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RESONANT COUPLING OF THE ELECTRON CLOUD WITH THE NUCLEUS IN HEAVY ATOMS

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Abstract. The paper envisages the interaction of the laser radiation with the atomic nucleus in heavy atoms. Within a linearized Thomas-Fermi theory we show that the electron cloud in heavy atoms exhibits giant dipolar vibrations as eigenmodes with a frequency in the range $15ZeV$, where Z is the atomic number. These eigenmodes screen to a large extent the electric field of the optical-laser radiation (within the dipolar approximation). At the same time, there may appear a resonant coupling of these modes with the nucleus, which increases considerably the strength of the internal electric field acting upon the electrons and the nucleus. We estimate the effect of this electric field on the atom ionization and the emission rate of nucleons from nucleus and find that, although the nucleon emission rate may be enhanced appreciably, the nucleon emission process is, in fact, stopped by the faster ionization rate, which spoils the resonance regime.

Key words: giant atomic vibrations, screening, resonant coupling, rate of nucleon emission.

1. INTRODUCTION

It was suggested that high-power laser radiation may be used for the treatment of the radioactive waste, by enhancing the rate of spontaneous or induced nuclear processes like alpha decay or proton emission. This seems to be one of the main objectives of the european project Extreme Light Infrastructure-Nuclear Physics at Magurele-Bucharest (ELI-NP) [1]. We show in this paper that, while the radiation of the optical lasers is appreciably screened by the vibrations of the electron cloud surrounding the atomic nucleus in heavy atoms, there may exist a resonant coupling between the nucleus and the electron cloud in such atoms for free-electron laser frequencies of the order $15ZeV$, which may enhance to a great extent the rate of atom ionization and the rate of nucleon emission from nucleus (Z being the atomic number of the heavy atom). These processes occur under the action of a high internal electric field generated by the resonant coupling. However, although the rate of nucleon emission may be enhanced appreciably, the fast process of nucleon emission is, in fact, stopped by the faster ionization process, which spoils the resonance regime.

2. THOMAS-FERMI THEORY OF HEAVY ATOMS

We start with heavy atoms with the atomic numbers $Z \gg 1$. The most convenient approach to these atoms is the Thomas-Fermi theory [2]. The kinetic energy of a free electron gas is $V(\hbar^2 k_F^5/10\pi^2 m)$, where V is the volume, k_F is the Fermi wavevector and m is the mass of the electron ($\hbar$ denotes the Planck constant); if k_F varies in space and if the gas is sufficiently dense, we have a local free electron gas with the kinetic energy $\Delta V(\hbar^2 k_F^5/10\pi^2 m)$; then, the total kinetic energy may be written as

$$E_{kin} = \int d\mathbf{r} \frac{\hbar^2 k_F^5}{10\pi^2 m} \ , \tag{1}$$

or, using the density $n = k_F^3/3\pi^2$,

$$E_{kin} = \int d\mathbf{r} \frac{3(3\pi^2)^{2/3}}{10} \frac{\hbar^2}{m} n^{5/3} \ ; \tag{2}$$

the total energy of a heavy atom can be written as

$$E = \int d\mathbf{r} \varepsilon_{kin}(n) - Ze^2 \int d\mathbf{r} \frac{n(\mathbf{r})}{r} + \frac{1}{2} e^2 \int d\mathbf{r} d\mathbf{r}' \frac{n(\mathbf{r}) n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \ , \tag{3}$$

where $\varepsilon_{kin}(n) = [3(3\pi^2)^{2/3}/10]\hbar^2 n^{5/3}/m$ is the local kinetic energy; the second term on the right in equation (3) is the Coulomb electron-nucleus attraction and the third term is the Coulomb electron-electron repulsion; $-e$ is the charge of the electron.

The first-order variation of this energy is

$$\delta E^{(1)} = \int d\mathbf{r} \frac{\partial \varepsilon_{kin}}{\partial n} \delta n - Ze^2 \int d\mathbf{r} \frac{\delta n}{r} + e^2 \int d\mathbf{r} d\mathbf{r}' \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \delta n \ ; \tag{4}$$

at equilibriun it should be zero, *i.e.*

$$\frac{\partial \varepsilon_{kin}}{\partial n} - \frac{Ze^2}{r} + e^2 \int d\mathbf{r}' \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} = 0 \ ; \tag{5}$$

this equation gives the equilibrium density $n_0(\mathbf{r})$. A convenient way of solving equation (5) is to introduce the self-consistent potential

$$\varphi(\mathbf{r}) = \frac{Ze}{r} - e \int d\mathbf{r}' \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \tag{6}$$

and notice that it satisfies the Poisson equation

$$\Delta \varphi = -4\pi Ze \delta(\mathbf{r}) + 4\pi e n(\mathbf{r}) \ ; \tag{7}$$

equation (5) becomes

$$\frac{\partial \varepsilon_{kin}}{\partial n} - e\varphi = 0 \ , \tag{8}$$

i.e.

$$(3\pi^2)^{2/3}\frac{\hbar^2}{2m}n^{2/3} - e\varphi = 0 \ , \tag{9}$$

or

$$\frac{\hbar^2 k_F^2}{2m} - e\varphi = 0 \ ; \tag{10}$$

this is the basic equation of the Thomas-Fermi theory model; since it gives $k_F \sim \varphi^{1/2}$ and $n \sim \varphi^{3/2}$ we call it the $3/2$-Thomas-Fermi model. As it is well known, this model does not bind the electrons about the nucleus [3, 4].

The basic assumption of the Thomas-Fermi model is a slightly inhomogeneous electron gas; in accordance with this assumption, we write $k_F^2 = 2\overline{k}_F k_F$ and $k_F^3 = 3\overline{k}_F^2 k_F$, where $\overline{k}_F$ is a parameter; also, we introduce the parameter $q^2 = 4\overline{k}_F/\pi a_H$, where $a_H = \hbar^2/me^2$ is the Bohr radius; we get $n = (q^2/4\pi e)\varphi$ and a linearized Poisson equation

$$\Delta\varphi = -4\pi Ze\delta(\mathbf{r}) + q^2\varphi \tag{11}$$

(equation (7)) with the solution $\varphi = (Ze/r)e^{-qr}$ and $n = (Zq^2/4\pi r)e^{-qr}$.

Making use of this solution in the total energy given by equation (3) and linearizing its expresion with respect to k_F, we get the total energy (equation (3))

$$E_q = \frac{\pi^2}{32}a_H^3 Ze^2 q^4 - \frac{3}{4}Z^2e^2 q \ , \tag{12}$$

where the first term on the right in equation (12) is the kinetic energy and the second term is the potential energy. This energy reaches the minimum value

$$E = -\frac{27}{8(6\pi)^{2/3}}Z^{7/3}\frac{e^2}{a_H} = -0.42Z^{7/3}\frac{e^2}{a_H} = -11.4Z^{7/3}eV \tag{13}$$

for the optimal value

$$q = (6/\pi^2)^{1/3}\frac{Z^{1/3}}{a_H} = 0.85\frac{Z^{1/3}}{a_H} \tag{14}$$

of the variational parameter q; (another useful formula is $E = -(9/16)Z^2e^2q$, where q is given by equation (14)). Adding quantum-mechanical corrections, we get the binding energy $-16Z^{7/3}eV$, in agreement with the empirical binding energy [5, 6]. The self-consistent potential $\varphi = (Ze/r)e^{-qr}$ and the equlibrium density $n_0 = (Zq^2/4\pi r)e^{-qr}$ correspond to q given by equation (14). We can see that the electron density in heavy atoms is concentrated in the region $r < R$, where R is given by $qR = 1$, *i.e.* $R = (\pi^2/6)^{1/3}a_H/Z^{1/3}$, which is a much smaller radius of the atom than the Bohr radius a_H ($Z \gg 1$). The parameter q given by equation (14) is the Thomas-Fermi screening wavevector and the radius R is the Thomas-Fermi screening wavelength.

3. GIANT DIPOLE VIBRATIONS

The motion of the electrons as a whole implies a small change δR in the radius R, given by a displacement $u = \delta R$, such that the equilibrium condition $qR = 1$ is preserved and $\delta q = -q^2 u$. The change in the energy given by equation (13) (which arises solely from the kinetic energy) is

$$\delta E = \frac{27}{4\pi^2} \frac{Z^3 e^2}{a_H^3} u^2 \; ; \tag{15}$$

this energy corresponds to a frequency ω_0 given by $\delta E = \frac{1}{2} M \omega_0^2 u^2$, where $M = Zm$ is the total mass of the electrons; we get the frequency

$$\omega_0 = \left(\frac{27}{2\pi^2} \right)^{1/2} \frac{Ze}{\sqrt{m a_H^3}} \simeq 4.5 Z \times 10^{16} s^{-1} \; ; \tag{16}$$

it corresponds to an energy $\hbar\omega_0 \simeq 28Z(eV)$, which is in the range of moderate X-rays. This is a plasma frequency. The wavelength $\lambda_0 = 2\pi c/\omega_0 \simeq (4.2/Z) \times 10^{-6} cm$ is still much longer than the dimension of the atom ($c = 3 \times 10^{10}$cm/s is the speed of light). For numerical estimates we use the values $e = 4.8 \times 10^{-10} esu$, $m = 10^{-27} g$, $\hbar = 10^{-27} erg \cdot s$.

The ω_0-vibrations are breathing eigenmodes of the electron cloud; they are subject to a small, natural, damping caused by their radiating energy. Since they imply the motion of the electron cloud as a whole with respect to the nucleus, we may call them giant atomic dipolar vibrations. Higher-order terms in the expansion of the energy with respect to the parameter q gives non-linearities which may lead to ionization [6].

Similar considerations can be made for ions; if the ion charge is $-ze$ the frequency given by equation (16) is modified by $Z \to Z + z/3$, which does not imply an appreciable change [6].

4. SCREENING AND RESONANCE

Let us assume that the atom is placed in an external laser field; in the dipolar approximation only the oscillating electric field $\mathbf{E} = \mathbf{E}_0 \sin\omega t$ is relevant, where $\mathbf{E}_0$ is the amplitude of the electric field and ω is the frequency of the laser radiation. Let us assume that the electric field is directed along the z-axis; it produces a displacement u of the electron cloud as a whole, directed along the z-axis; the local radial displacement is $u\cos\theta$, where θ is the angle made by the radius with the z-axis; the integration of $u^2 \cos^2\theta$ over θ in the total energy (equation (15)) gives a factor $1/3$, which means that the eigenfrequency is changed from ω_0 given by equation (16) to

$\omega_0/\sqrt{3}$ ($\simeq 15Z(eV)$). We write the square of this frequency as

$$\omega_0^2 = \frac{9}{2\pi^2}\frac{Z^2e^2}{ma_H^3} = \frac{4\pi\overline{n}e^2}{m} \ , \tag{17}$$

where the average density is $\overline{n} = (9/8\pi^3)Z^2/a_H^3$. Indeed, the average density is given by $\overline{n} = \overline{k}_F^3/3\pi^2$, which, using $\overline{k}_F = \pi a_H q^2/4$ and q given by equation (14), becomes $\overline{n} = (3\pi^2/16)Z^2/a_H^3$; it is similar with the average density given by equation (17). We can see that the average density corresponds to the number of electrons Z confined to a volume with the dimension of the order $a_H/Z^{1/3}$ (screening wavelength). Making use of the eigenfrequency given by equation (17), we can write the equation of motion for the displacement u as

$$\ddot{u} + \omega_0^2 u = -\tfrac{eE_0}{m}\sin\omega t \ ; \tag{18}$$

the solution of this equation (with vanishing initial conditions) is

$$u = \tfrac{eE_0}{m(\omega^2-\omega_0^2)}\left(\sin\omega t - \tfrac{\omega}{\omega_0}\sin\omega_0 t\right) \ . \tag{19}$$

Let us assume an assembly of electrical charges q at equilibrium, subject to a local displacement $\mathbf{u}$; such a displacement produces a density change $\delta n = -ndiv\mathbf{u}$, where n is the equilibrium density; it follows that we have a charge density imbalance $\delta\rho = -nqdiv\mathbf{u}$ and a current density $\delta\mathbf{j} = nq\dot{\mathbf{u}}$; the Gauss equation reads $div\mathbf{E} = -4\pi nqdiv\mathbf{u}$, where $\mathbf{E}$ is the electric field generated by this charge imbalance; or $div(\mathbf{E} + 4\pi nq\mathbf{u}) = 0$. Therefore, the polarization of the assembly is $\mathbf{P} = nq\mathbf{u}$. If the time variations of $\mathbf{u}$ are slow, *i.e.* if the frequency ω of the displacement $\mathbf{u}$ is such that $\omega \ll c/l$, where l is the dimension of the assembly, then the Gauss equation has the solution $\mathbf{E}_i = -4\pi\mathbf{P} = -4\pi nq\mathbf{u}$; $\mathbf{E}_i$ is the internal (depolarizing) field. We note that this field appears even if the displacement is uniform, due to the variation of the displacement at the surface of the assembly (if any); it is a dipolar field. For atoms, it is due to the displacement of the electron cloud with respect to the atomic nucleus [7]. We apply these considerations to the displacement given by equation (19), with $q = -e$ and $n = \overline{n}$; we get the polarization

$$P = -\tfrac{\overline{n}e^2E_0}{m(\omega^2-\omega_0^2)}\left(\sin\omega t - \tfrac{\omega}{\omega_0}\sin\omega_o t\right) \tag{20}$$

(whence we can derive the dynamic polarizability of the atom), the internal electric field

$$E_i = \tfrac{\omega_0^2E_0}{\omega^2-\omega_0^2}\left(\sin\omega t - \tfrac{\omega}{\omega_0}\sin\omega_o t\right) \tag{21}$$

and the total electric field inside the atom

$$E_t = E_i + E = \frac{\omega^2}{\omega^2-\omega_0^2}E_0\sin\omega t - \frac{\omega\omega_0}{\omega^2-\omega_0^2}E_0\sin\omega_0 t \ ; \tag{22}$$

for $\omega \ll \omega_0$, it can be written as

$$E_t \simeq -\frac{\omega^2}{\omega_0^2}E_0 \sin\omega t + \frac{\omega}{\omega_0}E_0 \sin\omega_0 t \ . \tag{23}$$

We can see that the electric feld of the optical-laser radiation, with frequency $\omega \ll \omega_0$, is appreciably reduced by the screening caused by the motion of the atomic electron cloud; its amplitude is of the order $\simeq (\omega^2/\omega_0^2)E_0$; for $\omega = 10^{15}s^{-1}$ and $\omega_0 = 2.6 \times 10^{16}Zs^{-1}(\simeq 15 \times ZeV)$ the amplitude of the total field becomes $\simeq (10^{-3}/Z^2)E_0$, which, for heavy atoms, is much smaller than the amplitude E_0 of the external field. At the same time, we notice the occurrence inside the atom of an electric field oscillating with the (higher) eigenfrequency ω_0, induced by the external field; its amplitude is reduced by a factor $\simeq (\omega/\omega_0)E_0$. We note that the use of various averaging procedures does not affect the result, since the restoring force per unit mass in the equation of motion (18) is $\omega_0^2 u = eE_i/m$, irrespective of the various values of the average density, a condition satisfied by equations (19) and (21).

The frequency $\omega_0 = 2.6 \times 10^{16}Zs^{-1}$ (equation (17)) corresponds to a radiation wavelength $\simeq (7.25/Z) \times 10^{-6}cm$; the dipolar approximation is still valid for such wavelengths. It is easy to see that the limit $\omega \to \omega_0$ in equation (22) gives

$$E_r = \frac{1}{2}(\sin\omega_0 t + \omega_0 t \cos\omega_0 t) \times E_0 \ ; \tag{24}$$

we can see that the total field increases indefinitely at resonance in a short time interval, as expected. It arises from the internal field generated by the electrons, displaced as a whole (also, the displacement, the polarization and the internal field given by equations (19)-(21) are very large at resonance). Therefore, we may consider a high electric field acting upon the nucleus, oscillating with the (high) eigenfrequency ω_0 of the electron cloud.

5. LOCAL DYNAMICS OF THE ELECTRON CLOUD

For higher values of Z (higher frequencies ω_0) local variations of the electron cloud may appear, which bring corrections to the global dynamics described above (which assumed that the electrons move as a whole). This amounts to go beyond the dipolar approximation in treating the atomic electron cloud. It is worth noting in this case that the magnetic field must be included in the dynamics of the electron; in fact, the electromagnetic field behaves in this case as a collection of photons (*e.g.*, X-rays or gamma rays), and their interaction with the electrons is quantum-mechanical; in addition, we note that the linearized Thomas-Fermi theory is not valid for distances too close to the atomic nucleus, where quantum-mechanical corrections should be included. Consequently, we may leave aside such small spatial variations of the electron density. On the other hand, we notice that the number of electrons inside the sphere with radius R is $Z(1-2/e) \simeq 0.27Z$, while the number of electrons outside this sphere is $2Z/e \simeq 0.73Z$; therefore, we can see that it is meaningful to consider electromagnetic fields with small spatial variations (long wavelengths), which affect mainly the tail of the electron density in heavy atoms; in this case, we may neglect the magnetic field, a treatment which may be termed a quasi-dipolar approximation. The resonance described above corresponds to the dipolar approximation, or, at most, to the quasi-dipolar approximation. We present below a description of the local dynamics of the atomic electron cloud in the quasi-dipolar approximation.

The second-order variation of the total energy E given by equaton (3) is

$$\delta E^{(2)} = \frac{1}{2}\int d\mathbf{r}\frac{\partial^2 \varepsilon_{kin}}{\partial n^2}(\delta n)^2 + \frac{1}{2}e^2\int d\mathbf{r}d\mathbf{r}'\frac{\delta n(\mathbf{r})\delta n(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \ , \tag{25}$$

where the derivatives are taken for $n = n_0$. We represent the density variations as $\delta n = -n_0 div\mathbf{u}$, where $\mathbf{u}(t,\mathbf{r})$ is a displacement field; these variations ensure the conservation of the total number of electrons; the second-order variation of the total energy becomes

$$\begin{gathered}\delta E^{(2)} = \tfrac{1}{2}\int d\mathbf{r}\tfrac{\partial^2 \varepsilon_{kin}}{\partial n^2}n_0^2(div\mathbf{u})^2 + \\ +\tfrac{1}{2}e^2\int d\mathbf{r}d\mathbf{r}'n_0(r)n_0(r')\tfrac{div[\mathbf{u}(\mathbf{r})]\cdot div[\mathbf{u}(\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|} \ ;\end{gathered} \tag{26}$$

together with the kinetic energy

$$T = \frac{m}{2}\int d\mathbf{r} n_0 \dot{u}^2 \tag{27}$$

of the displacement field $\mathbf{u}$, we get the Lagrangian of this field

$$\begin{gathered}L = \tfrac{m}{2}\int d\mathbf{r}n_0\dot{u}^2 - \tfrac{1}{2}\int d\mathbf{r}\tfrac{\partial^2 \varepsilon_{kin}}{\partial n^2}n_0^2(div\mathbf{u})^2 - \\ -\tfrac{1}{2}e^2\int d\mathbf{r}d\mathbf{r}'n_0(r)n_0(r')\tfrac{div[\mathbf{u}(\mathbf{r})]\cdot div[\mathbf{u}(\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|} \ .\end{gathered} \tag{28}$$

Therefore, the equation of motion for the field $\mathbf{u}$ is

$$\begin{gathered}mn_0\ddot{\mathbf{u}} - grad\left[\left(\tfrac{\partial^2 \varepsilon_{kin}}{\partial n^2}\right)n_0^2 div\mathbf{u}\right] - \\ -e^2 grad\left[n_0\int d\mathbf{r}'\tfrac{n_0(r')div[\mathbf{u}(\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|}\right] = n_0\mathbf{F}_{ex} \ ,\end{gathered} \tag{29}$$

where $\mathbf{F}_{ex}$ is the external force; or

$$\begin{gathered}m\ddot{\mathbf{u}} - \tfrac{(3\pi^2)^{2/3}\hbar^2}{3mn_0}grad\left(n_0^{5/3}div\mathbf{u}\right) - \\ -\tfrac{e^2}{n_0}grad\left[n_0\int d\mathbf{r}'\tfrac{n_0(r')div[\mathbf{u}(\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|}\right] = \mathbf{F}_{ex} \ .\end{gathered} \tag{30}$$

In the quasi-dipolar approximation (long-wavelength limit) we may take for the density n_0 the average density $\overline{n}=\overline{k}_F^3/3\pi^2=3Z^2/16\pi^3a_H^3$ (according to equation (17)); equation (30) becomes

$$m\ddot{\mathbf{u}}-\frac{(3\pi^2)^{2/3}\hbar^2}{3m}n_0^{2/3}grad\cdot div\mathbf{u}-e^2n_0grad\int d\mathbf{r}'\frac{div[\mathbf{u}(\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|}=\mathbf{F}_{ex}\ ; \tag{31}$$

for an external plane wave $\mathbf{F}_{ex}=-e\mathbf{E}_0e^{-i\omega t+i\mathbf{kr}}$ $(c/\omega\gg a_H/Z^{1/3})$ the solution is $\mathbf{u}=\mathbf{u}_0e^{-i\omega t+i\mathbf{kr}}$; the amplitude of the displacement field is given by

$$\omega^2\mathbf{u}_0-\omega_0^2\frac{(\mathbf{ku}_0)\mathbf{k}}{k^2}-v^2(\mathbf{ku}_0)\mathbf{k}=e\mathbf{E}_0\ , \tag{32}$$

where

$$\omega_0^2=\frac{3}{4\pi^2}\frac{Z^2e^2}{ma_H^3} \tag{33}$$

and

$$v^2=\frac{3^{1/3}}{(16\pi)^{2/3}}\frac{\hbar^2Z^{4/3}}{m^2a_H^2}\ . \tag{34}$$

We can see that the longitudinal displacement is

$$\mathbf{u}_0=\frac{e\mathbf{E}_0}{\omega^2-\Omega^2} \tag{35}$$

with the frequency of the eigenmodes given by

$$\Omega^2=\omega_0^2+v^2k^2\ ; \tag{36}$$

these modes are dispersive plasmons; they are the breathing modes derived above (now dispersive) with corrections arising from local dynamics. The plasma frequency ω_0 given by equation (33) is comparable with the plasma frequency given by equations (16) and (17); it arises from the Coulomb repulsion (at equilibrium; the Fourier transform of the Coulomb potential is involved in its expression). The velocity v $(v\ll c)$ arises from the variation of the kinetic energy. Equation (35) shows that the screening is present, as described above; the full solution of equation (31) includes the excitation of the eigenmodes too. From equation (32) we can see that the transverse modes are free. Therefore, the screening and the resonance effects of the dipolar approximation to the global vibrations of the electron cloud are preserved in the quasi-dipolar approximation, slightly modified by the local dynamics of the electron cloud. Also, we note that a global displacement u in equation (31) implies $grad\cdot div\mathbf{u}$ of the order u/a^2, where $a\simeq a_H/Z^{1/3}$; in this case the Coulomb repulsion in equation (31) is vanishing and the kinetic term gives a frequency $\omega\simeq\omega_0$.

6. FAST RATE OF NUCLEAR CHARGE EMISSION

According to the discussion above (equation (24)), we assume an electric field

$$\mathbf{E} = \frac{1}{2}\mathbf{E}_{ex}\left(\omega t \cos\omega t + \sin\omega t\right) \ , \tag{37}$$

where $\mathbf{E}_{ex}$ is the amplitude of the external field and ω stands for $\omega_0 = 2.6 \times 10^{16} Zs^{-1}$ given by equation (17); in order to preserve the dipolar approximation over the dimension a of the nucleus ($a \simeq 10^{-13}cm$), we restrict the time t according to the inequality $\omega t \ll \lambda/a \simeq (7.25/Z) \times 10^7$, where λ is the wavelength corresponding to the frequency ω (ω_0); this inequality ensures a low magnetic field, which may be neglected. We can see that the amplitude of the electric field acting upon the nucleus may reach values as high as $(10^7/Z)\mathbf{E}_{ex}$. The condition $\omega t \ll \lambda/a$ is the dipolar condition in the context of the presence of the factor ωt in the electric field. Indeed, the dipolar condition is $\omega \ll c/a$, which follows by comparing $\frac{1}{c}\frac{\partial E}{\partial t}$ with spatial derivatives of the form $\frac{\partial E}{\partial r}$; for a typical wave-like behavior we get $\frac{\omega}{c}\Delta E$ for the term with the time derivative and $\Delta E/l$ for the latter, where l $(= a)$ is the relevant distance. The inequality $\omega \ll c/a$ is, in fact, $\lambda \gg a$, where we recognize the familiar dipolar condition (compare with the non-relativistic condition $\omega t \gg a/\lambda$). In equation (37), the time variation ΔE acquires an additional factor ωt, such that the dipolar condition reads $\omega(\omega t) \ll c/a$, *i.e.* $\omega t \ll \lambda/a$. In fact, this condition is weak enough to include many oscillations ($\omega t \gg 1$), since $\lambda \gg a$.

Also, we assume a simplified model of atomic nucleus, where the nucleons move in a mean field given by a potential $V(\mathbf{r})$. In the non-relativistic approximation the dipole Hamiltonian of a proton with charge q and mass m is

$$H_d = H_0 - q\mathbf{r}\mathbf{E} \ , \ H_0 = \frac{1}{2m}p^2 + V(\mathbf{r}) \ , \tag{38}$$

where $\mathbf{r}$ denotes the charge position and $\mathbf{p}$ is the charge momentum. We consider the associated Schrödinger equation $i\hbar\partial\psi/\partial t = H_d\psi$ and introduce the unitary transformation

$$\psi = e^{iS_1}\phi \ , \ S_1 = \frac{1}{\hbar}\int_0^t dt' q\mathbf{r}\mathbf{E} = (q\mathbf{r}\mathbf{E}_{ex}/2\hbar\omega)\,\omega t \sin\omega t \ , \tag{39}$$

where we recognize the vector potential $\mathbf{A} = -(c\mathbf{E}_{ex}/2\omega)\omega t \sin\omega t$ ($\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$). The transformation given by equation (39) leads to $\mathbf{p} \to \widetilde{\mathbf{p}} = \mathbf{p} - q\mathbf{A}/c$ and the standard non-relativistic Hamiltonian

$$\widetilde{H}_d = e^{-iS_1}H_d e^{iS_1} = \frac{1}{2m}\left(\mathbf{p} - \frac{q}{c}\mathbf{A}\right)^2 + V(\mathbf{r}) \ , \tag{40}$$

with the associated Schrödinger equation $i\hbar\partial\phi/\partial t = \widetilde{H}_d\phi$. The transformation given by equation (39) is the well-known known Goeppert-Mayer transformation [8].

Let us write

$$\widetilde{H}_d = H_0 - \frac{q}{mc}\mathbf{p}\mathbf{A} + \frac{q^2}{2mc^2}A^2 \tag{41}$$

and continue with the unitary transformations [9]-[12]

$$\begin{gathered}\phi = e^{iS_2}e^{iS_3}\chi \ , \ S_2 = -\frac{q^2}{2\hbar mc^2}\int_0^t dt' A^2 \ , \\ S_3 = \frac{q}{\hbar mc}\int_0^t dt' \mathbf{p}\mathbf{A} = \frac{q}{2\hbar m\omega^2}\mathbf{E}_{ex}\mathbf{p}\left(\omega t\cos\omega t - \sin\omega t\right) \ ;\end{gathered} \tag{42}$$

the generator S_2 is irrelevant for our discussion below, so we give not its explicit expression. These transformations lead to the Schrödinger equation

$$i\hbar\frac{\partial\chi}{\partial t} = \left[\frac{1}{2m}p^2 + \widetilde{V}(\mathbf{r})\right]\chi \tag{43}$$

with the radiation-dressed potential

$$\widetilde{V}(\mathbf{r}) = e^{-\frac{q}{2m\omega^2}(\omega t\cos\omega t - \sin\omega t)\cdot\mathbf{E}_{ex}grad}V(\mathbf{r}) \ . \tag{44}$$

We proceed now to apply these results to the charge emission from the nucleus. Let us assume that the electric field $\mathbf{E}$ is directed along the z-axis. Then, equation (44) gives

$$\begin{gathered}\widetilde{V}(x,y,z) = V + \zeta(t)V_1 + \frac{1}{2!}\zeta^2(t)V_2 + ... = \\ = V\left(x,y,z-\zeta(t)\right) \ ,\end{gathered} \tag{45}$$

where $V_1 = \partial V/\partial z$, $V_2 = \partial^2 V/\partial z^2$, ... and

$$\zeta(t) = \frac{qE_{ex}}{2m\omega^2}\left(\omega t\cos\omega t - \sin\omega t\right) \ . \tag{46}$$

We can see that the potential $\widetilde{V}$ at the position of the charge is the original potential V at the position coordinate $z-\zeta(t)$, as if the charge is displaced in the potential $V(\mathbf{r})$ by $\zeta(t)$ along the z-coordinate [13]. The displacement $\zeta(t)$ given by equation (46) is, in fact, the displacement

$$\zeta(t) = -\frac{q}{m}\int_0^t dt_1\int_0^{t_1} dt_2 E(t_2) \ , \tag{47}$$

where $E(t)$ is the electric field given by equation (37), which shows that the equation of motion for the displacement ζ is the classical equation of motion $m\ddot{\zeta} = -qE$.

It is convenient to write the displacement $\zeta(t)$ given by equation (46) as

$$\begin{gathered}\zeta(t) = \zeta_0 f(\omega t) \ , \ \zeta_0 = \frac{qE_{ex}}{2m\omega^2} \ , \\ f(\omega t) = \omega t\cos\omega t - \sin\omega t \ .\end{gathered} \tag{48}$$

The function $f(\omega t)$ oscillates between ωt and $-\omega t$, increasing in time; with our numerical data the amplitude ζ_0 can be written as

$$\zeta_0 \simeq (1.7 \times 10^{-19}/Z^2)E_{ex} \tag{49}$$

(in cm; for proton mass $m = 2 \times 10^{-24}g$). This amplitude is much smaller than the dimension a of the nucleus; it follows that $\zeta(t)$ may become comparable with a after a long time τ_0 , given by

$$\mid f(\omega\tau_0) \mid = \frac{a}{\zeta_0} \simeq (5.6Z^2 \times 10^5)/E_{ex} \ . \tag{50}$$

For $\omega t \gg 1$ the maximum value of the function $\mid f(\omega t) \mid$ is ωt for $\omega t = n\pi$, where n is any (large) positive integer; from equation (50) we get the time

$$\tau_0 \simeq \frac{a}{\omega\zeta_0} \simeq 2Z \times 10^{-11}/E_{ex} \tag{51}$$

(in s^{-1}). Equation (51) is valid for

$$7.7Z^3 \times 10^{-3} \ll E_{ex} \ll 5.6Z^2 \times 10^5 \tag{52}$$

(in esu); the first inequality in equation (52) is the non-relativistic condition and the second inequality is $\omega\tau_0 \gg 1$. The upper limit in equation (52) is sufficiently high to cover all the relevant external fields. The inequalities in equation (52) imply the inequalities

$$(3.5/Z) \times 10^{-17} \ll \tau_0 \ll (2.6/Z^2) \times 10^{-9} \tag{53}$$

(in s).

The time τ_0 in the range given by equation (53) is much longer than the characteristic time t_0 of the nucleon states; for instance, $t_0 \simeq 10^{-21}s$ for energy $1MeV$. For lower external fields than those indicated in equation (52) the dynamics becomes more complex (due to the presence of the magnetic field) and more slow. Consequently, since $\tau_0 \gg t_0$, the nucleons suffer a re-arrangement (re-configuration) process, the nuclear mean field changes accordingly, and the dynamics amounts to the adiabatic introduction of the external field. The stationary nucleon states are slowly modified in time, the protons do not radiate electromagnetic energy and all the energy given by the external field is absorbed by the nucleus.

We pass now to estimate the energy loss of the electric field during the displacement of the charge. The electric field given by equation (37) can be written as $E(t) = (E_{ex}/2)g(\omega t)$, where $g(\omega t) = \omega t \cos\omega t + \sin\omega t$. The function $g(\omega t)$ oscillates between ωt and $-\omega t$ (for $\omega t = \pi n$, n being any integer greater than 0), with zeros placed approximately at $x_n = \omega t_n = (2n+1)\pi/2$ for large n. The electric field passes through two neighbouring zeros during a semi-period; during this time the electric field is felt by the nucleons as a (non-vanishing) perturbation, which lasts $\Delta(\omega t) = \pi$; the rapid re-arrangement processes accommodate this perturbation during this interval of time. Therefore, we compute the energy transferred to the nucleons in a semi-period and sum these amounts of energy. The mechanical work done by the electric field in a semi-period from $\omega t_n = (2n+1)\pi/2$ to $\omega t_{n+1} = \omega t_n + \pi$ is given by

$$\begin{gathered} W_n = q\int_{t_n}^{t_{n+1}} dt E(t)\dot{\zeta}(t) = \frac{q^2}{m}\int_{t_n}^{t_{n+1}} dt E(t)\int_0^t dt' E(t') = \\ = \frac{q^2E_{ex}^2}{4m\omega^2}\int_{x_n}^{x_{n+1}} dx(x\cos x + \sin x)\int_0^x dx'(x'\cos x' + \sin x') = \\ = \frac{q^2E_{ex}^2}{8m\omega^2}x^2\sin^2 x\,|_{x_n}^{x_{n+1}} = \frac{\pi^2q^2E_{ex}^2}{4m\omega^2}(n+1)\ , \end{gathered} \tag{54}$$

where $x = \omega t$. The summation over n gives the total mechanical work

$$W \simeq \frac{\pi^2 q^2 E_{ex}^2}{8m\omega^2}(\omega\tau)^2 \tag{55}$$

for $\omega\tau \gg 1$. When this energy is greater than the binding energy E_b ($\simeq \hbar/t_0$) of the nucleon, the nucleon is ejected from the nucleus. The total energy gained by the protons is ZW; it is shared by all the nucleons, so we are left with ZW/A on the average for one nucleon, where A is the mass number. It follows that the ejection time is

$$\tau \simeq \frac{2}{\pi q E_{ex}}\left[mE_b(A/Z)\right]^{1/2}\ ; \tag{56}$$

with our numerical data we get

$$\tau \simeq 2\times 10^{-3}\frac{\left[E_b(A/Z)\right]^{1/2}}{E_{ex}} \tag{57}$$

(in s); it is useful to give this formula for binding energies in eV,

$$\tau \simeq (2.5\times 10^{-9})\frac{\left[E_b(eV)(A/Z)\right]^{1/2}}{E_{ex}(esu)}\ . \tag{58}$$

The rate of nucleon ejection from the nucleus is $w = 1/\tau$. We can see that this rate is enhanced to a large extent, irrespective of whether the nucleus is stable or spontaneously decaying; the result may be applied to spontaneous alpha decay, where the pre-formation process of the alpha particle reduces appreciably the parameter E_b in equation (58) and contributes to the enhancement of the decay rate. This is in contrast with the small effects of low external fields on the spontaneous alpha decay rate [14]. Also, we note that the ejection process holds both for protons and neutrons, as a consequence of the energy sharing. Moreover, the nucleon capture is favoured at resonance.

However, the high resonant electric field acts on the electrons too; the above calculations are valid in this case, the ionization time being given by

$$\tau_{el} \simeq (5.6 \times 10^{-11}) \frac{[E_{be}(eV)]^{1/2}}{E_{ex}(esu)} \ , \tag{59}$$

where E_{be} is the electron binding energy. We can see that the ionization time τ_{el} is much shorter than the time of nucleon emission τ. During the ionization process the resonance frequency changes (according to $Z \to Z + z/3$, where z is the ion charge; see discussion above), and the ionization and the nucleon emission are stopped; by tuning the external frequency such as to maintain the resonant regime, the ionization process may continue, up to a maximum ion charge of the order $z \simeq Z^{2/3}$, when the Thomas-Fermi theory is not valid anymore; at this point, the screening and the resonant regime disappear [13]. Since the nucleon emission time τ is much longer than the ionization time, we can see that, practically, the fast nucleon emission cannot be achieved.

7. CONCLUSION

In conclusion, it is shown in this paper that heavy atoms (or ions) may exhibit giant dipolar vibrations of their electron cloud with a frequency of the order $\omega_0 = 15ZeV$ (in the moderate X-ray range); the description of these eigenmodes is made here by means of a linearized Thomas-Fermi theory, which provides accurate binding energies. The dynamics of these vibrations implies the displacement of the electron cloud as a whole, with respect to the atomic nucleus (dipolar approximation). For shorter wavelengths (quasi-dipolar approximation) the local dynamics of the electron cloud reveals dispersive (propagating) plasmons with the same basic frequency ω_0. As a consequence of the high frequency ω_0 of its dipolar eigenmodes, the electron cloud screens to a great extent the electric field of an external optical-laser radiation, such that the electric field seen by the atomic nucleus is greatly reduced. At the same time, the presence of these eigenmodes indicates a resonant regime for an external field with frequencies close to the frequency ω_0, which enhances appreciably the internal electric field acting upon the electrons and the nucleus; this is the internal electric field of the electrons coupled to the nucleus. The (high) rates of atom ionization and nucleon ejection from the nucleus under the action of such an enhanced electric field are estimated in this paper. It is shown that the much faster ionization process spoils the resonant regime very quickly and precludes, in fact, the fast nucleon emission.

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