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Error estimation for the magnetic field of the accelerators magnets

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Abstract

A few types of errors produced by defects in coils are estimated for the magnetic field in the accelerator magnets.

The circular, almost constant magnetic field B created by a toroidal magnet is affected by the defective field due to small interruptions in the coil. The total field can be represented as

$$B(\theta) = B + \sum_{i=1}^{M} b(\theta - \theta_i) \quad , \tag{1}$$

where M is the number of interruptions (and of regular coils) and $b(\theta - \theta_i)$ is the defective contribution of an interruption in the coil placed at θ_i. The mean square deviation of $B(\theta)$ is given by

$$\overline{(\Delta B)^2} = B_1 - B_2 \quad , \tag{2}$$

where

$$B_1 = \frac{1}{2\pi} \int d\theta \cdot \left\langle \left[\sum_{i=1}^{M} b(\theta - \theta_i) \right]^2 \right\rangle \tag{3}$$

and

$$B_2 = \left[\frac{1}{2\pi} \int d\theta \cdot \left\langle \sum_{i=1}^{M} b(\theta - \theta_i) \right\rangle \right]^2 \quad , \tag{4}$$

$\langle ... \rangle$ denoting the average over the distribution of θ_i. Making use of the Fourier representation

$$b(\theta) = \frac{1}{2\pi} \sum_n b_n e^{in\theta} \tag{5}$$

the mean defective field is readily computed as

$$\begin{aligned} \tfrac{1}{2\pi} \int d\theta \cdot \left\langle \textstyle\sum_{i=1}^{M} b(\theta - \theta_i) \right\rangle = \tfrac{1}{2\pi} \textstyle\sum_n b_n \cdot \tfrac{1}{2\pi} \int d\theta e^{in\theta} \cdot \textstyle\sum_{i=1}^{M} \left\langle e^{-in\theta_i} \right\rangle = \\ = M b_0 / 2\pi = \tfrac{M}{2\pi} \int d\theta \cdot b(\theta) \quad . \end{aligned} \tag{6}$$

Since the interruptions are small the integral in (6) can be approximated by $b\Delta\theta = b\Delta l / R$, where Δl is the length of the interruption and R is the torus radius. We obtain therefore

$$\frac{1}{2\pi} \int d\theta \cdot \left\langle \sum_{i=1}^{M} b(\theta - \theta_i) \right\rangle = M b \frac{\Delta l}{L} \quad , \tag{7}$$

where L is the length of the torus. In addition, we can also use the estimation $b/B=-\Delta l/L$, so that

$$B_2=B^2\left(\Delta l/L\right)^4 M^2 \quad . \tag{8}$$

In a similar way we have

$$\begin{aligned} B_1 &= \frac{1}{2\pi}\int d\theta\cdot\frac{1}{(2\pi)^2}\sum_{nm} b_n b_m^* e^{i(n-m)\theta}\cdot\left\langle \sum_{ij=1}^M e^{-i(n\theta_i-m\theta_j)}\right\rangle = \\ &= \frac{1}{(2\pi)^2}\sum_n \left|b_n\right|^2\cdot\left|\sum_{i=1}^M\left\langle e^{-in\theta_i}\right\rangle\right|^2 \quad . \end{aligned} \tag{9}$$

We shall assume that θ_i is normally distributed around the mean value $\overline{\theta_i}$ with a small dispersion δ, so that

$$\left\langle e^{-in\theta_i}\right\rangle=\frac{1}{\sqrt{2\pi}\delta}\int d\theta_i\cdot e^{-in\theta_i}\cdot e^{-\left(\theta_i-\overline{\theta_i}\right)^2/2\delta^2}=e^{-in\overline{\theta_i}}\cdot e^{-n^2\delta^2/2} \quad ; \tag{10}$$

we get therefore

$$B_1=\frac{1}{(2\pi)^2}\sum_n\left|b_n\right|^2\cdot e^{-n^2\delta^2}\cdot\left|\sum_{i=1}^M\left\langle e^{-in\overline{\theta_i}}\right\rangle\right|^2 \quad . \tag{11}$$

The summation over i in (11) gives M and $n=M\times integer$, as a consequence of a symmetrically distributed interruptions, so that (11) becomes

$$B_1=\frac{M^2}{(2\pi)^2}\sum_n\left|b_{Mn}\right|^2\cdot e^{-M^2\delta^2n^2}=\frac{M^2}{(2\pi)^2}\int d\theta d\theta' b(\theta)b(\theta')\sum_n e^{-M^2\delta^2n^2-iMn(\theta-\theta')} \quad . \tag{12}$$

The summation over n may be replaced by an integral and we obtain

$$B_1=\frac{M^2}{(2\pi)^2}\cdot\frac{\sqrt{\pi}}{M\delta}\int d\theta d\theta' b(\theta)b(\theta')\cdot e^{-\left(\theta-\theta'\right)^2/4\delta^2} \quad ; \tag{13}$$

since δ is very small we may approximate the integral over the gaussian in (13) by

$$B_1=\frac{M^2}{(2\pi)^2}\cdot\frac{\sqrt{\pi}}{M\delta}\cdot 2\sqrt{\pi}\delta\int d\theta\cdot b^2(\theta)=\frac{M}{2\pi}\int d\theta\cdot b^2(\theta) \quad , \tag{14}$$

and we can see that the result does not depend on δ. Followed a similar estimation as that employed in (7) and (8) we obtain

$$B_1=\frac{M}{2\pi}\cdot b^2\cdot\frac{\Delta l}{R}=Mb^2\cdot\frac{\Delta l}{L}=B^2\left(\frac{\Delta l}{L}\right)^3 M \quad . \tag{15}$$

From (8) and (15) we get the relative error

$$\varepsilon=\sqrt{\overline{(\Delta B)^2}}/B=\left(\Delta l/L\right)^{3/2}\left[M\left(1-M\cdot\Delta l/L\right)\right]^{1/2} \quad , \tag{16}$$

produced by the defective field, or

$$\varepsilon=\left(\Delta l/L\right)^{3/2}\sqrt{M} \tag{17}$$

since $M\cdot\Delta l/L\ll 1$.

A similar estimation holds for the magnetic field produced by the cylindrical magnets distributed regularly along the accelerator ring. In this case the defective field is due to the end coils of the magnets, and the ratio b/B is of the order r/l, where r is the radius of the magnet and l is its length, *i.e.* we may replace Δl by r. On the other hand we have $M = L/(l + \Delta l) \approx L/l$, since $\Delta l \approx r \ll l$. From (7), (8) and (15) we obtain

$$B_1 \cong B^2 (r/l)^3 \quad , \quad B_2 \cong B^2 (r/l)^4 \quad , \tag{18}$$

so that we get the error

$$\varepsilon \cong (r/l)^{3/2} \; . \tag{19}$$

For typical values $r \cong 3cm$ and $l \cong 15m$ the error is $\varepsilon \cong 10^{-2}\%$.

Another type of error can appear in the magnetic field near the central line of a cylindrical magnet from defects in the winding of the longitudinal coil. The magnetic field near the centre of the magnet, produced by a coil which extends from $-\Delta\theta$ to $+\Delta\theta$, is given by

$$B = \frac{I}{2\pi r} \cdot \frac{1}{\Delta\theta_0} \cdot \int_{-\Delta\theta}^{\Delta\theta} d\theta \cdot \cos\theta = \frac{I}{2\pi r} \cdot \frac{1}{\Delta\theta_0} \cdot 2\sin\Delta\theta \quad , \tag{20}$$

where I is the current through the coil and $\Delta\theta_0$ is the angle of a wire in the coil. The defective field due to a defect of extension $\delta\theta$ placed at θ_i is

$$b_i = \frac{I}{2\pi r} \cdot \frac{\delta\theta}{\Delta\theta_0} \cdot \cos\theta_i = b_0 \cos\theta_i \quad , \tag{21}$$

so that the mean square deviation of the field $B(\theta) = B + \sum_i b_0 \cos\theta_i$ is

$$\overline{(\Delta B)^2} = b_0^2 \cdot M \cdot \left[\overline{\cos^2\theta} - \left(\overline{\cos\theta}\right)^2\right] \quad , \tag{22}$$

where M is the number of defects distributed independently. The averages in (24) are straightforwardly computed, and we obtain

$$\overline{\cos^2\theta} - \left(\overline{\cos\theta}\right)^2 = \frac{1}{2} + \frac{\sin 2\Delta\theta}{4\Delta\theta} - \frac{\sin^2\Delta\theta}{(\Delta\theta)^2} \quad ; \tag{23}$$

on the other hand, from (20) and (21) we have $b_0/B = \delta\theta/2\sin\Delta\theta$, and we obtain the error

$$\varepsilon = \frac{\delta\theta}{2\sin\Delta\theta} \cdot M^{1/2} \cdot \left[\overline{\cos^2\theta} - \left(\overline{\cos\theta}\right)^2\right]^{1/2} \quad . \tag{24}$$

For $\Delta\theta = \pi/4$ the error given by (24) is $\varepsilon \cong 0.1 \cdot \delta\theta \cdot (M/2)^{1/2}$, and for typical values $\delta\theta \sim 1/30$ and $M \sim 10 - 20$ we get $\varepsilon \sim 1\%$.