA_0}{c} \cos(\omega t - kx), \quad p_y = 0$$

and

$$\mathcal{E} = mc^2 + \frac{q^2 A_0^2}{2mc^2} \cos^2(\omega t - kx), \quad (15)$$

where  $\lambda = c/\omega$  is the radiation wavelength. For time-averaged quantities an effective mass can be defined by

$$\mathcal{E}^2/c^2 - p_x^2 = m^{*2} c^2, \quad m^{*2} = m^2 \left( 1 + \frac{q^2 A_0^2}{2m^2 c^4} \right), \quad (16)$$

by analogy with the free particle [24, 25]. Equations (14) and (15) coincide with equations (6), (3) and, respectively, (5) (we recall  $p^i = mcu^i$ ).

We can see that, apart from oscillations, the charge exhibits a drift motion along the direction of propagation of the wave, governed by the ratio of the field energy  $qA_0$  to the rest energy  $mc^2$ . There exist also solutions with negative energy  $\mathcal{E} = c(-\alpha - p_x)$  and negative momentum  $p_x$  (corresponding to  $\alpha = -mc$  in the above calculations), which move in opposite direction. The oscillations of the charge give rise to radiation [9, 26, 27] (Compton effect); the Lorentz reaction force is extremely small [28]; in the limit of high fields, where  $x$  is approximately  $ct$ , the phase is very small and the oscillations and the radiation are fading out.

We introduce the parameter

$$\eta = \frac{qA_0}{2mc^2} \quad (17)$$

(or  $\eta = qE_0/2m\omega c$ , where  $E_0 = \omega A_0/c$  is the electric field,  $q > 0$ ); from Eq. (14) the drift velocity of the charge is given approximately by

$$v_x \simeq \frac{\eta^2}{1 + \eta^2} c, \quad (18)$$

and the coordinates  $x$  and  $z$  can be written as

$$\begin{aligned} x &\simeq v_x t + \frac{1}{2} \frac{v_x \lambda}{c} \sin 2(\omega - kv_x)t = vt + \frac{1}{2} \frac{v_x \lambda}{c} \sin 2\omega(1 - \frac{v_x}{c})t, \\ z &= -2\eta\lambda \sin \omega(1 - \frac{v_x}{c})t; \end{aligned} \quad (19)$$

a current  $J = qv_x$  occurs, along the direction of propagation of the radiation. The charge gets rapidly ultrarelativistic, as shown by Eqs. (19) [29].

Usually, the parameter  $\eta$  is small ( $\eta \ll 1$ ). However, a laser high intensity  $I = 10^{22}$  w/cm<sup>2</sup>, focalized in a pulse of dimension  $d$ , generates an electric field  $E_0 = \sqrt{4\pi I/c} \simeq 10^{10}$  statvolt/cm ( $E_0^2 d^3 = 4\pi I d^2 \tau = 4\pi I d^3/c$ , where  $\tau = d/c$  is the duration of the pulse). For a linear dimension of the pulse  $d = 10 \mu\text{m}$ ,  $\tau \simeq 30$  fs; the intensity  $10^{22}$  w/cm<sup>2</sup> corresponds to  $\simeq 10$  Pw, *i.e.* a total energy per pulse  $\simeq 300$  w. The vector potential is  $A_0 = cE_0/\omega = 10^{-5} E_0 = 10^5$  statvolt for the optical frequency  $\omega \simeq 3 \times 10^{15}$  s<sup>-1</sup> ( $\nu = \omega/2\pi$ ,  $\lambda = 0.5 \mu\text{m}$ ); the corresponding energy for an electron is  $qA_0 = 4.8 \times 10^{-5}$  erg  $\simeq 30$  MeV. This energy is much higher than the rest energy of the electron  $mc^2 = 0.5$  MeV, such that the ratio  $\eta = qA_0/2mc^2 = 30$  is much larger than unity. It follows that an electron can be accelerated, during the short duration  $\tau$  of the pulse, up to velocities close to the speed of light, along the direction of propagation of the radiation field, and up to energies of the order 1 GeV.

If the radiation is propagated in gaseous plasmas [30], then a radiation pulse is a wavepacket; when focalized, it is a three-dimensional wavepacket which distributes the electrons over its surface, such as to compensate the radiation field. Under such circumstances, the charges are accelerated by the transport motion of the wavepacket (pulse; pulsed polariton) [31]. The mechanism of charge acceleration presented here is distinct from, and complementary to the mechanism of charge acceleration in plasmas.

#### 4. DAMPING FORCE

The general solution of the equations of motion of a charge driven by an electromagnetic plane wave and the damping (reaction) force has been given in Ref. [32]. We estimate here, by a direct method, the magnitude of the damping force and show that in realistic cases the effect of the damping is small. The damping force [23] is given by

$$g^i = \frac{2q^3}{3mc^3} \frac{\partial F^{ik}}{\partial x^l} u_k u^l - \frac{2q^4}{3m^2 c^5} \left[ F^{il} F_{kl} u^k - (F_{kl} u^l)(F^{km} u_m) u^i \right]; \quad (20)$$

it must be much smaller than  $q^2/c\gamma a^2$ , where  $a = q^2/mc^2$  is the classical electromagnetic radius of the charge; this condition, which indicates the limits of applicability of the electromagnetism, does not imply always that the damping (reaction) force is smaller than the electromagnetic (Lorentz) force; in the extreme limit  $v \simeq c$  the damping may be higher than the driven force. Making use of the field given by equations (1) we get the reaction force

$$\begin{aligned} g^0 &= \frac{2q^3}{3mc^3}(u^0 - u^1) \left\{ \frac{\omega^2}{c^2} Au^3 + \frac{qE^2}{mc^2} [1 - (u^0 - u^1)u^0] \right\}, \\ g^1 &= \frac{2q^3}{3mc^3}(u^0 - u^1) \left\{ \frac{\omega^2}{c^2} Au^3 + \frac{qE^2}{mc^2} [1 - (u^0 - u^1)u^1] \right\}, \\ g^3 &= \frac{2q^3}{3mc^3}(u^0 - u^1)^2 \left( \frac{\omega^2}{c^2} A - \frac{qE^2}{mc^2} u^3 \right) \end{aligned} \quad (21)$$

(we can check  $u_i g^i = 0$ ). Making use of the four-velocity given in equation (3) we get

$$\begin{aligned} g^0 &= -\frac{2a^2\omega^2 A^2}{3c^3} \left( 1 + a \frac{E^2}{2m\omega^2} \right), \\ g^1 &= -\frac{2a^2\omega^2}{3c^3} \left[ A^2 - \frac{c^2 E^2}{\omega^2} \left( 1 - \frac{q^2 A^2}{2m^2 c^4} \right) \right], \\ g^3 &= \frac{2aqA\omega^2}{3c^3} \left( 1 - a \frac{E^2}{m\omega^2} \right), \end{aligned} \quad (22)$$

or, in the limit of high-intensity fields,

$$g^0 = g^1 \simeq -\frac{a^3 A^2 E^2}{3mc^3}, \quad g^3 \simeq -\frac{2a^2 q A E^2}{3mc^3}. \quad (23)$$

Using the numerical data given above ( $E_0 = 10^{10}$  statvolt/cm,  $A_0 = 10^5$  statvolt,  $\omega \simeq 3 \times 10^{15} \text{ s}^{-1}$  ( $\lambda = 0.5 \text{ }\mu\text{m}$ )) for an electron, we can see that the damping force ( $g^0 = g^1 \simeq 10^{-12} \text{ g/s}$ ,  $g^3 \simeq 10^{-13} \text{ g/s}$ ) is much smaller than the Lorentz force ( $qEu_0^3/c \simeq 10^{-8} \text{ g/s}$ ,  $qE_0/c \simeq 10^{-10} \text{ g/s}$ ;  $\gamma \simeq 10^4$ ) and the validity condition  $g^i \ll q^2/c\gamma a^2$  is satisfied ( $q^2/c\gamma a^2 \simeq 10^{-8} \text{ g/s}$ ).

## 5. "STOPPING" POINT

Let us assume that initially the charge moves with momentum  $f^1 = -bmc$ ,  $b > 0$ , in direction opposite to the laser beam. Then, from Eqs. (11)-(13), we get

$$x = \frac{1+2\eta^2-\beta^2}{1+2\eta^2+\beta^2} ct + \frac{\eta^2}{1+2\eta^2+\beta^2} \lambda \sin \frac{2\omega}{c} s, \quad z = -\frac{2\eta}{\beta} \lambda \sin \frac{\omega}{c} s, \quad (24)$$

$$p_x = \frac{mc}{2\beta} \left( 1 - \beta^2 + 4\eta^2 \cos^2 \frac{\omega}{c} s \right), \quad \mathcal{E} = \frac{mc^2}{2\beta} \left( 1 + \beta^2 + 4\eta^2 \cos^2 \frac{\omega}{c} s \right),$$

where  $\beta = b + \sqrt{1 + b^2}$ . We can see that there exists a value  $\beta^2 = 1 + 2\eta^2$  for which

$$x = \frac{\eta^2}{2(1+2\eta^2)} \lambda \sin \frac{2\omega}{c} s, \quad p_x = \frac{mc}{\sqrt{1+2\eta^2}} \eta^2 \cos \frac{2\omega}{c} s, \quad (25)$$

$$\mathcal{E} = \frac{mc^2}{\sqrt{1+2\eta^2}} \left( 1 + \eta^2 + 2\eta^2 \cos^2 \frac{\omega}{c} s \right),$$

where the drift motion ceases. If the initial action of the launched particle is maintained, the charge oscillates indefinitely at the stopping point, and radiates (mainly with frequencies  $\omega$  and  $2\omega$ ); if not, this is a turning point, where the charge is reflected. The associated stochastic photon emission in the so-called radiation-dominated regime (non-linear Compton effect) was considered recently by numerical simulations [33, 34].

## 6. QUANTUM CHARGE

As it is well known [35, 36], the wavefunction of a relativistic quantum charge in an electromagnetic plane wave is

$$\psi = \left[ 1 + \frac{q}{2c} \frac{(\gamma k)(\gamma A)}{(pk)} \right] e^{\frac{i}{\hbar} S} u, \quad (26)$$

where

$$S = -px - \int^{\xi} d\xi' \left[ \frac{q}{c(pk)} (pA) - \frac{q^2}{2(pk)c^2} A^2 \right] \quad (27)$$

is the mechanical action,  $p$  is the charge momentum four-vector,  $k$  is the field four-wavevector,  $\gamma$  denotes the Dirac matrices and  $u$  is a constant bispinor; the notation  $(pk)$  stands for the scalar product of the two four vectors  $p$  and  $k$  (Volkov wavefunction; compare with the classical action given by Eq. (10), where  $p$  is replaced by  $f$ ). The electromagnetic wave depends only on the phase  $s = (kx)$ . We assume that the interaction is introduced adiabatically; then  $u$  is the solution of the free Dirac equation  $(\gamma p - mc)u = 0$ , *i.e.* it is the plane wave constant bispinor. Therefore, the wavefunction can be written as

$$\psi_{p\sigma} = \frac{1}{\sqrt{2\varepsilon V}} \left[ 1 + \frac{q}{2c} \frac{(\gamma k)(\gamma A)}{(pk)} \right] e^{\frac{i}{\hbar} S} u_{p\sigma}, \quad (28)$$

where  $\sigma = \pm 1$  is the spin label,  $V$  is the volume,  $\varepsilon = (m^2 c^4 + p^2 c^2)^{1/2}$  and  $u_{p\sigma}$  is normalized such as  $\bar{u}_{p\sigma} u_{p\sigma'} = 2mc^2 \delta_{\sigma\sigma'}$ ,  $\bar{u}_{-p\sigma} u_{-p\sigma'} = -2mc^2 \delta_{\sigma\sigma'}$  [37]. We give here these constant bispinors

$$u_{p\sigma} = \begin{pmatrix} (\varepsilon + mc^2)^{1/2} w_\sigma \\ (\varepsilon - mc^2)^{1/2} (\mathbf{n}\boldsymbol{\sigma}) w_\sigma \end{pmatrix}, \quad u_{-p-\sigma} = \begin{pmatrix} (\varepsilon - mc^2)^{1/2} (\mathbf{n}\boldsymbol{\sigma}) w'_\sigma \\ (\varepsilon + mc^2)^{1/2} w'_\sigma \end{pmatrix}, \quad (29)$$

where  $\mathbf{n} = \mathbf{p}/p$ ,  $w'_\sigma = -\sigma_y w_{-\sigma}$  and  $w_\sigma$  can be taken as the eigenvectors of the matrix  $\sigma_z$ ;  $\sigma$  denotes the spin matrices. We note that  $u_{p\sigma}^* u_{p\sigma'} = \bar{u}_{p\sigma} \gamma^0 u_{p\sigma'} = 2\varepsilon \delta_{\sigma\sigma'}$ . The wavefunctions  $\psi_{p\sigma}$  are orthonormal [38]; also, the completeness of these wavefunctions can be proved (see, for instance, Refs. [39–41]). The phase  $S$  given by Eq. (27) is the classical mechanical action [38]; it contains the drift motion of the charge along the propagation of the radiation wave, while the pre-exponential factor in the wavefunction given by Eq. (26) includes the oscillations of the charge in the radiation field. The current  $j^i = c\bar{\psi}\gamma^i\psi$  (with the probability density  $\rho = \bar{\psi}\gamma^0\psi = \psi^*\psi = j^0/c$ ) and the momentum  $q^i = \psi_{p\sigma}^*(p^i - \frac{q}{c}A^i)\psi_{p\sigma}$  can be computed straightforwardly, and an effective mass  $m^*$  can be derived, identical with the effective mass in the classical case [42].

Let us consider an electromagnetic wave propagating along the  $x$ -direction,  $k^i = (1, 1, 0, 0)$ ,  $s = k_i x^i = ct - x$ , with the electromagnetic potentials  $A^i = (0, 0, 0, A)$ ,  $A = A_0 \cos \frac{\omega}{c}s$  (linear polarization) and a charge moving initially along the  $y$ -direction,  $p^i = (mc, 0, p_y, 0)$ , with a small  $p_y$ ; then, the pre-exponential factor of the wavefunction in Eq. (26) is

$$1 + \frac{q}{2c} \frac{(\gamma k)(\gamma A)}{(pk)} = 1 - \frac{qA_0 \cos \frac{\omega}{c}s}{2mc^2} \begin{pmatrix} -i\sigma_y & \sigma_z \\ \sigma_z & -i\sigma_y \end{pmatrix} \quad (30)$$

(we note that the matrix  $(\gamma^0 - \gamma^1)\gamma^3$  entering the pre-exponential factor is twofold degenerate); the mechanical action given by Eq. (27) becomes

$$S = - \left( mc^2 + \frac{q^2 A_0^2}{4mc^2} \right) t + p_y y + \frac{q^2 A_0^2}{4mc^3} x - \frac{q^2 A_0^2}{8mc^2 \omega} \sin \frac{2\omega}{c} s. \quad (31)$$

The charge acquires an average drift momentum

$$P_x \simeq \frac{q^2 A_0^2}{4mc^3} = m\eta^2, \quad (32)$$

an energy

$$\mathcal{E} \simeq mc^2 + \frac{q^2 A_0^2}{4mc^2} = mc^2(1 + \eta^2) \quad (33)$$

and a phase velocity

$$v_x \simeq \frac{\mathcal{E}}{P_x} = \frac{1 + e^2 A_0^2 / 4m^2 c^4}{e^2 A_0^2 / 4m^2 c^4} c = \frac{1 + \eta^2}{\eta^2} c, \quad (34)$$

which is higher than the speed of light in vacuum  $c$ ; the group velocity may attain values as large as  $c$ .

We can see that for high-intensity fields [43],  $\eta = qA_0/2mc^2 \gg 1$ , the charge becomes ultrarelativistic; leaving aside the pre-exponential factor given by Eq. (30), the wavefunction can be written as

$$\psi_{p\sigma} \simeq \frac{1}{\sqrt{2V}} \begin{pmatrix} w_\sigma \\ \sigma_y w_\sigma \end{pmatrix} e^{\frac{i}{\hbar}S}, \quad \psi_{-p-\sigma} \simeq -\frac{1}{\sqrt{2V}} \begin{pmatrix} w_{-\sigma} \\ \sigma_y w_{-\sigma} \end{pmatrix} e^{-\frac{i}{\hbar}S}, \quad (35)$$



where

$$S \simeq -\frac{q^2 A_0^2}{4mc^2} t + \frac{q^2 A_0^2}{4mc^3} x = -\frac{q^2 A_0^2}{4mc^3} (ct - x) = -mc\eta^2 (ct - x); \quad (36)$$

(the two bispinors are not independent, which corresponds to the ultrarelativistic case). The extremely fast oscillations produced by the large phase  $S$  indicate that we may view the charge as being in the quasi-classical limit (ray approximation); for practical purposes the charge may be viewed as a classical charge. A similar result can be obtained by taking the limit  $m \rightarrow 0$ , either formally in the Volkov wavefunction given by Eq. (26), or in the initial Dirac equation (Weyl equation, with only one spinor) [36]. We can check that the current is  $\bar{\psi}_{p\sigma} \gamma^i \psi_{p\sigma} = \frac{1}{V} (1, 1, 0, 0)$ , corresponding to a plane wave which describes an ultrarelativistic particle. The spinor  $\psi_{-p-\sigma}$  corresponds to negative energy (and momentum). The negative-energy electrons in the Dirac Fermi sea get lower and lower (negative) energy (as if they would have a negative mass); such that the gap between the negative-energy states and positive-energy states is increased by radiation. Similar considerations are valid for pair creation in laser fields in the presence of a Coulomb potential (Bethe-Heitler process [44, 45]). It is also worth noting that the accelerated charge “feels” not anymore the radiation for very strong fields; in the rest frame of an ultrarelativistic particle the electromagnetic fields are vanishing. From Eq. (36) we get the charge wavelength  $\lambda \simeq \hbar/\eta^2 mc$ ; we can see that for large  $\eta$  the wavelength is much shorter than the Compton wavelength; it follows again that for many practical purposes we may use the classical approach for the accelerated charge [46–48]. Within this approach small interference effects or rapidly varying spin dynamics are lost; in the limit of high-intensity fields these effects are small, so they can be disregarded. An ultrarelativistic particle is practically a “radiation” field; as such, it does not radiate, and does not “feel” the accelerating electromagnetic field.

## 7. STANDING WAVE

Let us consider now a classical relativistic charge in a standing electromagnetic wave with the vector potential

$$A = A_z = \frac{1}{2} A_0 [\cos(\omega t - kx) + \cos(\omega t + kx)] = A_0 \cos \omega t \cos kx \quad (37)$$

(linear polarization); the frequency of the wave is in the optical range,  $\nu = \omega/2\pi \simeq 10^{15} \text{ s}^{-1}$ , and the wavelength is  $\lambda = 2\pi/k = c/\nu \simeq 3 \times 10^{-5} \text{ cm} = 0.3 \text{ }\mu\text{m}$ . A charge in a standing wave spends there more time than the wave period ( $1/\nu = 10^{-15} \text{ s}$ ); consequently, in the Hamilton-Jacobi equation

$$\frac{1}{c^2} (\partial S / \partial t)^2 = (\nabla S - \frac{q}{c} \mathbf{A})^2 + m^2 c^2 \quad (38)$$

we may take the average of  $\mathbf{A}$  and  $\mathbf{A}^2$  with respect to the time; Eq. (38) becomes

$$\frac{1}{c^2}(\partial S/\partial t)^2 = (\partial S/\partial x)^2 + (\partial S/\partial y)^2 + (\partial S/\partial z)^2 + \frac{q^2 A_0^2}{2c^2} \cos^2 kx + m^2 c^2 ; \quad (39)$$

(a similar result holds for a circularly polarized wave). The solution of this equation is the mechanical action

$$S = \pm \frac{qA_0}{\sqrt{2\omega}} \cos kx + p_y y + p_z z - \mathcal{E} t , \quad (40)$$

where  $p_{y,z}$  are constant transverse momenta and  $\mathcal{E}$  is the energy, given by

$$\mathcal{E}^2 = m^2 c^4 + \frac{1}{2} q^2 A_0^2 + (p_y^2 + p_z^2) c^2 . \quad (41)$$

We note that the energy acquires the form of the energy of a free particle, at rest along the longitudinal  $x$ -direction (wave direction), with a renormalized mass. This indicates that the assumption of a relativistic classical charge in a standing electromagnetic wave is not warranted, as expected. In particular, we can see that the coordinate  $x$  is not determined.

Indeed, contradictions can appear from such an assumption. For instance, the longitudinal momentum is

$$P_x = p_x = \frac{\partial S}{\partial x} = \pm \frac{qA_0}{\sqrt{2}c} \sin kx , \quad (42)$$

whence, making use of  $p_x = mv_x/(1 - v_x^2/c^2)^{1/2}$ , we get a “velocity”

$$\frac{dx}{dt} = v_x = c \frac{\frac{qA_0}{\sqrt{2}mc^2} \sin kx}{\sqrt{1 + \frac{q^2 A_0^2}{2m^2 c^4} \sin^2 kx}} \quad (43)$$

and a “force”

$$\frac{dp_x}{dt} = v_x \frac{dp_x}{dx} = \frac{1}{2} c \frac{d}{dx} \sqrt{m^2 c^2 + p_x^2} = mc^2 k \frac{2\eta^2 \sin kx \cos kx}{\sqrt{1 + 2\eta^2 \sin^2 kx}} , \quad (44)$$

which looks like a ponderomotive force. However, the motion under the action of such a force is meaningless. In particular, both the “velocity” and the “force” given by Eqs. (43) and, respectively, (44), vanish at  $kx = n\pi$ , where  $n$  is any integer.

The situation is different for a non-relativistic charge; in this case the energy reads

$$\mathcal{E} = \frac{1}{2m} (\mathbf{P} - \frac{q}{c} \mathbf{A})^2 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} - \frac{q}{mc} p_z A + \frac{q^2}{2mc^2} A^2 , \quad (45)$$

or, taking the temporal average,

$$\mathcal{E} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{e^2 A_0^2}{4mc^2} \cos^2 kx ; \quad (46)$$

we can see that the charge moves in a periodic potential

$$U(x) = \frac{q^2 A_0^2}{4mc^2} \cos^2 kx ; \quad (47)$$

the standing wave acts as an optical lattice for the charge. It follows that the quantum-mechanical motion of a charge in a standing electromagnetic wave generates energy bands, as it is well known [49]; and, in addition, a charge may suffer diffraction on a standing wave (Kapitza-Dirac effect [50]). The classical motion is confined within the potential wells centered at  $kx = (2n + 1)\pi/2$ , according to the potential energy included in the expression of the Hamiltonian given by Eq. (46), subject to a ponderomotive force  $mc^2 k \eta^2 \sin 2kx$ .

## 8. CONCLUDING REMARKS

Finally, it is worthwhile commenting upon the Compton effect in a classical electromagnetic wave. It is instructive at this moment to have an estimation of the (mean) photon density in a laser pulse with moderate intensity  $I = 10^{18}$  w/cm<sup>2</sup>; this intensity corresponds to an electric field of the order  $E_0 \simeq \sqrt{4\pi I/c} = 10^8$  statvolt/cm (and a similar magnetic field). The energy density is of the order  $w \simeq I/c = 10^{14}$  erg/cm<sup>3</sup>, with a density of photons  $n \simeq 10^{25}$  cm<sup>-3</sup> with energy, say,  $\hbar\omega = 1$  eV; the photon flow (flux) is  $cn \simeq 10^{35}$ /cm<sup>2</sup>·s. Currently, electrons may be injected in an electromagnetic waves to give an electric current of, at most, the order  $\simeq 100$  mA, which corresponds to  $\simeq 10^{17}$  electrons per second (we may admit that such an electric current can be produced experimentally over a cross-sectional area 1 cm<sup>2</sup>); it follows that we may have an electron flow  $\simeq 10^{17}$ /cm<sup>2</sup>·s. We may see that electron flow is much weaker than the photon flow. Therefore, we may conclude that the disruption of an electromagnetic wave by electron beams is unlikely and the Compton effect is not likely to disturb appreciably either the electromagnetic wave or the electron dynamics. We may view the Compton scattering in an electromagnetic wave as a statistical-mechanical effect, where the mean free path of the electron is of the order of the mean separation distance between the photons ( $\simeq 10^{-8}$  cm), the Compton cross-section  $\sigma$  is of the order of the square of the classical electromagnetic radius of the electron ( $r_e = e^2/mc^2 \simeq 2.8 \times 10^{-13}$  cm), for the radiation wavelength  $\simeq 3 \times 10^{-5}$  cm ( $\hbar\omega = 1$  eV).

In conclusion, we may say that the acceleration of a classical relativistic charge by a traveling high-intensity electromagnetic wave is analyzed here. Such a charge acquires quickly a fast drift motion and becomes ultrarelativistic, moving with energies in the GeV's range; under these circumstances it ceases to radiate and "feels" not anymore the carrying wave. Also, it is shown that a relativistic quantum charge in high-intensity laser radiation (Volkov state) becomes rapidly "localized", such that

a (quasi-) classical treatment is more adequate for many practical purposes. Similarly, it is shown that a charge in a standing electromagnetic wave behaves quantum-mechanically and non-relativistically. This situation leads to energy bands and charge diffraction on a standing electromagnetic wave (Kapitza-Dirac effect), as it is well known in an optical lattice.

*Acknowledgements.* The authors are indebted to the members of the Laboratory of Theoretical Physics at Magurele-Bucharest for many fruitful discussions. This work has been supported by the Scientific Research Agency of the Romanian Governments through Grants 04-ELI / 2016 (Program 5/5.1/ELI-RO), PN 16 42 01 01 / 2016, PN (ELI) 16 42 01 05 / 2016 and #09370108-2009 / Ph12 / 2014 / ELI-NP.

#### REFERENCES

1. V. Yanovski, V. Chvykov, G. Kalinchenko, P. Rousseau, T. Planchon, T. Matsuoka, A. Maksimchuk, J. Nees, G. Cheriaux, G. Mourou and K. Krushelnik, "Ultra-high intensity-300-Tw laser at 0.1Hz repetition rate", *Opt. Express* **16** 2109-2114 (2008).
2. TPL (Texas Petawatt Laser) <http://texaspetawatt.ph.utexas.edu/overview.php> (2011).
3. Vulcan 10 Pw, <http://www.clf.rl.ac.uk/New+Initiatives/The+Vulcan+10+Petawatt+Project/14684.aspx> (2011).
4. Astra Gemini, <http://www.clf.rl.ac.uk/Science/12258.aspx> (2011).
5. J. P. Chambaret, P. Georges, G. Cheriaux, G. Rey, C. Le Blanc, P. Audebert, D. Douillet, J. L. Paillard, P. Cavillac, D. Fournet, F. Mathieu and G. Mourou, "The Extreme Light Infrastructure Project ELI and its Prototype APOLLON/ILE: The Associated Laser Bottlenecks", in *Frontiers in Optics, Opt. Soc. Am., Washington*, p. FMI2, DC (2009) (doi:10.1364/FIO.2009.FMI2).
6. PHELIX (Petawatt High-Energy Laser for heavy Ion eXperiments), <https://www.gsi.de/work/forschung/appamml/plasmaphysikhelix/phelix.htm> (2011).
7. ELI (Extreme Light Infrastructure), [http://www.eli-beams.eu/\(2011\)](http://www.eli-beams.eu/(2011)).
8. A. V. Korzhimanov, A. A. Gonoskov, E. A. Khazanov and A. M. Sergeev, "Horizons of petawatt laser technology", *Phys. Usp.* **54** 9-28 (2011).
9. Y. I. Salamin, S. X. Hu, K. Z. Hatsagortsyan and C. H. Keitel, "Relativistic high-power laser-matter interaction", *Phys. Repts.* **427** 41-155 (2006).
10. A. Di Piazza, C. Muller, K. Z. Hatsagortsyan and C. H. Keitel, "Extremely high-intensity laser interactions with fundamental quantum systems", *Revs. Mod. Phys.* **84** 1177- 1228 (2012).
11. C. E. Clayton, J. E. Ralph, F. Albert, R. A. Fonseca, S. H. Glenzer, C. Joshi, W. Lu, K. A. Marsh, S. F. Martins, W. B. Mori, A. Pak, F. S. Tsung, B. B. Pollock, J. S. Ross, L. O. Silva and D. H. Froula, "Self-guided laser wakefield acceleration beyond 1GeV using ionization induced injection", *Phys. Rev. Lett.* **105** 105003 (2010).
12. V. Malka, "Laser plasma accelerators", *Phys. Plasmas* **19** 055501 (2012).
13. D. Haberberger, S. Tochitsky, F. Fiuza, C. Gong, R. A. Fonseca, L. O. Silva, W. B. Mori and C. Joshi, "Collisionless shocks in laser-produced plasma generate monoenergetic high-energy proton beams", *Nature Phys.* **8** 95-99 (2012).
14. D. L. Burke, R. C. Field, G. Horton-Smith, J. E. Spencer, D. Walz, S. C. Berridge, W. M. Bugg, K. Shmakov, A. W. Weidemann, C. Bula, K. T. McDonald, E. J. Prebys, C. Bamber, S. G. Boege, T. Koffas, T. Kotseroglou, A. C. Melissinos, D. D. Meyerhofer, D. A. Reiss and W. Ragg, "Positron production in multiphoton light-by-light scattering", *Phys. Rev. Lett.* **79**, 1626-1629 (1997).

15. C. Bamber, S. J. Boege, T. Koffas, T. Kotseroglou, A. C. Melissinos, D. D. Meyerhofer, D. A. Reiss and W. Ragg, C. Bula, K. T. McDonald, E. J. Prebys, D. L. Burke, R. C. Field, G. Horton-Smith, J. E. Spencer, D. Walz, S. C. Berridge, W. M. Bugg, K. Shmakov, A. W. Weidemann, "Studies of nonlinear QED in collisions of  $46.6\text{GeV}$  electrons with intense laser pulses", *Phys. Rev.* **D60** 092004 (1999).
16. M. Marklund and P. K. Shukla, "Nonlinear collective effects in photon-photon and photon-plasma interactions", *Revs. Mod. Phys.* **78** 591-640 (2006).
17. Y. I. Salamin, "Electron acceleration from rest in vacuum by an axicon gaussian laser beam", *Phys. Rev.* **A73** 043402 (2006).
18. M. Marklund, "Fundamental optical physics: probing the quantum vacuum", *Nature Photon.* **4** 72-74 (2010).
19. F. Ehlotzky, K. Krajewska and J. Z. Kaminski, "Fundamental processes of quantum electrodynamics in laser fields of relativistic power", *Reps. Progr. Phys.* **72b**, 046401 (2009).
20. J. H. Eberly, "Interaction of very intense light with free electrons", in *Progr. Opt.*, ed. E. Wolf, **7**, 359-415, NH, Amsterdam (1969).
21. W. J. Brown, S. G. Anderson, C. P. J. Barty, S. M. Betts, R. Booth, J. K. Crane, R. R. Cross, D. N. Fittinghoff, D. J. Gibson, F. V. Hartemann, E. P. Hartouni, J. Kuba, G. P. Le Sage, D. R. Slaughter, A. M. Tremaine, A. J. Wootton, P. T. Springer and J. B. Rosenzweig, "Experimental characterization of an ultrafast Thomson scattering x-ray source with three-dimensional time and frequency-domain analysis", *Phys. Rev. ST Accel. Beams* **7** 060702 (2004).
22. C. T. L. Smeenk, L. Arissian, B. Zhou, A. Mysyrowicz, D. M. Villeneuve, A. Staudte and P. B. Corkum, "Partitioning of the linear photon momentum in multiphoton ionization", *Phys. Rev. Lett.* **106** 193002 (2011).
23. L. Landau and E. Lifshitz, *Course of Theoretical Physics*, vol. 2, *The Classical Theory of Fields*, Elsevier (2004).
24. T. W. B. Kibble, "Mutual refraction of electrons and photons", *Phys. Rev.* **150** 1060-1069 (1966).
25. H. K. Avetissian, *Relativistic Nonlinear Electrodynamics. Interaction of Charged Particles with Strong and Super Strong Laser Fields*, Springer Series in Optical Sciences **120**, ed. W. T. Rhodes, Springer, NY (2006).
26. Y. I. Salamin and F. H. M. Faisal, "Harmonic generation by superintense light scattering from relativistic electrons", *Phys. Rev.* **A54** 4383-4395 (1996).
27. Y. I. Salamin and F. H. M. Faisal, "Harmonic generation by scattering circularly polarized light of arbitrary intensity from free electrons of arbitrary initial velocity", *Phys. Rev.* **A55** 3964-3967 (1997).
28. E. S. Sarachik and G. T. Scappert, "Classical theory of the scattering of intense laser radiation by free electrons", *Phys. Rev.* **D1** 2738-2753 (1970).
29. Y. I. Salamin and C. H. Keitel, "Electron acceleration by a tightly focused laser beam", *Phys. Rev. Lett.* **88** 095005 (2002).
30. M. Apostol, "Propagation of electromagnetic pulses through the surface of dispersive bodies", *Roum. J. Phys.* **58** 1298-1318 (2013).
31. M. Apostol and M. Ganciu, "Polaritonic pulse and coherent X- and gamma rays from Compton (Thomson) backscattering", *J. Appl. Phys.* **109** 013307 (2011).
32. A. di Piazza, "Exact solution of the Landau-Lifshitz equation in a plane wave", *Lett. Math.Phys.* **83** 305-313 (2008).
33. J.-X. Li, K. Z. Hatsagortsyan, B. J. Galow and C. H. Keitel, "Attosecond gamma-ray pulses via non-linear Compton scattering in the radiation-dominated regime", *Phys. Rev. Lett.* **115** 204801 (2015).

34. J.-X. Li, Y.-Y. Chen, K. Z. Hatsagortsyan and C. H. Keitel, "Angle-resolved stochastic photon emission in the quantum radiation-dominated regime", arXiv: 1612.06796v1 (physics.plasm-ph), 20 Dec. 2016.
35. D. M. Volkov, "Über eine Klasse von Lösungen der Diracschen Gleichung", *Z. Phys.* **94** 250-260 (1935).
36. L. Landau and E. Lifshitz, *Course of Theoretical Physics*, vol. 4, V. B. Berestetskii, E. M. Lifshitz and L. P. Pitaevskii, *Quantum Electrodynamics*, Elsevier (2002).
37. M. Apostol, "Dynamics of electron-positron pairs in vacuum polarized by an external electromagnetic field", *J. Mod. Optics* **58** 611-618 (2011).
38. V. I. Ritus, "Quantum effects of the interaction of elementary particles with an intense electromagnetic field", *J. Sov. Las. Res.* **6** 497-617 (1985).
39. O. von Roos, "Frequency shift in high-intensity Compton scattering", *Phys. Rev.* **150** 1112-1118 (1966).
40. Z. Fried, A. Baker and D. Korff, "Comments on intensity-dependent frequency shift in Compton scattering and its possible detection", *Phys. Rev.* **151b** 1040-1048 (1966).
41. S. Zakowicz, "Square-integrable wave packets from the Volkov solutions", *J. Math. Phys.* **46b** 032304 (2005).
42. W. Greiner and J. Reinhardt, *Quantum Electrodynamics*, Springer, Berlin (2009).
43. M. Parry, "Electron in the ultrashort laser pulse", arXiv: hep-ph/0207274v1 (2002).
44. H. Bethe and W. Heitler, "On the stopping of fast particles and on the creation of positive electrons", *Proc. Roy. Soc.* **A146** 83-112 (1934).
45. W. Heitler, *The Quantum Theory of Radiation*, Dover, NY (1984).
46. J. S. Roman, L. Roso and H. R. Reiss, "Evolution of a relativistic wavepacket describing a free electron in a very intense laser field", *J. Phys. B: At. Mol. Opt. Phys.*, **33** 1869-1880 (2000).
47. G. R. Mocken and C. H. Keitel, "Quantum dynamics of relativistic electrons", *J. Comput. Phys.* **199** 558-588 (2004).
48. M. Mahmoudi, Y. I. Salamin and C. H. Keitel, "Free-electron quantum signatures in intense laser fields", *Phys. Rev.* **A72** 033402 (2005).
49. W. Becker, M. Meckbach and H. Mitter, "Motion of relativistic particles in standing wave fields", *J. Phys.* **A12** 799-809 (1979).
50. P. L. Kapitza and P. A. M. Dirac, "The reflection of electrons from standing light waves", *Proc. Cambridge Phil. Soc.* **29** 297-300 (1933).