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Electric flow through an ideal ferromagnet–superconductor junction

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A B S T R A C T

It is investigated the possibility of controlling the electric flow through a ferromagnet–superconductor junction by spin polarization, within a simple, ideal model of a perfect ferromagnetic–superconductor junction. The ferromagnetic and superconducting properties as well as the Andreev reflection are briefly reviewed and the electrical resistance of the junction is computed both in the diffusive and ballistic regime for the ferromagnetic sample. It is shown that the resistance of the junction increases with increasing magnetization, including both positive or negative jumps on passing from the ballistic to the diffusive regime.

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1. Introduction

In spite of numerous past and recent extensive attempts [1–19], which provide valuable experimental and theoretical information, we still lack a simple, operational model for a basic understanding of the electric flow through a ferromagnet–superconductor junction. It is the aim of this paper to attempt a formulation of such a model. We consider an ideal, perfect ferromagnet–superconductor junction, of sufficient in-plane extension, as one realized by an insulating non-magnetic thin film [20], such as to avoid the complications arising from proximity effects [21]. It is assumed that the ferromagnet and the s-wave superconductor are homogeneous. We compute herein the electrical resistance of such a junction, affected mainly by the well-known Andreev reflection, both in the diffusive regime and ballistic regime for the ferromagnetic sample. It is shown that the junction resistance increases with increasing magnetization, including both positive and negative jumps on passing from the ballistic into the diffusive regime.

2. Ferromagnet

We adopt a simple Fermi liquid picture for the charge carriers in the ferromagnet; the charge carriers are assumed to be electrons, with an isotropic single-particle energy spectrum $\varepsilon(\mathbf{k})$ labelled by the wavevector $\mathbf{k}$ in the normal (non-ferromagnetic) state; their number is given by $N = Vk_F^3/3\pi^2$, where k_F denotes the Fermi wavevector and V is the volume of the sample; the quasi-particles have a Fermi velocity

$$v_n = \partial\varepsilon/\hbar\partial k|_{k=k_F} = \hbar k_F/m^*, \tag{1}$$

where m^* is their effective mass (and $\hbar$ is Planck's constant); the Fermi level is given by $\mu_n = \varepsilon(k_F)$ (which defines the Fermi surface by fixing μ_n from the number of particles).

Below a critical temperature T_m the ferromagnetic state begins to set up; it is characterized by a temperature-dependent gap Δ_m in the single-particle energy spectrum, which reads now

$$\begin{aligned} \varepsilon_1(\mathbf{k}) &= -\Delta_m/2 + \varepsilon(k), \\ \varepsilon_2(\mathbf{k}) &= \Delta_m/2 + \varepsilon(k), \end{aligned} \tag{2}$$

as corresponding to spin up (label 1) and spin down (label 2), respectively. The number of electrons is given now by

$$N = Vk_{F1}^3/6\pi^2 + Vk_{F2}^3/6\pi^2, \tag{3}$$

and the magnetization reads

$$M = \mu_B(Vk_{F1}^3/6\pi^2 - Vk_{F2}^3/6\pi^2), \tag{4}$$

where $\mu_B = e\hbar/2mc$ is Bohr's magneton (with usual notations – e is the electron charge, m is the electron mass and c denotes the

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velocity of light). It is convenient to introduce a reduced magnetization defined as $m = M/\mu_B N$, which leads to

$$k_{F1} = k_F(1+m)^{1/3},\quad k_{F2} = k_F(1-m)^{1/3} \tag{5}$$

for the two Fermi wavevectors in Eqs. (3) and (4); obviously, the relative magnetization varies between 0 and 1, $0 < m < 1$. In the ferromagnetic state there are two types of quasi-particles, corresponding to spin up and spin down, moving with velocities

$$v_{F1,2} = v_{1,2} = \partial\varepsilon_{1,2}/\hbar\partial k|_{k=k_{F1,2}} = \hbar k_{F1,2}/m^* = v_n(1 \pm m)^{1/3}; \tag{6}$$

this is the main point through which the dependence on magnetization is introduced in the thermal or electric flows through the ferromagnet–superconductor junction, together with the m-dependence of the Fermi wavevectors $k_{F1,2}$ given by Eq. (5). The Fermi level of the ferromagnetic state is given by

$$\mu_m = -\Delta_m/2 + \varepsilon(k_{F1}) = \Delta_m/2 + \varepsilon(k_{F2}), \tag{7}$$

hence,

$$\Delta_m = \varepsilon\left(k_F(1+m)^{1/3}\right) - \varepsilon\left(k_F(1-m)^{1/3}\right). \tag{8}$$

This equation determines the temperature dependence of the magnetization m. Indeed, the ferromagnetic gap has a typical dependence $\Delta_m = \Delta_{m0}(1 - T/T_m)^{1/2}$ on temperature T close to T_m; for lower temperatures its temperature slope is vanishing, as for a typical mean-field theory. Since $k_{F1,2}$ have a slow dependence on magnetization (except for $k_{F2} = k_F(1-m)^{1/3}$ for $m \sim 1$), we may use the expansion

$$\Delta_m = \frac{2}{3}\hbar v_n k_F m \tag{9}$$

for Eq. (8); similarly, the Fermi level reads

$$\mu_m = -\Delta_m/2 + \mu_n + \frac{1}{3}\hbar v_n k_F m + O(m^2) = \mu_n + O(m^2), \tag{10}$$

whence one can see that it does not change appreciably in the ferromagnetic state. These m-expansions are used, for small values of m.

3. Superconductor

For later convenience we review here briefly the Gorkov equations for a simple model of superconductivity [22]. Let $c_{\mathbf{k}}$ be the destruction operator of a quasi-particle state in a normal conductor; it obeys the Heisenberg's equation

$$i\hbar\partial c_{\mathbf{k}}/\partial t = \varepsilon(\mathbf{k})c_{\mathbf{k}} = [\mu_n + \hbar\mathbf{v}_n(\mathbf{k} - \mathbf{k}_F)]c_{\mathbf{k}}, \tag{11}$$

or, introducing the field operator $\psi(\mathbf{r}) = \left(1/\sqrt{V}\right)\Sigma_{\mathbf{k}} c_{\mathbf{k}} e^{i\mathbf{kr}}$ for $\mathbf{k}$ close to $\mathbf{k}_F$,

$$i\hbar\partial\psi/\partial t = (\mu_n - \hbar\mathbf{v}_n\mathbf{k}_F - i\hbar\mathbf{v}_n\partial/\partial\mathbf{r})\psi. \tag{12}$$

We write the effective quasi-particle interaction as

$$H_{int} = \frac{1}{2}\int d\mathbf{r}d\mathbf{r}' \cdot g(\mathbf{r}-\mathbf{r}')\psi^+_\alpha(\mathbf{r})\psi^+_\beta(\mathbf{r}')\psi_\beta(\mathbf{r}')\psi_\alpha(\mathbf{r}), \tag{13}$$

where α, β are spin labels and $g(\mathbf{r}-\mathbf{r}')$ is an interaction potential (here chosen as spin-independent for simplicity); Eq. (12) reads now

$$i\hbar\partial\psi_\alpha/\partial t = (\mu_n - \hbar\mathbf{v}_n\mathbf{k}_F - i\hbar\mathbf{v}_n\partial/\partial\mathbf{r})\psi_\alpha + \int d\mathbf{r}' \cdot g(\mathbf{r}-\mathbf{r}')\psi^+_\beta(\mathbf{r}')\psi_\beta(\mathbf{r}')\psi_\alpha(\mathbf{r}); \tag{14}$$

such an interaction may lead to superconductivity, by a macroscopic occupation $\langle\psi_\alpha(\mathbf{r})\psi_\beta(\mathbf{r})\rangle \neq 0$ of the pair states. Here we assume a δ-type interaction $g(\mathbf{r}-\mathbf{r}') = g\delta(\mathbf{r}-\mathbf{r}')$, which makes the pair wavefunction a spin singlet $\langle\psi_{-\alpha}(\mathbf{r})\psi_\alpha(\mathbf{r})\rangle$, and define $F_\alpha = g\langle\psi_{-\alpha}(\mathbf{r})\psi_\alpha(\mathbf{r})\rangle$. According to its definition $F_{-\alpha} = -F_\alpha$, while $F^*_{-\alpha} = F_\alpha$ by time reversal symmetry; it follows that we may define the superconducting gap parameter $\Delta_\alpha = \Delta^*_\alpha = -\Delta_{-\alpha}\ (>0)$ through $F_\alpha = i\Delta_\alpha$. In addition, we include the basic time-dependence $\psi_\alpha \sim e^{-i\mu_n t/\hbar}$ in Eq. (14) (and subtract $\mu_n N$ from the hamiltonian). Eq. (14) becomes

$$i\hbar\partial\psi_\alpha/\partial t = (-\hbar\mathbf{v}_n\mathbf{k}_F - i\hbar\mathbf{v}_n\partial/\partial\mathbf{r})\psi_\alpha + i\Delta_\alpha(\mathbf{r})\psi^+_{-\alpha}(\mathbf{r}); \tag{15}$$

in addition we assume a constant $\Delta_\alpha = \Delta_\alpha(\mathbf{r})$ as for a s-wave pair state. The above equation leads to

$$i\hbar\partial c_{\mathbf{k}\alpha}/\partial t = \hbar v_n(k-k_F)c_{\mathbf{k}\alpha} + i\Delta_\alpha c^+_{-\mathbf{k}-\alpha},\quad -i\hbar\partial c^+_{-\mathbf{k}-\alpha}/\partial t = \hbar v_n(k-k_F)c^+_{-\mathbf{k}-\alpha} + i\Delta_\alpha c_{\mathbf{k}\alpha} \tag{16}$$

for $\mathbf{k}$ along $\mathbf{v}_n$, which are solved for the well-known superconducting spectrum

$$\varepsilon_\pm(\mathbf{k}) = \mu_n \pm \sqrt{\Delta^2_\alpha + \hbar^2 v^2_n(k-k_F)^2}, \tag{17}$$

with (the original) operators $c_{\mathbf{k}\alpha} \sim e^{-i(\mu_n+\hbar\omega)t/\hbar}$, $c^+_{-\mathbf{k}-\alpha} \sim e^{i(\mu_n-\hbar\omega)t/\hbar}$ and $\hbar\omega = \pm\sqrt{\Delta^2_\alpha + \hbar^2 v^2_n(k-k_F)^2}$; the lower branch joints smoothly the rest of the original energy spectrum, so that the superconducting Fermi level is given by

$$\mu_s = \mu_n - \Delta_\alpha. \tag{18}$$

It is convenient to measure the wavevectors with respect to the Fermi wavevector, *i.e.* $\varepsilon_\pm(\mathbf{k}) = \mu_n \pm \sqrt{\Delta^2_\alpha + \hbar^2 v^2_n k^2}$, so that the solutions of Eq. (16) read

$$c_{\mathbf{k}\alpha} = u_k b_{\mathbf{k}\alpha} + i v_k b^+_{-\mathbf{k}-\alpha},\quad c_{-\mathbf{k}-\alpha} = u_k b_{-\mathbf{k}-\alpha} - i v_k b^+_{\mathbf{k}\alpha}, \tag{19}$$

where $u_k = |\cos\theta_k|$, $v_k = |\sin\theta_k|$, $\tan\theta_k = -(E_k - \hbar v_n k)/\Delta_\alpha$, $E_k = \hbar\omega = \sqrt{\Delta^2_\alpha + \hbar^2 v^2_n k^2}$, or

$$u^2_k = \frac{1}{2}(1 + \hbar v_n k/E_k),\quad v^2_k = \frac{1}{2}(1 - \hbar v_n k/E_k) \tag{20}$$

for the energy branch $\varepsilon(\mathbf{k}) = \mu_n + sgn(k)E_k$, and $u_k v_k = \Delta_\alpha/2E_k$. The self-consistency condition $\Delta_\alpha = -ig\langle\psi_{-\alpha}(\mathbf{r})\psi_\alpha(\mathbf{r})\rangle$ leads to the well-known gap equation

$$1 = -\frac{gk^2_F}{2\pi^2}\int dk\frac{\tanh\beta E_k}{2E_k}, \tag{21}$$

whence one may see that interaction must be attractive, *i.e.* $g < 0$; one obtains the well-known critical temperature

$$T_c \simeq \hbar v_n k_c e^{-1/Dg}, \tag{22}$$

where $D = k^2_F/2\pi^2\hbar v_n$ is the density of states (per spin) at the Fermi surface, k_c is a wavevector cutoff (the scale energy $\hbar v_n k_c$ is of the order of the Debye energy $\hbar\omega_D$ for a phonon–electron superconducting interaction), and the sign of the interaction has been changed. Similarly, one obtains the temperature dependence of the gap $\Delta_\alpha = \Delta_{\alpha 0}(1 - T/T_c)^{1/2}$ for temperatures close to the critical temperature, the gap $\Delta_{\alpha 0}$ being given by $\Delta_{\alpha 0} \simeq 2\hbar v_n k_c e^{-1/Dg}$. We may neglect the temperature dependence of the superconducting gap and Fermi level Eq. (18), assuming the temperature is sufficiently low for superconductivity be well developed.

4. Andreev reflection

Further on, we give a brief description of the Andreev reflection [23]. We focus first on the superconducting Eqs. (15) and (16), where we drop out the label n for the Fermi velocity $\mathbf{v}_n$, and the superconducting gap Δ_α is assumed constant and positive. In addition we introduce the one-particle amplitudes

$$\varphi_\alpha = \langle 0|\psi_\alpha|1\alpha\rangle, \quad \chi_\alpha = \langle 0|\psi^+_{-\alpha}|1\alpha\rangle, \tag{23}$$

where $|1\alpha\rangle$ is an excited one-particle state. The amplitude φ_α is the wavefunction of a $\mathbf{k}, \alpha$-quasi-particle, while χ_α represents a $-\mathbf{k}, -\alpha$-quasi-hole in a superconducting pair. Indeed, for instance,

$$\varphi_\alpha = \langle 0|\psi_\alpha|1\alpha\rangle = \frac{1}{\sqrt{V}}\sum e^{i\mathbf{k}'\mathbf{r}}\langle 0|\mathbf{c}_{\mathbf{k}'\alpha'}\mathbf{c}^+_{\mathbf{k}\alpha}|0\rangle = \frac{1}{\sqrt{V}}e^{i\mathbf{kr}} \tag{24}$$

is the wavefunction of a quasi-particle, and similarly for χ_α for the superconducting state. The connection of the amplitudes above with the canonical transform Eq. (19) is obvious; Eq. (15) reads now

$$\begin{aligned} &i\hbar\partial\varphi_\alpha/\partial t = (-\hbar\mathbf{v}\mathbf{k}_F - i\hbar\mathbf{v}\partial/\partial\mathbf{r})\varphi_\alpha + i\Delta_\alpha\chi_\alpha, \\ &-i\hbar\partial\chi_\alpha/\partial t = (-\hbar\mathbf{v}\mathbf{k}_F - i\hbar\mathbf{v}\partial/\partial\mathbf{r})\chi_\alpha + i\Delta_\alpha\varphi_\alpha, \end{aligned} \tag{25}$$

and it is easy to check up the continuity equation

$$\partial\left(|\varphi_\alpha|^2 + |\chi_\alpha|^2\right)\Big/\partial t + \mathbf{v}\partial\left(|\varphi_\alpha|^2 - |\chi_\alpha|^2\right)\Big/\partial\mathbf{r} = 0 \tag{26}$$

for each spin orientation α. Since $\Delta_{-\alpha} = -\Delta_\alpha$ and, similarly, $\chi_{-\alpha} = -\chi_\alpha$ according to its definition, Eq. (25) are the same for each spin orientation, so we may drop out the label α for the superconducting gap and amplitudes. Eq. (26) tells that the localization probability $|\varphi|^2 + |\chi|^2$ of a quasi-particle in the superconducting state changes in time according to the divergence of the current $\mathbf{j} = \mathbf{v}\left(|\varphi|^2 - |\chi|^2\right)$. The current $\mathbf{j}$ consists of two contributions, $\mathbf{v}|\varphi|^2$ which flows along the velocity, and $-\mathbf{v}|\chi|^2$ which flows in the opposite direction; this latter contribution is the Andreev reflection. The quasi-particles encounter a potential barrier on their attempt of entering a superconductor, and, consequently, they are reflected by the superconductor gap, as well as transmitted through. Eq. (25) can also be written as

$$\begin{aligned} &\hbar(\omega + \mathbf{v}\mathbf{k}_F + i\mathbf{v}\partial/\partial\mathbf{r})\varphi = i\Delta\chi, \\ &\hbar(\omega - \mathbf{v}\mathbf{k}_F - i\mathbf{v}\partial/\partial\mathbf{r})\chi = -i\Delta\varphi \end{aligned} \tag{27}$$

for $\varphi, \chi \sim e^{-i\omega t}$. In addition, we remove the $\mathbf{v}\mathbf{k}_F$-term by introducing

$$\xi = e^{-i\mathbf{k_F r}}\varphi, \quad \eta = e^{-i\mathbf{k_F r}}\chi, \tag{28}$$

so that Eq. (27) become

$$\begin{aligned} &\hbar(\omega + i\mathbf{v}\partial/\partial\mathbf{r})\xi = i\Delta\eta, \\ &\hbar(\omega - i\mathbf{v}\partial/\partial\mathbf{r})\eta = -i\Delta\xi; \end{aligned} \tag{29}$$

for $\xi, \eta \sim e^{i\mathbf{kr}}$ one can check out the superconducting spectrum $\hbar\omega = \pm\sqrt{\Delta^2 + \hbar^2(\mathbf{vk})^2}$, while the reduced wavefunctions are given by

$$\begin{aligned} &\xi = \frac{C_\alpha}{\sqrt{2}}\sqrt{1 + \mathbf{vk}/\omega}e^{i\mathbf{kr}}, \\ &\eta = \frac{-iC_\alpha}{\sqrt{2}}\sqrt{1 - \mathbf{vk}/\omega}e^{i\mathbf{kr}}, \end{aligned} \tag{30}$$

where C_α is a constant, $\hbar\mathbf{vk} = \pm\sqrt{(\hbar\omega)^2 - \Delta^2}$ and $\hbar\omega > \Delta$ (otherwise the quasi-particle does not propagate, and the wavefunctions decay exponentially with the distance). The wavevectors $\mathbf{k}$ in Eq. (30) are small in comparison with the Fermi wavevector $\mathbf{k}_F$ (where the velocity $\mathbf{v}$ is calculated), so the wavefunctions ξ, η vary slowly in space. The constant C_α bears temporarily a spin label, for the sake of generality. Before passing to the ferromagnet–superconductor junction we note that the transmitted (tunneling) current in the superconductor is

$$\mathbf{j}_{t\alpha} = \mathbf{v}\left(|\xi|^2 - |\eta|^2\right) = |C_\alpha|^2\mathbf{v}(\mathbf{vk}/\omega). \tag{31}$$

We may pass now to the Andreev reflection in a ferromagnet–superconductor junction. We assume that the Fermi level in ferromagnet is close to the Fermi level in superconductor, as for a perfect contact. Under these circumstances Eq. (25) hold for the ferromagnet by simply dropping out the superconducting-gap contribution. Obviously, the remaining part depends on the spin orientation, through both the Fermi velocity and Fermi wavevector. In addition, χ vanishes for the non-superconducting sample (indeed, $\eta \to 0$ in Eq. (30) for $\Delta \to 0$, as expected), so that we may write down Eq. (27) (or (29)) as

$$[\omega + \mathbf{v}_{1,2}(\mathbf{k}_{F1,2} - \mathbf{k}_F) + i\mathbf{v}_{1,2}\partial/\partial\mathbf{r}]\xi_{1,2} = 0, \tag{32}$$

where the velocities $\mathbf{v}_{1,2} = \mathbf{v}(1 \pm m)^{1/3}$ and the Fermi wavevectors $\mathbf{k}_{F1,2} = (1 \pm m)^{1/3}$ correspond to spin up and down, respectively, as defined in (6) and (5), m being the reduced magnetization. In addition, we may note that the term $\mathbf{v}_{1,2}(\mathbf{k}_{F1,2} - \mathbf{k}_F) = \pm(1/3)vk_Fm \sim \Delta_m$ is small according to the previous discussion, *i.e.* it is comparable to Δ with respect to the Fermi energy. Consequently it is immaterial in Eq. (32). It follows that the corresponding Eqs. (27) and (29) for the ferromagnet reduce to

$$[\omega + i\mathbf{v}_{1,2}\partial/\partial\mathbf{r}]\xi_{1,2} = 0, \tag{33}$$

whose solution is

$$\xi_{1,2} = A_{1,2}e^{i\mathbf{k}_{1,2}\mathbf{r}} \tag{34}$$

for

$$\mathbf{v}_{1,2}\mathbf{k}_{1,2} = \mathbf{v}(1 \pm m)^{1/3}\mathbf{k}_{1,2} = \omega. \tag{35}$$

The continuity condition of the wavefunctions ξ given by Eqs. (30) and (34) leads to

$$A_{1,2} = \frac{C_{1,2}}{\sqrt{2}}\sqrt{1 + \mathbf{vk}/\omega} \tag{36}$$

for a boundary placed at $x = 0$. On the other hand, the incoming current is given by

$$\mathbf{j}_i = \mathbf{v}_1|A_1|^2 + \mathbf{v}_2|A_2|^2 = \mathbf{v}\left[(1 + m)^{1/3}|A_1|^2 + (1 - m)^{1/3}|A_2|^2\right]. \tag{37}$$

Making use of Eq. (31) we may define the transmission coefficient

$$\begin{aligned} w &= (j_{t1} + j_{t2})/j_i = \frac{|C_1|^2 + |C_2|^2}{(1 + m)^{1/3}|A_1|^2 + (1 - m)^{1/3}|A_2|^2}(\mathbf{vk}/\omega) \\ &= \frac{2(|A_1|^2 + |A_2|^2)}{(1 + m)^{1/3}|A_1|^2 + (1 - m)^{1/3}|A_2|^2}\frac{\mathbf{vk}/\omega}{1 + \mathbf{vk}/\omega}. \end{aligned} \tag{38}$$

The asymptotic spin amplitudes in the superconductor are equal, *i.e.* $|A_1|^2 = |A_2|^2$ (and $|C_1|^2 = |C_2|^2$), so we get

$$w = \frac{4}{(1 + m)^{1/3} + (1 - m)^{1/3}}\frac{\mathbf{vk}/\omega}{1 + \mathbf{vk}/\omega}, \tag{39}$$

or

$$w = \frac{2}{(1 + m)^{1/3} + (1 - m)^{1/3}}w_0, \tag{40}$$

where

$$w_0 = 2\frac{\mathbf{vk}/\omega}{1 + \mathbf{vk}/\omega} \tag{41}$$

is the transmission coefficient for zero magnetization. Within the present approximation

$$w_0 \simeq 2\sqrt{2}\sqrt{\hbar\omega/\Delta - 1}. \tag{42}$$

One can see that the transmission coefficient in the Andreev reflection increases slowly with increasing magnetization,

$$w = (1 + m^2/9)w_0 \tag{43}$$

for small values of m.

5. Electrical resistance of the junction. Diffusive regime

For a voltage drop U a charge

$$-\frac{\partial n}{\partial \varepsilon} e^2 U \frac{2 \cdot d\mathbf{p}}{(2\pi\hbar)^3} \tag{44}$$

is transported per unit volume by a quasi-particle, where n denotes the Fermi distribution. During the quasi-particles lifetime τ the charge flux (charge per unit area) along the x-direction is

$$-\frac{\partial n}{\partial \varepsilon} e^2 U v_x \tau \frac{2 \cdot d\mathbf{p}}{(2\pi\hbar)^3}, \tag{45}$$

while the total flow (charge per unit area and unit time) is

$$j = -\frac{2e^2}{(2\pi\hbar)^3} \int d\mathbf{p} \cdot \frac{\partial n}{\partial \varepsilon} v_x^2 \tau (\partial U / \partial x). \tag{46}$$

From $j = \sigma E$, where $E = -\partial U/\partial x$ is the electric field, we obtain the electric conductivity

$$\sigma = \frac{e^2}{3\pi^2\hbar} k_F^2 v\tau, \tag{47}$$

in the low-temperature limit. In the derivation given above the statistical equilibrium is assumed, as well a mean-free path much shorter than the size of the sample, a low, uniform electric field, and a lifetime free of finite-size contributions or other geometric effects.

Eq. (47) shows that the electric conductivity of a ferromagnet does not depend essentially on magnetization. Indeed, the dependence on the magnetization comes through the velocity v and Fermi wavevector k_F in Eq. (47), which gives $(1/2)(v_1 k_{F1}^2 + v_2 k_{F2}^2) = (1/2) v k_F^2 (1 + m + 1 - m) = v k_F^2$. A weak magnetization dependence may be included in the lifetime, but its contribution is uncertain. This point is supported by the fact that flows are proportional to density of states $\sim k_F^2/v$ multiplied by velocity v multiplied by mean-free path $v\tau$ in the diffusive regime, hence their $\sim k_F^3$ proportionality to density, and the independence of magnetization.

The electric conductivity corresponding to the tunneling current in a superconductor can be derived in a similar way. The flow involves now the quantum probability beside the statistical one, *i.e.* it is given by

$$j = -\frac{2e^2}{(2\pi\hbar)^3} \int d\mathbf{p} \cdot \frac{n}{T} v_x^2 \tau \left[|\varphi|^2 - |\chi|^2 \right] (\partial U / \partial x), \tag{48}$$

where the temperature is so small in comparison with the superconducting gap that we may use $n = e^{-\hbar\omega/T}$, $\hbar\omega > \Delta$, for the Fermi distribution. The wavefunctions φ and χ are those given by (28) and (30) for $|C_\alpha|^2 = 1$; one can see that

$$|\varphi|^2 - |\chi|^2 = \mathbf{vk}/\omega \simeq \sqrt{2}\sqrt{\hbar\omega/\Delta - 1} \tag{49}$$

is much lesser than unity (which corresponds to a normal conductor), as due to the Andreev reflection. Making use of (48) and (49) one can compute the tunneling electric conductivity of a superconductor as

$$\sigma_s = \frac{e^2}{3\pi^2\hbar} \frac{\sqrt{2} k_F^2 v\tau}{T} \int_\Delta^\infty d\xi \cdot \sqrt{\frac{\xi - \Delta}{\Delta}} e^{-\xi/T} = \frac{e^2}{3\pi^2\hbar} k_F^2 v\tau \sqrt{\pi T / 2\Delta} e^{-\Delta/T}, \tag{50}$$

or

$$\sigma_s = \sigma \sqrt{\pi T / 2\Delta} e^{-\Delta/T} \tag{51}$$

for the tunneling conductivity of the superconducting state, where σ is the electric conductivity of the normal state. One can note in Eq. (51) a drastic reduction in the electric conductivity, in comparison with the normal state, as a consequence of the Andreev reflection. In addition, from

$$j = \sigma \frac{U - U_0}{l_f}, \quad j = \sigma_s \frac{U_0}{l_s} \tag{52}$$

for a ferromagnet–superconductor junction, where $l_{f,s}$ denote the lengths of the ferromagnet and superconducting samples, respectively, one obtains the electric resistance of the junction

$$R_j = l_f/\sigma + l_s/\sigma_s = R + R_s \tag{53}$$

for unit area, whence one can see that it is independent of magnetization; U_0 denotes the voltage drop at the junction. The superconducting resistance $R_s = l_s/\sigma_s$ is very high in comparison with the normal resistance $R = l_{f,s}/\sigma$. In particular

$$R_s = l_s/\sigma_s = R\sqrt{2\Delta/\pi T} e^{\Delta/T} \tag{54}$$

is the additional, large electric resistance due to the Andreev reflection in the superconductor, similar to the one computed originally for a thermal flow [23,24].

6. Electrical resistance of the junction. Ballistic regime for the ferromagnet

It is easy to see that for a mean-free path $\Lambda = v\tau$ comparable with the sample length Eq. (46) leads to an electric current

$$I = \frac{e^2}{3\pi^2\hbar} k_F^2 A \cdot U, \tag{55}$$

through the cross-sectional area A. By this equation quanta $\sim 2e^2/h$ of electric conductance can be inferred. Actually, in such a ballistic regime of transport the lifetime τ does not appear anymore in Eq. (45), and the average over angle integration gives 1 instead of 2/3 in Eq. (46); we obtain therefore

$$j = \frac{e^2 k_F^2}{2\pi^2\hbar} U, \tag{56}$$

i.e. an electric resistance

$$R = \frac{2\pi^2\hbar}{e^2 k_F^2} \tag{57}$$

for unit area. It is worth noting that in a ballistic transport regime the resistance may depend on the voltage drop, in some cases. Indeed, for a normal conductor we have obviously $\hbar \mathbf{v} \delta \mathbf{k}_F = -eU$, and the current $j = \left(-ek_F^2 / 2\pi^2\right) \int du (\mathbf{v}\delta\mathbf{k}_F) = \left(e^2 k_F^2 / 2\pi^2\hbar\right) U$, hence the ballistic resistance Eq. (57). For a superconductor the current is reduced by $\mathbf{vk}/\omega$, according to Eq. (41), where $\hbar\omega = -eU$. One obtains $R_s^{-1} = R^{-1}\sqrt{e^2U^2 - \Delta^2} / eU$, which is the typical behaviour for the tunneling resistance in superconductors [25–27].

We turn now the attention to a ferromagnet–superconductor junction where the ferromagnet is in the ballistic or quasi-ballistic regime. We assume that the superconducting sample is in the diffusive regime, *i.e.*

$$j = \frac{1}{R} \sqrt{\pi T / 2\Delta} e^{-\Delta/T} U_0, \tag{58}$$

where $R = 3\pi^2\hbar l_s / e^2 k_F^2 \Lambda$ as given by Eq. (50). Let us assume that the temperature is sufficiently low and the ferromagnetic sample is sufficiently thin that the length l_f is much shorter than the mean-free path $\Lambda = v\tau$ in the normal state of the ferromagnet, $l_f < \Lambda$. Increasing the magnetization the spin-up electron fluid increases its mean-free path $\Lambda_1 = \Lambda(1 + m)^{1/3}$, so that it transports in the ballistic regime; therefore, we may write down

$$j_1 = \frac{e^2 k_{F1}^2}{4\pi^2\hbar}(U - U_0) = \frac{e^2 k_F^2}{4\pi^2\hbar}(1+m)^{2/3}(U - U_0), \quad (59)$$

according to the discussion above. The spin-down electron fluid decreases its mean-free path $\Lambda_2 = \Lambda(1-m)^{1/3}$ on increasing magnetization. Up to a threshold magnetization $m_t = 1 - (l_f/\Lambda)^3$ it is still in the ballistic regime, so that

$$j_2 = \frac{e^2 k_{F2}^2}{4\pi^2\hbar}(U - U_0) = \frac{e^2 k_F^2}{4\pi^2\hbar}(1-m)^{2/3}(U - U_0); \quad (60)$$

it follows

$$j = j_1 + j_2 = \frac{e^2 k_F^2}{4\pi^2\hbar}\left[(1+m)^{2/3} + (1-m)^{2/3}\right](U - U_0), \quad (61)$$

which means a resistance

$$R_f = R\frac{2}{(1+m)^{2/3} + (1-m)^{2/3}}, \quad m < m_t, \quad (62)$$

where $R = 2\pi^2\hbar/e^2k_F^2$ as given above. For $m > m_t$ the mean-free path Λ_2 gets shorter than the length l_f of the sample and the spin-down fluid flows in the diffusive regime. In this case

$$j_2 = \frac{e^2}{6\pi^2\hbar}k_{F2}^2 v_2 \tau \frac{U - U_0}{l_f} = \frac{e^2 k_F^2}{6\pi^2\hbar}\frac{v\tau}{l_f}(1-m)(U - U_0)$$
$$= \frac{e^2 k_F^2}{6\pi^2\hbar}\frac{\Lambda}{l_f}(1-m)(U - U_0) = \frac{1}{3R}\frac{1-m}{(1-m_t)^{1/3}}(U - U_0); \quad (63)$$

it follows the resistance

$$R_f = R\frac{2}{(1+m)^{2/3} + \frac{2}{3}\frac{1-m}{(1-m_t)^{1/3}}}, \quad m > m_t \quad (64)$$

for the ferromagnetic sample. The two resistances given by Eqs. (62) and (64) are discontinuous at the threshold magnetization $m = m_t$, as a consequence of the distinct numerical factors in the ballistic and diffusive conductivities. This negative jump in the resistance is in fact round-off (by geometric effects, for instance), and it may be viewed as a negative resistance for magnetization values close to magnetization threshold. Apart from this jump the resistance R_f exhibits a monotonous increase with magnetization over the entire range $0 < m < 1$. In addition, as previously discussed, the Andreev reflection may greatly be diminished for values of the magnetization m close to unity for the spin-down quasi-particle fluid (in the sense that the corresponding electric flow may drastically be reduced), but its contribution to the conductivity is small for $m \sim 1$. We note two limiting behaviours for R_f, namely $R_f \sim R(1 + m^2/9)$ for $m \sim 0$ (which is similar to the behaviour of the transmission coefficient w as given by (43)), and $R_f \sim 2^{1/3}R\left\{1 + \frac{1}{3}\left[\frac{2^{1/3}}{(1-m_t)^{1/3}} - 1\right](m-1)\right\}$ for $m \sim 1$. We note also that the resistance of the junction is $R_j = R_f + R_s$, and it depends on magnetization through R_f.

For $\Lambda < l_f < 2^{1/3}\Lambda$ there exists another threshold $m_t = (l_f/\Lambda)^3 - 1$ below which both spin-up and spin-down fluids flow diffusively, while for $m > m_t$ the spin-up fluid flows ballistically (we call this regime quasi-ballistic). The ferromagnetic resistance is then given by

$$R_f = \frac{3}{2}R(1 + m_t)^{1/3}, \quad m < m_t \quad (65)$$

in the former case, and

$$R_f = \frac{3}{2}R(1 + m_t)^{1/3}\frac{2}{1 - m + \frac{3}{2}(1+m_t)^{1/3}(1+m)^{2/3}}, \quad m > m_t \quad (66)$$

in the latter, where R is the same as above. For small values of the magnetization the resistance is constant (and close to the value R corresponding to $l_f < \Lambda$), while for higher values of magnetization it increases up to the same value $2^{1/3}R$ as above. At the threshold it has a positive jump, in contrast to the case $l_f < \Lambda$, where the jump is negative. The reduced ferromagnetic resistance is shown in Fig. 1 for both cases.

7. Discussion and concluding remarks

It is worth stressing the complexity brought about by the ferromagnet–superconductor interface. Herein we have assumed a perfect, ideal junction, leaving aside the proximity effects (and neglecting the potential barrier it may contribute). In particular, the matching conditions of the Andreev reflection may be appreciably affected by a more realistic, complex interface. at the ferromagnet–superconductor junction. It is very likely that two very dissimilar solids (including appreciably different Fermi levels) may diffuse largely into one another, such that an extended contact is built up at the interface. Such a contact acts like a "third solid" in-between the former two, with its own properties. Along such an extended contact the physical properties vary slowly, and the Andreev reflection may work in principle. However, such an extended contact may put certain limitations on the ballistic regime of transport in the ferromagnetic sample, as due to proximity effects. If the two solids are similar they diffuse into each other over a rather limited scale length. It is reasonable to assume, as we did herein, that the two Fermi levels are close to each other. It is also reasonable to assume that both the ferromagnetic and superconducting gaps do not change appreciably this picture.

It was shown that under such ideal conditions the electric transport through a ferromagnet–superconductor junction is not affected by spin polarization in the diffusive regime. On the contrary, it depends on magnetization in the ballistic regime for the ferromagnetic sample. It is our opinion that a ballistic regime for the superconducting sample may prove to be inconsistent (in particular, a spin-polarized injected current may destroy the superconductivity over small length scales). The change in magnetization may be performed by slight changes in temperature just below the magnetic critical temperature, but much lower than the superconducting critical temperature. The results are valid for small values of the relative magnetization m. For high values of the magnetization ($m \to 1$) the spin-down fluid of quasi-particles in the ferromagnet ceases to fulfill the Andreev matching conditions. However, the two spin fluids of quasi-particles act like two conductors coupled in parallel, and the spin-up contribution dominates the junction resistance.

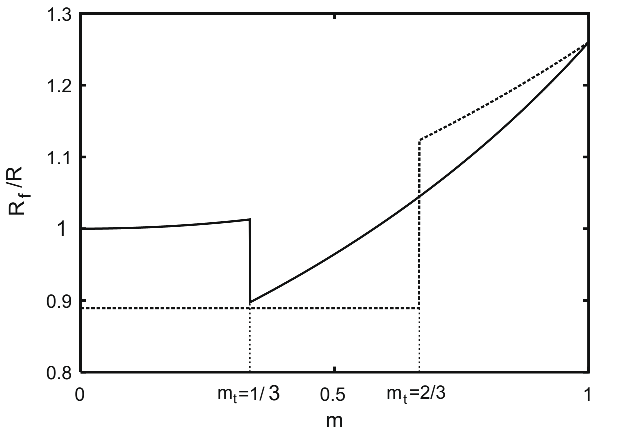

Fig. 1. Reduced resistance of a ferromagnetic sample vs. magnetization in the ballistic (solid line) and quasi-ballistic (dotted line) regime for two arbitrary values of the threshold magnetization m_t.

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