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Weather dynamics

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Weather is a non-equilibrium atmospheric statistical ensemble, at least in a determined region and over a determined lapse of time. Consequently, it is characterized by a set of variables $x^i(r,t)$, which can be viewed as a vectorial local field of position r and time t. The infinitesimal changes $\delta x^i(r,t)$ (also called fluctuations) are of utmost importance, as they serve for forecasting the weather by continuity. Among $x^i(r,t)$, it is convenient to employ those variables which determine the thermodynamic functions, like energy for instance, whose local change reads

$$\delta e = X^i \delta x^i \ , \tag{1}$$

where $X^i(r,t)$ are the corresponding generalized forces. These forces are the dynamical variables of interest. Their motion contributes to the change in the global entropy

$$-\int dr \cdot m^i (\dot{X}^i)^2/2 \ , \tag{2}$$

where the integration is extended over a determined region and m^i are constants coefficients. Similarly, the global entropy is a functional of the variation

$$\delta E = \int dr \cdot X^i \delta x^i \ , \tag{3}$$

in the global energy, so that it can be written as

$$S = S_0 - \int dr \cdot m^i (\dot{X}^i)^2/2 + a(\delta E)^2/2 + b(\delta E)^4 ..., \tag{4}$$

where a and b are constant coefficients. It is worth noting that S is an even functional of δE in order to ensure an extremum. Indeed, the evolution of weather is supposed to proceed such as to achieve a maximum value of the global entropy. The extremum condition for (4) leads to the equations of motion

$$m^i \ddot{X}^i + \int dr' \cdot X^j(r') \delta x^j(r',t) \delta x^i(r,t) = 0 \tag{5}$$

in the linear approximation, where m^i has been renormalized through $m^i \rightarrow m^i/a$. Equations (5) are the basic coupled-oscillators equations of the present model of weather dynamics. They can also be written as

$$m^i \ddot{X}^i + \int dr' \cdot C^{ij}(r,r',t) X^j(r') = 0 \ , \tag{6}$$

where $C^{jj}(r,r',t) = \delta x^i(r,t) \delta x^j(r',t)$ is called the covariant matrix. It is easy to see that this matrix is positive definite, its eigenvalues being $(\delta x(r,t))^2$. In general, solutions of (6) are oscillatory functions of time, controlled by spatial and state-variables correlations of variances (covariance matrix), in qualitative agreement with empirical evidence.

In order to simplify discussion, we limit ourselves to only one state variable, and discretize the spatial region such as (6) becomes

$$m_\alpha \ddot{X}_\alpha + C_{\alpha\beta}(t) X_\beta = 0 \ , \tag{7}$$

where $C_{\alpha\beta}(t) = \delta x_\alpha(t)\delta x_\beta(t)$ is the covariance matrix over a spatial grid α, β (as, for instance, weather recordings from various meteorological stations). Equations (7) in their general form is a system of coupled Hill equations. For $C_{\alpha\beta} = \delta x_\alpha \delta x_\beta$ independent of time, they reduce to coupled linear oscillators with eigenfrequencies ω_α given by $\omega_\alpha^2 = (\delta x_\alpha)^2/m_\alpha$. It is worth noting the case $\delta x_\alpha = a_\alpha[1 + \varepsilon \cos(\omega_\alpha t + \varphi_\alpha)]$, where ε is a small parameter and φ_α is a phase (averaging over such a random phase leads to $C_{\alpha\beta} = a_\alpha a_\beta = const$), which is a system of coupled Mathieu equations, and which may lead to parametric resonance. Dissipation can be included in equations (7) through $\gamma_\alpha \dot{X}_\alpha$, as usually, driving external forces can also be considered, and non-linearities included. A rich phenomenology can thereby obtained for the weather dynamics.