ELI-NP at Magurele - “Pulse and Impulse of ELI”

1) "**Polaritonic pulse** and coherent X- and gamma rays from Compton (Thomson) backscattering" (MA&MGan), J. Appl. Phys. **109** 013307 (2011) (1-6)

2)”Dynamics of **electron–positron pairs** in a vacuum polarized by an external radiation field” (MA), Journal of Modern Optics, **58** 611 (2011)

3)”**Classical interaction** of the electromagnetic radiation with two-level polarizable matter” (MA), Optik **123** 193 (2012)

4)”**Coherent polarization** driven by external electromagnetic fields” (MA&MGan), Physics Letters **A374** 4848 (2010)

5)”Coupling of **(ultra-) relativistic atomic nuclei** with photons” (MA&MGan), AIP Advances **3** 112133 (2013)

6)”Propagation of **electromagnetic pulses** through the surface of dispersive bodies” (MA), Roum J. Phys. **58** 1298 (2013)

7)”**Giant dipole oscillations** and ionization of heavy atoms by intense electromagnetic pulses” (MA), Roum. Reps. Phys. **67** 837 (2015)

8)”**Parametric resonance**” in molecular rotation spectra” (MA&LCCun
Chem. Phys. **472** 262 (2016)

9)”**Motion of an electric charge under the action of laser fields**” (MA)-2016

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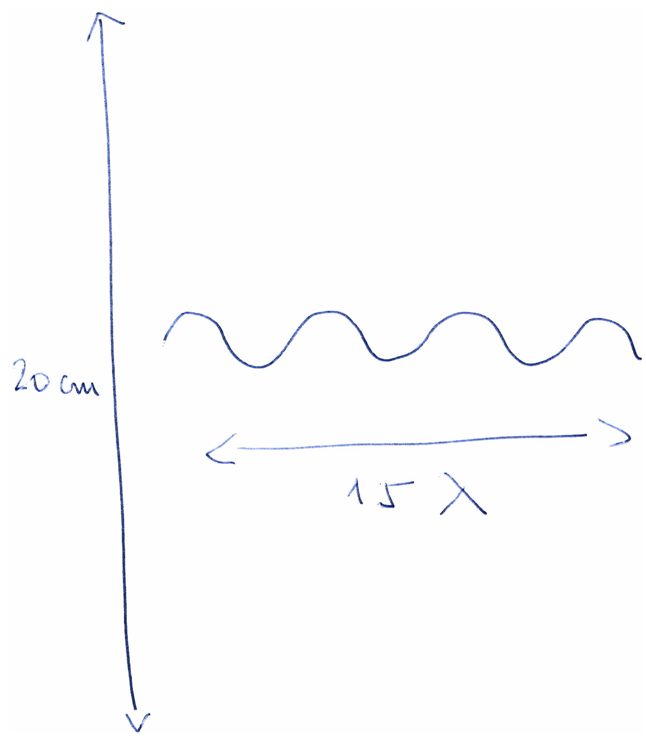
Motion of an electric charge under the action of laser fields

M Apostol

2016

- Lasers for physicists: resonant cavity, pumped, laser emission (diaphragm); usully in pulses, say $50fs$; width 15μ, pretty monochromatic ($1eV$) (quasi-plane wave; also Gaussian beams))



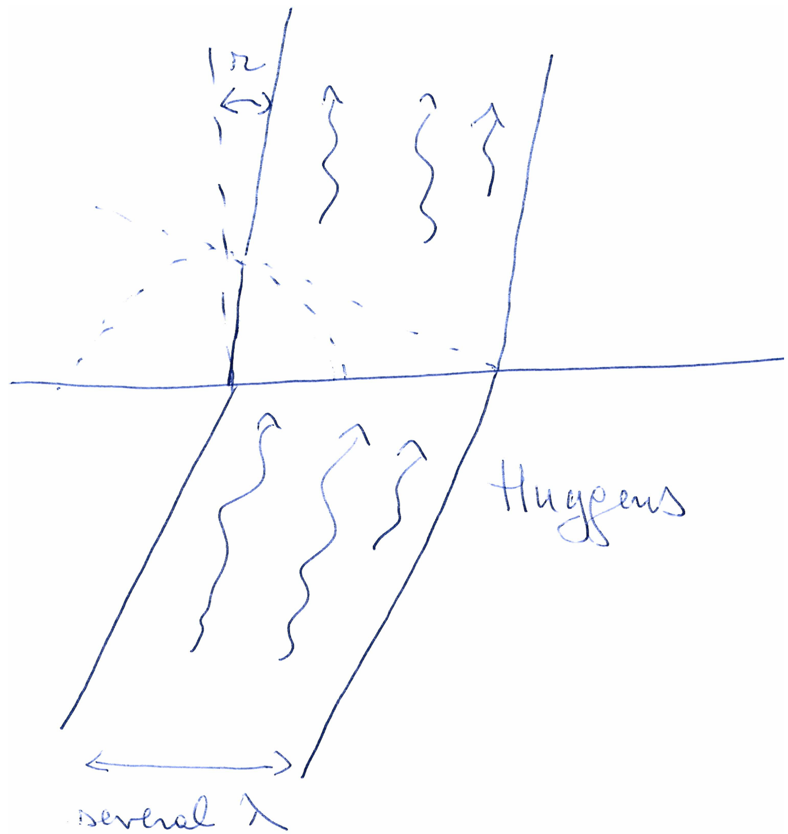
-Pay attention: the width (breath) of this thin slab of light as large as 20-40cm

-Propagates in (slightly) ionized gases (density cca $10^{19} cm^{-3}$)

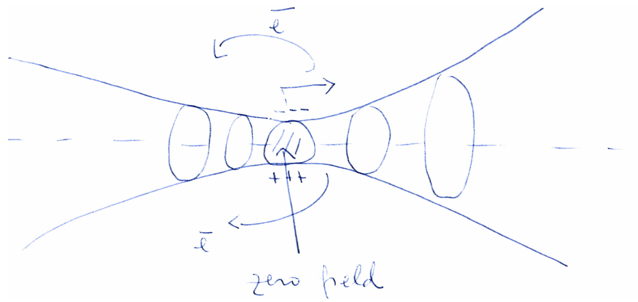
-Electrons may be accelerated by such quasi-plane waves (though in pulses)

-Big difference between plane waves and focused beams

-First, for instance, refraction requires a rather large beam width (Huygens construction). At the same time, "ghost" waves (Sommerfeld, Brillouin,1914), plasma oscillations (infinite wavelength); narrow beams (pencils, long), do not refract (but reflect) (MA, RJP (2013)); bullets refract and reflect like particles



-Focused pulses (beams): different. (Of course, a focused beam is not a superposition, a wavepacket! it is a “condensed” pulse).



A bullet in the focus (3-4cm); high polarization, compensating electric field, electrons accumulated on its surface and carried on by the pulse (for 3-4cm); this is the acceleration mechanism of the polaritonic pulse (MA&Gan, JAP (2011)). It is based on the exctinction theorem (Ewald, Oseen, 1914-1915). No field inside, relatively stable entity. Carry a macroscopic charge, coherent Compton scatt (Thomas, backscatt), γ-laser. The same happens behind and beyond the focal region, but it is necessary to have a zero slope for the beam envelope to be effective

I. Introduction

Breit-Wheeler process

$$\gamma + \gamma \to e^- + e^+$$

still unobserved

A similar process: multi-photon collision

$$\gamma + n\omega \to e^- + e^+$$

seen indirectly by multi-photon Compton effect

$$e^- + n\omega \to e^- + \gamma$$

Electron in laser field collides with n optical photons, produces γ which interact with n optical photons, releasing “dressed” positrons which are observed! (and “dressed” electrons)

1st issue: charge in laser field

2nd issue: Vacuum breakdown

Constant electric field: $eE \cdot \hbar/mc = mc^2$, Schwinger limit $E = m^2c^3/e\hbar \simeq 4.4 \times 10^{13} statvolt/cm$ $(1 statvolt/cm = 3 \times 10^4 V/m)$, breaks the vacuum

Constant magnetic field: similar, $(e\hbar/mc)H = mc^2$, no breakdown!

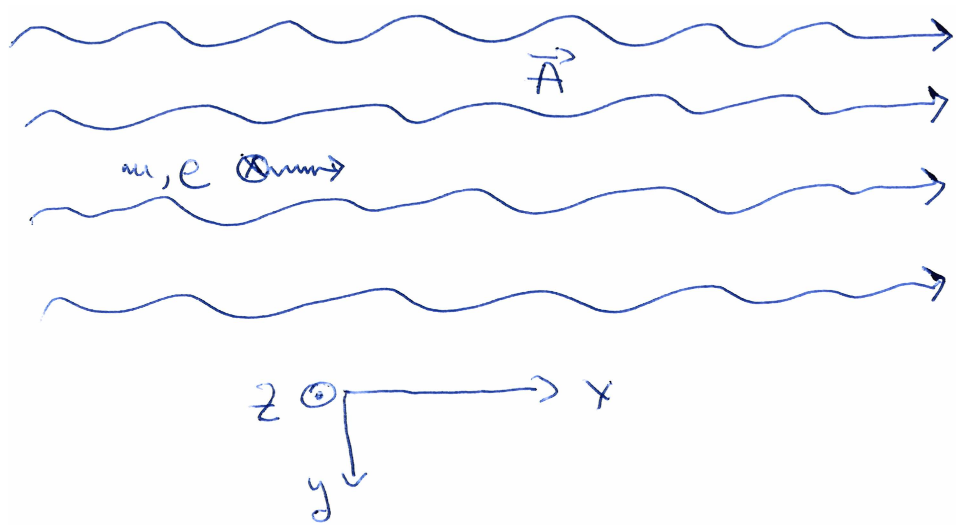
Klein paradox: more reflected electrons from vacuum, in high electric field

Non-linear Electrodynamics, two invariants $E^2 - H^2$ and EH; not in radiation!

Vacuum birefringence, photon splitting

Vacuum polarization (MA, JMO (2011))

II. Classical charge in an electromagnetic plane wave



Classical relativistic electron in radiation field

Classical relativistic electron

$$(\frac{1}{c}\mathcal{E} - \frac{e}{c}\Phi)^2 = m^2c^2 + (\mathbf{p} - \frac{e}{c}\mathbf{A})^2$$

Hamilton-Jacobi equation

$$g^{ij}\left(\frac{\partial S}{\partial x^i} + \frac{e}{c}A_i\right)\left(\frac{\partial S}{\partial x^j} + \frac{e}{c}A_j\right) = m^2c^2$$

Plane wave, A_i functions of the phase $\xi = k_i x^i$ only

Solution

$$S = -\frac{1}{2}\gamma(ct + x) - \frac{m^2c^2 + \kappa^2}{2\gamma}\xi + \boldsymbol{\kappa r} + \frac{e}{c\gamma}\int^{\xi} d\xi' \boldsymbol{\kappa A} - \frac{e^2}{2c^2\gamma}\int^{\xi} d\xi' A^2$$

$\gamma = k_i f^i$, $f_i f^i = m^2c^2$, f^i is the momentum of a free particle

Momenta: $\partial S/\partial x^i$, **coordinates:** $\partial S/\partial f^i = const$

$$y = \frac{\kappa_y}{\gamma}\xi - \frac{e}{c\gamma}\int^{\xi} d\xi' A_y$$

$$z = \frac{\kappa_z}{\gamma}\xi - \frac{e}{c\gamma}\int^{\xi} d\xi' A_z$$

$$x = \frac{1}{2}\left(\frac{m^2c^2+\kappa^2}{\gamma^2} - 1\right)\xi - \frac{e}{c\gamma^2}\int^{\xi} d\xi' \boldsymbol{\kappa A} + \frac{e^2}{2c^2\gamma^2}\int^{\xi} d\xi' A^2$$

and

$$p_y = \kappa_y - \frac{e}{c}A_y$$

$$p_z = \kappa_z - \frac{e}{c}A_z$$

$$p_x = -\frac{1}{2}\gamma + \frac{m^2c^2+\kappa^2}{2\gamma} - \frac{e}{c\gamma}\vec{\kappa}\vec{A} + \frac{e^2}{2c^2\gamma}A^2$$

Energy

$$\mathcal{E} = c(\gamma + p_x)\ ,\ (\gamma = mc)$$

Charge at rest at $x = y = z = 0$ at the initial moment of time $t = 0$ $A = A_z = A_0 \cos(\omega t - kx) = A_0 \cos \frac{\omega}{c}(ct - x) = A_0 \cos \frac{\omega}{c}\xi$ (lin pol)

$$z = -\frac{eA_0}{mc^2}\lambda \sin(\omega t - kx)\ ,\ y = 0\ ,$$

$$x = \frac{e^2A_0^2/4m^2c^4}{1+e^2A_0^2/4m^2c^4}\left[ct + \frac{\lambda}{2}\sin 2(\omega t - kx)\right]$$

$$p_x = \frac{e^2A_0^2}{2mc^3}\cos^2(\omega t - kx)\ ,\ p_z = -\frac{eA_0}{c}\cos(\omega t - kx)\ ,\ p_y = 0$$

and energy

$$\mathcal{E} = mc^2 + \frac{e^2A_0^2}{2mc^2}\cos^2(\omega t - kx)$$

Effective mass, time-average

$$\mathcal{E}^2/c^2 - p_x^2 = m^{*2}c^2 \ , \ m^{*2} = m^2\left(1 + \frac{e^2A_0^2}{2m^2c^4}\right)$$

Note: 1) oscillations 2ω along the propagation direction

2) drift along the propagation direction, $\eta = eA_0/2mc^2$

3) negative energy, negative momentum p_x,

$\mathcal{E} = c(-\gamma - p_x)$; Note the linear dep on p_x!

Application to laser fields, 1

Parameter

$$\eta = \frac{eA_0}{2mc^2}$$

Drift velocity of the charge

$$v \simeq \frac{\eta^2}{1+\eta^2}c$$

Coordinates:

$$x \simeq vt + \frac{1}{2}\frac{v\lambda}{c}\sin 2(\omega - kv)t = vt + \frac{1}{2}\frac{v\lambda}{c}\sin 2\omega(1-\frac{v}{c})t$$

$$z = -2\eta\lambda \sin \omega(1-\frac{v}{c})t \ , \ \mathcal{E} = mc^2(1+\eta^2)$$

Current $J = ev$ along the direction of propagation of the radiation

-Duration of the transient (accelerating) regime: $1 - e^{-t/\Delta t}$

Δt - time needed to introduce the charge in the beam

$\Delta t \simeq \omega^{-1}$ (pulse duration $\tau \simeq s\omega^{-1}$, $s \geq 10$

electron is accelerated very quickly

-high intensities, electron moves almost with the pulse velocity c

-oscillation phase $\simeq \omega\xi/c = \omega t/(1+\eta^2)$

for $t = \tau$, phase $\omega\tau/(1+\eta^2) = s/(1+\eta^2) \ll 1$: oscillations may be neglected (frequency very small, $\xi \ll \lambda$)

coordinate $z \simeq \lambda\eta s/(1+\eta^2) = d\eta/(1+\eta^2) \ll d$

el stays in the pulse

-coordinate $y = \kappa_y\xi/\gamma \ll \kappa_y\lambda/\gamma$, momentum κ_y should be sufficiently small for the electron to stay inside of the pulse

Estimation of η

Usually, $\eta \ll 1$

Laser intensity $I = 10^{22} w/cm^2$ (high), focalized in a pulse of dimension $d \Longrightarrow$ generates an electric field $E_0 = \sqrt{8\pi I/c} = 10^{10} statvolt/cm$, very high; vector potential $A_0 = cE_0/\omega = 10^{-5} E_0 = 10^5 statvolt$ for the optical frequency $\omega = 3 \times 10^{15} s^{-1}$; $eA_0 = 10^{-5} = 30 MeV \gg mc^2 = 0.5 MeV$; $\eta = eA_0/2mc^2 = 30$. This energy is much higher than the rest energy of the electron $mc^2 = 0.5 MeV$, so that the ratio $\eta = eA_0/2mc^2 = 30 \gg 1$

It follows: 1) electron accelerated up to ultra-relativistic velocities

Note: 2) in vacuum; in a gaseous plasma, pulse wavepacket (Ap, RJP (2013)), which distributes the electrons over its surface, such as to compensate the radiation field; no field available in the pulse to accelerate charges (the charges are accelerated by the the transport motion of the wavepacket (pulse; pulsed polariton) (Ap&Gan, JAP (2011)); distinct (and complementary) mechanism of acceleration

III. Quantum charge in an electromagnetic plane wave

For many purposes the rel el is classical

2nd order Dirac eq

$$[(p - \frac{e}{c}A)^2 - m^2c^2 - \frac{i}{2}\frac{e\hbar}{c}F_{\mu\nu}\sigma^{\mu\nu}]\psi = 0$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ - the strength of the elm field, $\sigma^{\mu\nu} = (\boldsymbol{\alpha}, i\boldsymbol{\Sigma})$,

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \sigma & 0 \end{pmatrix} \ , \ \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}$$

$$(\mathbf{a}, \mathbf{b}) = \begin{pmatrix} 0 & a_x & a_y & a_z \\ -a_x & 0 & -b_z & b_y \\ -a_y & b_z & 0 & -b_x \\ -a_z & -b_y & b_x & 0 \end{pmatrix}$$

Elm potentials functions of the phase $\xi = kx$ only

Solution (Volkov)

$$\psi_{p\sigma} = \frac{1}{\sqrt{2\varepsilon V}} \left[1 + \frac{e}{2c} \frac{(\gamma k)(\gamma A)}{(pk)} \right] e^{\frac{i}{\hbar} S} u_{p\sigma}$$

$$S = -px - \int^{\xi} d\xi' \left[\frac{e}{c(pk)} (pA) - \frac{e^2}{2(pk)c^2} A^2 \right]$$

$$u_{p\sigma} = \begin{pmatrix} (\varepsilon + mc^2)^{1/2} w_\sigma \\ (\varepsilon - mc^2)^{1/2} (\boldsymbol{n\sigma}) w_\sigma \end{pmatrix} \ , \ u_{-p-\sigma} = \begin{pmatrix} (\varepsilon - mc^2)^{1/2} (\boldsymbol{n\sigma}) w'_\sigma \\ (\varepsilon + mc^2)^{1/2} w'_\sigma \end{pmatrix}$$

where $\boldsymbol{n} = \mathbf{p}/p$, $w'_\sigma = -\sigma_y w_{-\sigma}$ (w_σ eigenvectors of σ_z); $\overline{u}_{p\sigma} u_{p\sigma'} = 2mc^2 \delta_{\sigma\sigma'}$, $\overline{u}_{-p\sigma} u_{-p\sigma'} = -2mc^2 \delta_{\sigma\sigma'}$

Dirac matrices are

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ , \ \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}$$

Note: 1) orthogonality $\psi_{p\sigma}$ (completeness) (wavepackets)

2)phase classical mechanical action; it contains the drift motion of the electron along the propagation direction of the wave

3)the pre-exponential factor includes the oscillations of the electron in the radiation field

The current $j^\mu = c\overline{\psi}\gamma^\mu\psi$

$$j^\mu = \frac{c}{\varepsilon V}\left\{p^\mu - \frac{e}{c}A^\mu + k^\mu\left[\frac{e}{c(pk)}(pA) - \frac{e^2}{2(pk)c^2}A^2\right]\right\}$$

The momentum

$$q^\mu = \psi^*_{p\sigma}(p^\mu - \tfrac{e}{c}A^\mu)\psi_{p\sigma} = p^\mu - \tfrac{e}{c}A^\mu + k^\mu\left[\tfrac{e}{c(pk)}(pA) - \tfrac{e^2}{2(pk)c^2}A^2\right] +$$

$$+k^\mu\tfrac{ie}{8\hbar(pk)\varepsilon}F_{\lambda\nu}(u^*\sigma^{\lambda\nu}u)$$

Time average

$$q^\mu = p^\mu - k^\mu \frac{e^2\overline{A^2}}{2(pk)c^2}$$

$$q^2 = p^2 - \frac{e^2\overline{A^2}}{c^2} = m^2c^2(1 - e^2\overline{A^2}/m^2c^4)$$

effective mass m^*, which increases with increasing interaction ($A^2 = -\mathbf{A}^2$).

Application to laser fields, 2

Consider an elm wave as before:

$$1 + \frac{e}{2c}\frac{(\gamma k)(\gamma A)}{(pk)} = 1 - \frac{eA_0 \cos\frac{\omega}{c}\xi}{2mc^2}\begin{pmatrix} -i\sigma_2 & \sigma_3 \\ \sigma_3 & -i\sigma_2 \end{pmatrix}$$

$$S = -\left(mc^2 + \frac{e^2A_0^2}{4mc^2}\right)t + p_y y + \frac{e^2A_0^2}{4mc^3}x - \frac{e^2A_0^2}{8mc^2\omega}\sin\frac{2\omega}{c}\xi$$

High-intensity interaction: drift momentum (in localization regions)

$$P_x \simeq \frac{e^2A_0^2}{4mc^3}$$

energy

$$\mathcal{E} \simeq mc^2 + \frac{e^2A_0^2}{4mc^2} = mc^2 + cP_x$$

phase velocity

$$v_x \simeq \frac{\mathcal{E}}{P_x} = \frac{1 + e^2 A_0^2/4m^2c^4}{e^2 A_0^2/4m^2c^4} > c$$

Group velocity $\simeq c$

Very high elm field, $eA_0/2mc^2 \gg 1$ (injection of electrons in a laser beam focalized in vacuum):

1) The charge becomes ultra-relativistic (group velocity)

2) Simplification

$$\psi_{p\sigma} \simeq \frac{1}{\sqrt{2V}} \begin{pmatrix} w_\sigma \\ \sigma_2 w_\sigma \end{pmatrix} e^{\frac{i}{\hbar}S} \ , \ \psi_{-p-\sigma} \simeq -\frac{1}{\sqrt{2V}} \begin{pmatrix} w_{-\sigma} \\ \sigma_2 w_{-\sigma} \end{pmatrix} e^{-\frac{i}{\hbar}S}$$

where

$$S \simeq -\frac{e^2 A_0^2}{4mc^2} t + \frac{e^2 A_0^2}{4mc^3} x = -\frac{e^2 A_0^2}{4mc^3}(ct - x)$$

3) very similar to a free particle (oscillations lost, $n\omega$?)

4) $\psi_{-p-\sigma}$ corresponds to negative energy (and momentum); negative electrons; they get lower and lower (negative) energy (as if they would have a negative mass); the gap between the negative- and positive-energy states increased by the radiation

El-Pos pair production, e.g. by

$$\gamma + n\omega \to e^{+} + e^{-}$$

γ injected in the laser focus

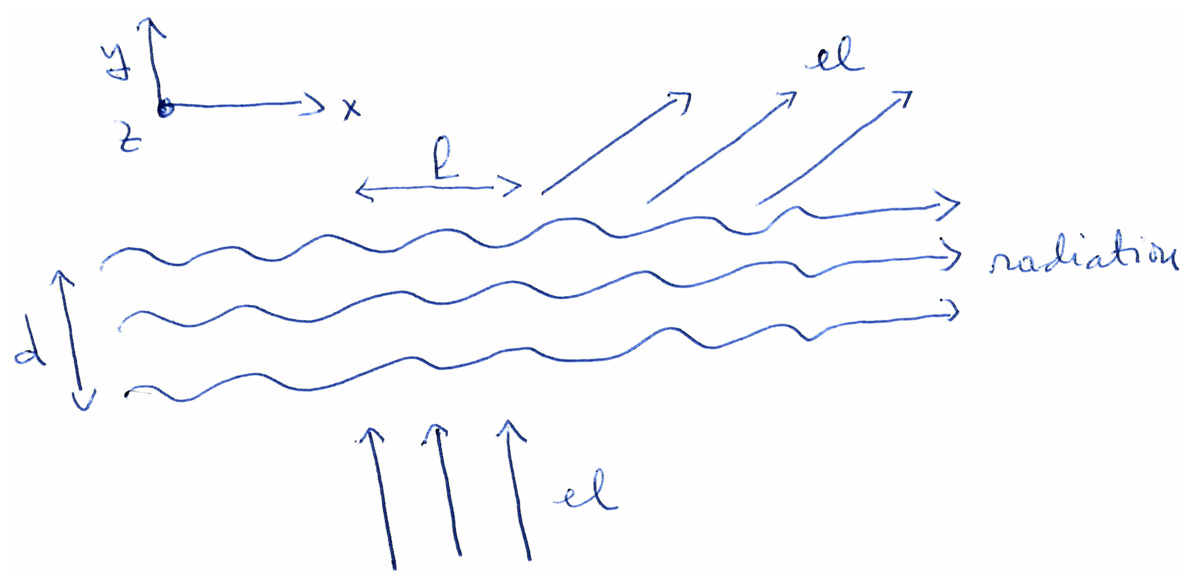
$$E^{-} = mc^2 + \frac{e^2 A_0^2}{4mc^2} \ (= mc^2 + cP_x)$$

$$E^{+} = -mc^2 - \frac{e^2 A_0^2}{4mc^2} \ (= -mc^2 - cP_x)$$

$$\Delta E = E^{-} - E^{+} = 2mc^2 + \frac{e^2 A_0^2}{2mc^2}$$

enhanced by radiation; pair production in radiation?

Injection of the electrons in the laser beam



Adiabatic injection of electrons in laser beam

Uncertainties: $\Delta t = dm/p_y$, $\Delta x = c\Delta t = cdm/p_y$, $\Delta P_x = \hbar p_y/cdm$, $\Delta\mathcal{E} = \hbar p_y/dm$, $\Delta v = \hbar p_y/dmE$ in velocity

Notes:

1) Electron transported by the radiation field with ultra-relativistic velocities along the direction of propagation of the radiation

2) Oscillations of the charge become slow in this case, the phase $\xi = ct - x$ is vanishing (phase velocity close to c)

3) Motion is uniform and the charge does not radiate (Lorentz reaction?)

4) Accelerated electron "feels" not anymore the radiation; fields in rest frame

$$E_z^{'} = \frac{\omega A_0}{c} \sin \frac{\omega}{c}(ct - x) \cdot \sqrt{\frac{1 - v/c}{1 + v/c}}\ ,\ H_y^{'} = -\frac{\omega A_0}{c} \sin \frac{\omega}{c}(ct - x) \cdot \sqrt{\frac{1 - v/c}{1 + v/c}}$$

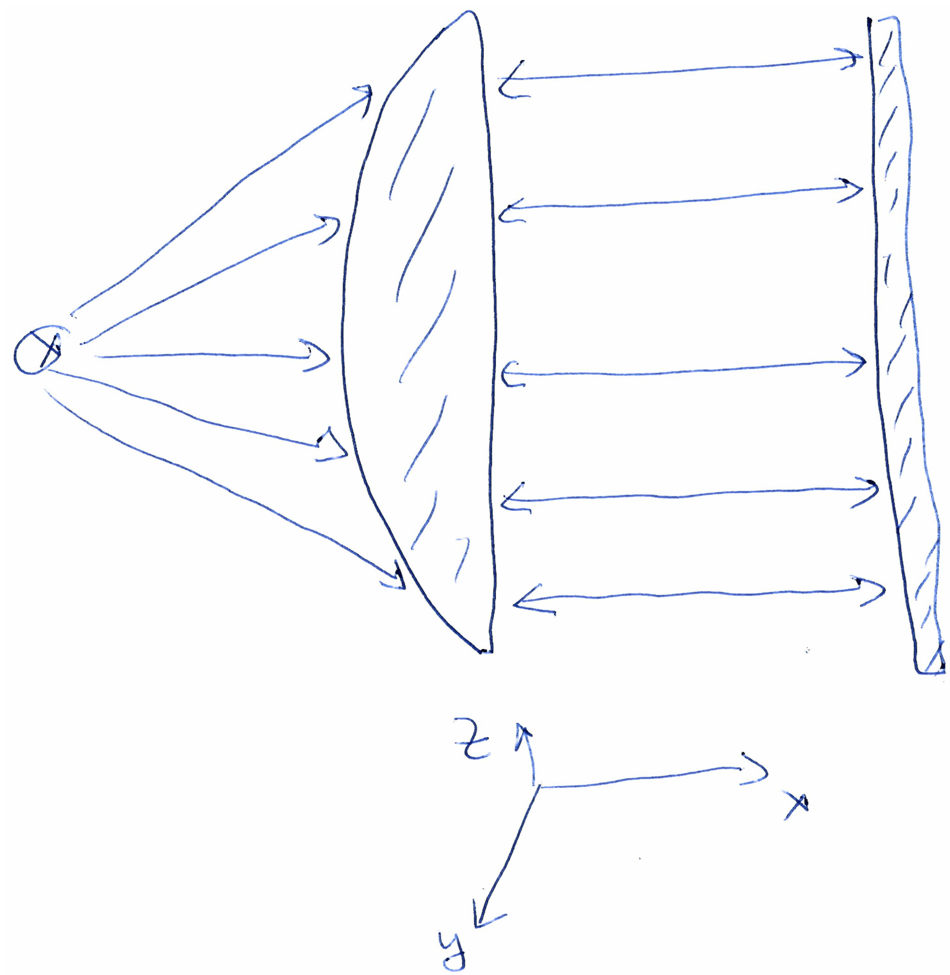
and frequency

$$\omega^{'} = \omega \sqrt{\frac{1 - v/c}{1 + v/c}}$$

(Doppler effect); for $v \to c$ these quantities vanish

Electron wavelength $\lambda \simeq \hbar/\eta mc \ll \lambda_{Compton}$: **Classical Approximation!**

IV. Standing electromagnetic wave



Vector potential

$$A = A_z = \frac{1}{2}A_0[\cos(\omega t - kx) + \cos(\omega t + kx)] = A_0 \cos \omega t \cos kx$$

optical frequency $\nu = \omega/2\pi \simeq 10^{15}s^{-1}$, wavelength $\lambda = 2\pi/k = c/\nu \simeq 3 \times 10^{-5}cm = 0.3\mu m$

More: electrons injected into the st wave, thickness, say, $1mm$, spent a time $10^{-1}/c = 3 \times 10^{-12}s$ which is much longer than the wave period $1/\nu = 10^{-15}s$; consequently: we may **average** the Hamilton-Jacobi equation

$$\frac{1}{c^2}(\partial S/\partial t)^2 = (gradS - \frac{e}{c}\mathbf{A})^2 + m^2c^2$$

with respect to the **time**:

$$\frac{1}{c^2}(\partial S/\partial t)^2 = (\partial S/\partial x)^2 + (\partial S/\partial y)^2 + (\partial S/\partial z)^2 + \frac{e^2A_0^2}{2c^2}\cos^2 kx + m^2c^2$$

Solution:

$$S = \pm\frac{eA_0}{\sqrt{2}\omega}\cos kx + p_y y + p_z z - Et$$

energy

$$\mathcal{E}^2 = m^2c^4 + \frac{1}{2}e^2A_0^2 + (p_y^2 + p_z^2)c^2$$

"Oscillating (space varying)" drift motion: longitudinal momentum (along the x-direction)

$$P_x = \frac{\partial S}{\partial x} = \pm \frac{eA_0}{\sqrt{2}c} \sin kx$$

El-ph collision: conservation of energy and momentum, **Compton effect**:

$$\mathcal{E}_0 + \hbar\omega = \sqrt{m^2c^4 + c^2p_t^2 + e^2A_0^2/2} + \hbar\omega' ,$$

$$p_{0x} + \hbar k = \pm \frac{eA_0}{\sqrt{2}c} \sin kx + \hbar k'_x$$

$$p_{0t} = p_t + \hbar k'_t ;$$

$\mathcal{E}_0 = \sqrt{m^2c^4 + c^2p_{0x}^2 + c^2p_{0t}^2}$ -energy of the incident electron

Obs: -spatial average of the electron momentum $P_x = (\pm eA_0/\sqrt{2}c)\sin kx$ inside the wave is zero

-for stability, we may suggests that spatial average of the photon momentum after collision must be zero, $\overline{k_x^{'}} = 0$

-since two photons with opposite momenta $\pm\hbar k$ are present in the standing wave in equal proportions, $\Longrightarrow$original momentum of the electron along the x-direction must also be vanishing, $p_{0x} = 0$

-in order the wave be stable, spatial average of the transverse momentum of the photon after collision must be zero, $\overline{k_t^{'}} = 0$; it follows $\overline{p}_t = p_{0t}$

However, we must allow for **fluctuations**, so that we have

$$\overline{p_t^2} = p_{0t}^2 + \hbar^2 \overline{k_t'^2}$$

and

$$\frac{e^2 A_0^2}{4c^2} = \hbar^2 \overline{k_x'^2}$$

Frequency shift

$$\hbar \Delta\omega = \hbar(\omega' - \omega) = -\frac{e^2 A_0^2/4 + \hbar^2\omega^2}{2(\mathcal{E}_0 + \hbar\omega)}$$

since $\overline{k_t'^2} > 0 \Longrightarrow eA_0/2 < \hbar\omega'$ $(eA_0 > 0)$, $\Longrightarrow eA_0/2 < \hbar\omega$

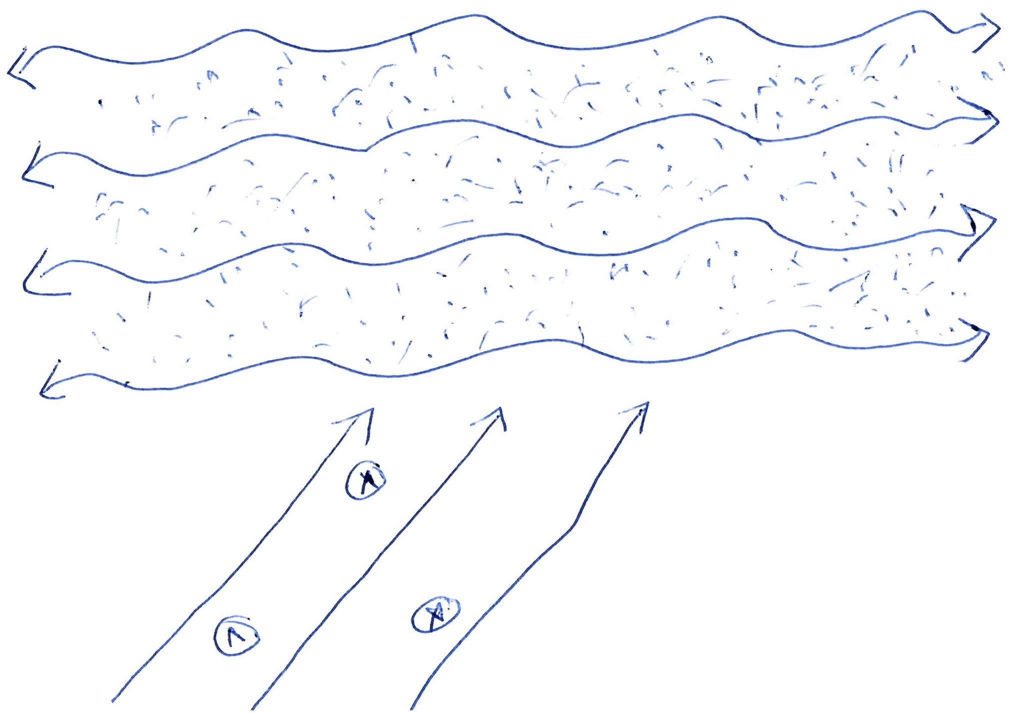
This indicates that: 1) either the electrons do not penetrate the standing wave for high values of the electromagnetic field ($eA/2 > \hbar\omega$), or 2) the standing wave is destroyed by the electron beam, or 3) the electrons simply suffer Compton collisions, without destroying the wave (and a "fluctuating" standing wave is a too strong requirement, not valid)

Usual electron beams have a very weak electron flow; consequently, the Compton effect they produce in a standing electromagnetic wave do not cause any damage to the wave

Estimation of the photon density: moderate intensity $I = 10^{18} w/cm^2$; electric field $\simeq E_0 \simeq \sqrt{I/c} = 10^7 statvolt/cm$ (and a similar magnetic field); energy density $\simeq w \simeq I/c = 10^{14} erg/cm^3$, density of photons $n \simeq 10^{25} cm^{-3}$ (energy $\hbar\omega = 1eV$); photon flow (flux) $cn \simeq 10^{35}/cm^2 \cdot s$

Relativistic electrons: accelerated to an electric current $\simeq 100mA$, $\Longrightarrow \simeq 10^{17}$ electrons per second; over a cross-sectional area $1cm^2$; $\Longrightarrow$ electron flow $\simeq 10^{17}/cm^2 \cdot s$

(Compton $\omega^{'} = \omega[1+(\hbar\omega/mc^2)(1-\cos\varphi 0]^{-1}$, $\Delta\omega = -(\hbar\omega\omega^{'}/mc^2)(1-\cos\varphi))$.



Conclusion: electron flows are much weaker than photon flows

Disruption of a standing electromagnetic wave by electron beams is highly unlikely

Electrons in a standing electromagnetic wave suffer multiple Compton collisions

Since the photon density is very high, an electron suffers **many collisions**, its **mean free path very short**, moving practically in a **straight line**; its **mean free path much shorter than the wavelength of the wave**; electron **does not "feel"** the structure of the standing wave; it behaves, practically, as a **free electron**, sufering **many collisions**; therefore, its intrusion in the standing wave has, practically, no effect (except the slight Compton effect)

Injection of electrons in a standing electromagnetic wave is practically a multiple Compton effect in vacuum

Mean free path of the electron is of the order of the mean separation distance between the photons ($\simeq 10^{-8} cm$), the Compton cross-section σ is of the order of the square of the classical electromagnetic radius of the electron ($r_e = e^2/mc^2 \simeq 2.8 \times 10^{-13} cm$), and the radiation wavelength is $\simeq 3 \times 10^{-5} cm$ ($\hbar\omega = 1 eV$)

Electron "stability" in a standing rad wave

Average, over Compton scatterings

$P_x = mv_x/(1 - v_x^2/c^2)^{1/2}$, oscillating acceleration

$$\frac{dp_x}{dt} = v_x \frac{dp_x}{dx} = \frac{1}{2} c \frac{d}{dx} \sqrt{m^2c^2 + p_x^2} = mc^2 k \frac{2\eta^2 \sin kx \cos kx}{\sqrt{1 + 2\eta^2 \sin^2 kx}}$$

coordinate $x(t)$ obtained from

$$\frac{dx}{dt} = v_x = c \frac{\pm \frac{eA_0}{\sqrt{2}mc^2} \sin kx}{\sqrt{1 + \frac{e^2 A_0^2}{2m^2c^4} \sin^2 kx}}$$

Solution: $x = \pi n \lambda$; delocalized over stable positions (optical lattice); q-mech, non-rel treatment justified

Creation of electron–positron pairs from vacuum in a standing electromagnetic wave of high-power lasers

-Since el or pos Compton wlngth much shorter than radiation wlngt: spatial variation of the standing wave disregarded

Left with a high electric field (variable in time), Schwinger limit?

No, far away

Moreover, magnetic field from sp variations diminish considerably the rate of pair production

-Catalytic creation by γ, enhance the effect of the electric field

$$(w \simeq (E_{schw}/E)^2(time = mc^2/\hbar)(vol = (mc/\hbar)^3]e^{-\frac{E_{schw}}{E}\cdot\frac{mc^2}{\hbar\omega\gamma}})$$

Time needed for creating a pair much longer than the wave period: time variation of the wave averaged!

-Apart from such dificulties, the **enlargement of the gap**:

$\sqrt{m^2c^4 + e^2A_0^2/2}$ and $-\sqrt{m^2c^4 + e^2A_0^2/2}$

-Similar considerations for electron-positron pairs created in laser fields in the presence of a Coulomb potential (Bethe-Heitler process); pair creation unlikely

V. Electron diffraction by a standing electromagnetic wave

Electron wavelengh $a \simeq \lambda \simeq 10^{-5} cm$ (radiation wvlgth)

Electron momentum $p \simeq 6 \times 10^{-22} g \cdot cm/s$, energy $cp \simeq 10 eV$

It follows: non-relativistic quantum electrons

Electron velocity $v = p/m \simeq 10^6 cm/s$; el motion takes a much longer time than the radiation period; it follows, we may average with the time

$$\mathcal{E} = \frac{1}{2m}(\mathbf{p} - \frac{e}{c}\mathbf{A})^2 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} - \frac{e}{mc} p_z A + \frac{e^2}{2mc^2} A^2$$

or

$$\mathcal{E} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{e^2 A_0^2}{4mc^2} \cos^2 kx$$

Periodic potential

$$U(x) = \frac{e^2 A_0^2}{4mc^2} \cos^2 kx$$

along the x-axis (with momentum $p_x = (eA_0/\sqrt{2}c) \sin kx$)

Energy bands; Diffraction (Kapitza-Dirac effect)

Born approximation

$$d\sigma = |f|^2 \, do = \left| \frac{m}{2\pi\hbar^2} \int d\mathbf{r}' U(\mathbf{r}') e^{i\mathbf{q}-\mathbf{q}')\mathbf{r}'} \right|^2 do$$

Bragg condition for $2k$ ($\lambda/2$)

Limitation of Born approx

$$\frac{mU_0}{\hbar^2}\frac{V}{r} \ll 1$$

(especially multiple scatterings)

-higher el energy and a thin standing wave, transmission diffraction, classical law $\frac{\lambda}{2}\sin\theta = na$, n any integer (diffraction grating)

-high radiation intensity, reflection diffraction, same law (truncated potential $U(x)$)