A model of Seismic Focus and Related Statistical Distributions of Earthquakes

Application to Vrancea, Romania

Some other issues in Theoretical Seismology

apoma laboratory, 2005

Summary

7 Vrancea earthquakes

8 Omori's law

9 Bath's law

10 Critical-point theory

11 Amplification factors

12 Non-linear elasticity

Introduction

-Little consensus

-A model of seismic focus, Omori-type parameter on geometrical grounds, time, energy and magnitude distributions, logarithmic distributions and recurrence law, seismicity rate and mean recurrence time; regular seisms

-Application to Vrancea, Romania

-Accompanying seismic activity, foreshocks and aftershocks, Omori's law, self-replication, Euler's relationship, , Bath's law, rate of released energy

-Amplification factors

-Non-linear elasticity; a non-linear wave equation with cubic anharmonicities

Gutenberg-Richter law. Critical zone

Gutenberg-Richter law (1956-1935)

$$\ln E = a + bM \;,\;\; E/E_0 = \exp(bM), \;\; a \simeq 10, \, b = 3.5$$

(E in J; errors up to a factor of 10)

Critical radius $E \sim R^3$

$$\ln(R/R_0) = bM/3 \simeq 1.17M$$

Threshold energy, radius

A model of Seismic Focus

$$\partial E/\partial t = -vgradE \sim (1/r)\frac{E+E_0}{t+t_0} \ , \ r = 1/3$$

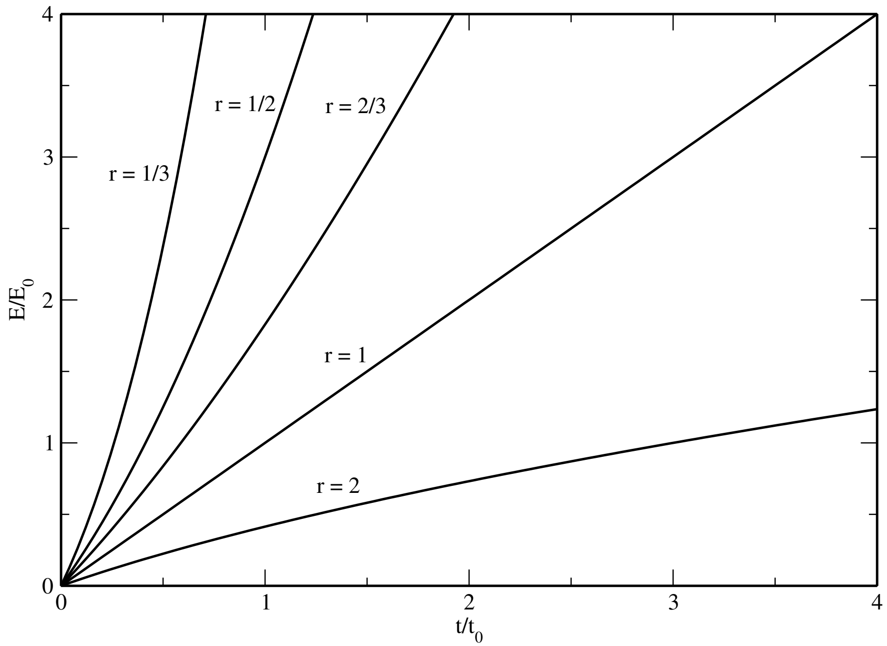
$$1 + t/t_0 = (1 + E/E_0)^r$$

(power-type, Omori-type law); geometric Omori-type parameter r

Mean recurrence time

$$t = t_0 e^{\beta M} \ , \ \beta = br = 1.17$$

Threshold time; rate of seismicity



Reduced energy *vs* reduced time for the focus model, for various r values

Temporal distribution

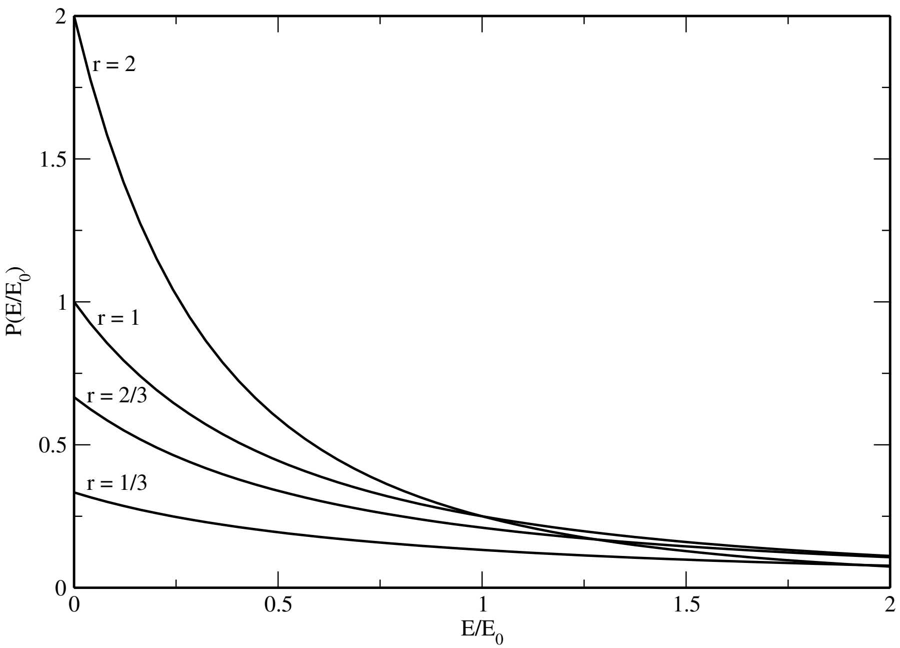
Temporal frequency: $N_0 = T/t_0$, $N = T/(t + t_0)$

$$N/N_0 \sim 1/(1 + t/t_0)$$

$$dN/N_0 = \frac{dt/t_0}{(1 + t/t_0)^2} = \frac{r}{E_0} \cdot \frac{dE}{(1 + E/E_0)^{1+r}}$$

Omori-type (power law) energy distribution

$$P(E)dE = \frac{r}{E_0} \cdot \frac{dE}{(1 + E/E_0)^{1+r}} \simeq \frac{rE_0^r}{E^{1+r}} dE$$



Energy probability distribution for various r values

Magnitude distribution

$$P(M)dM = \frac{\beta e^{bM} dM}{(1 + e^{bM})^{1+r}} \simeq \beta e^{-\beta M} dM$$

Log distribution $\lg(\Delta N/T) \sim A - BM$, agreement empirical data ($\Delta M = 0.1$)

$$A = \lg(\beta \Delta M/t_0) \simeq 4.6 \ , \ B \simeq 0.6 \ , \ 5.5 < M < 7.3$$

$$\beta = 2.3B \sim 1.38 \ vs \ \beta = 1.17$$

-agreement. Seismicity rate: $1/t_0 \sim 10^5 - 10^6$ per year (worldwide)

Recurrence law

$$P_{ex} = 1/(1 + e^{bM})^r \ \ , \ \ (> M)$$

$$\ln(N_{ex}/N_0) = -r \ln(1 + e^{bM}) \simeq -\beta M$$

$$\ln(N_{ex}/T) \simeq -\ln t_0 - \beta M$$

(a certain universality in $B = \beta/2.3 \simeq 0.6$)

Mean recurrence time

$$t_r = t_0 e^{\beta M} \ \ (> M) \ \ (or \ \frac{t_0}{\beta \Delta M} e^{\beta M} \ \ M \, to \, M + \Delta M)$$

$$P_r = (1/t_r) e^{-t/t_r}$$

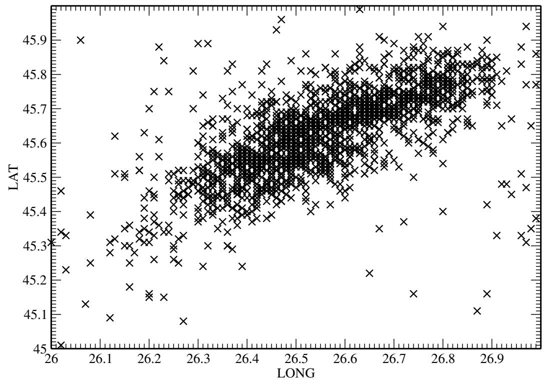
for fixed mean recurrence time (maximize the entropy!)

Accuracy $\delta t_r = (\sqrt{2} - 1) t_r \Rightarrow$ 41%; little practical usefulness for prediction

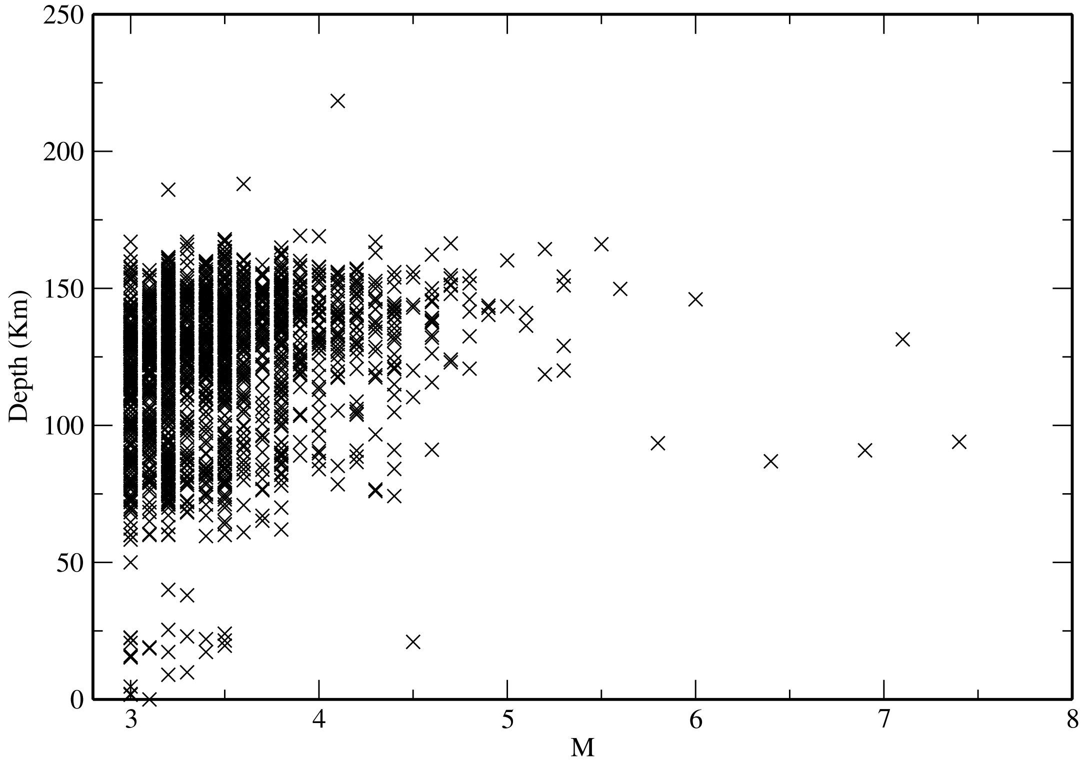
Vrancea Earthquakes

1999 Vrancea earthquakes, (moment) magnitude $M > 3$, from 1974 to 2004 (30 years)

(Romanian Earthquake Catalogue, updated 2005)



Geographical distribution (latitude 45.7N, longitude 26.6E)



Depth distribution

Magnitude distribution (error $\Delta M = 0.1$)

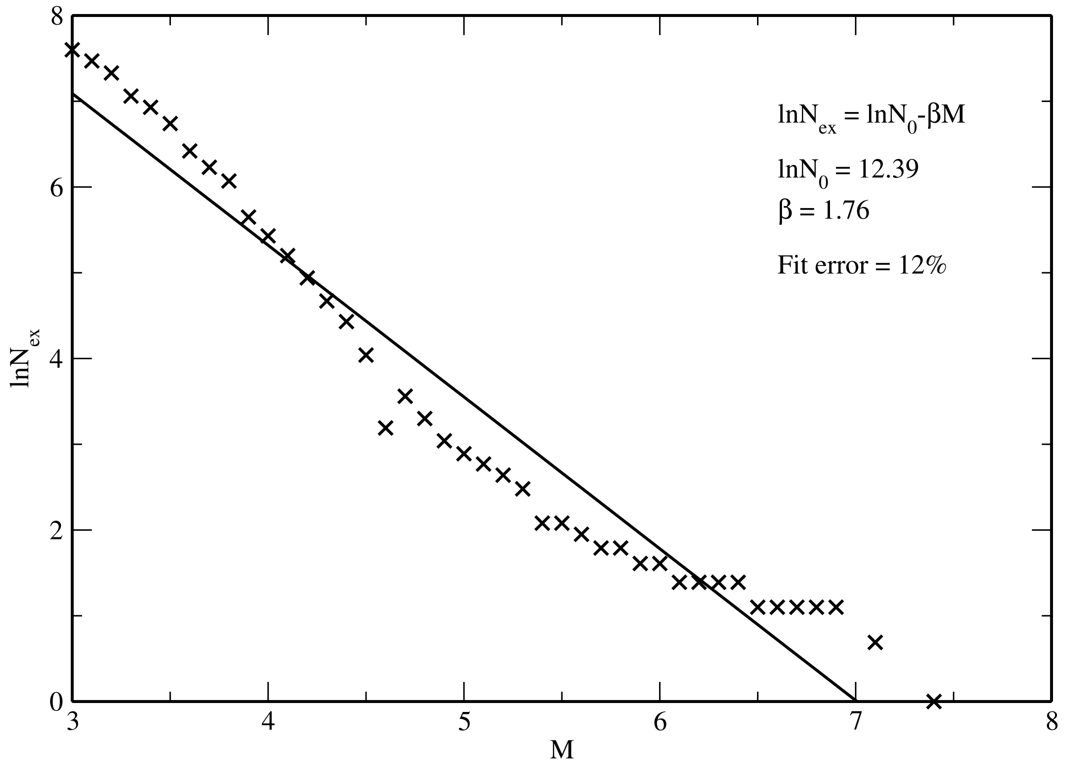
M=3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1
ΔN=245	230	362	143	176	230	109	72	147	56	48	41

M=4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3
ΔN=33	23	27	7	15	8	6	3	2	2	2	4

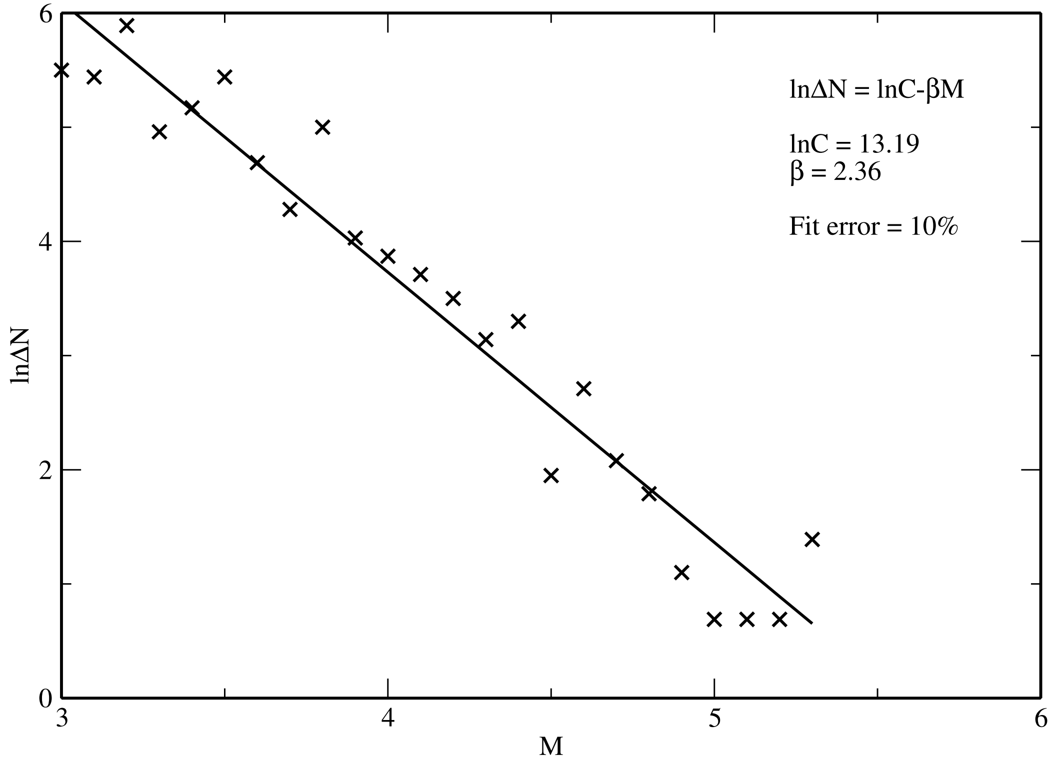
M=5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7
ΔN=0	1	1	0	1	0	1	0	0	0	1	0	0	0

M=6.8	6.9	-	7.1	-	7.4
ΔN=0	1	-	1	-	1

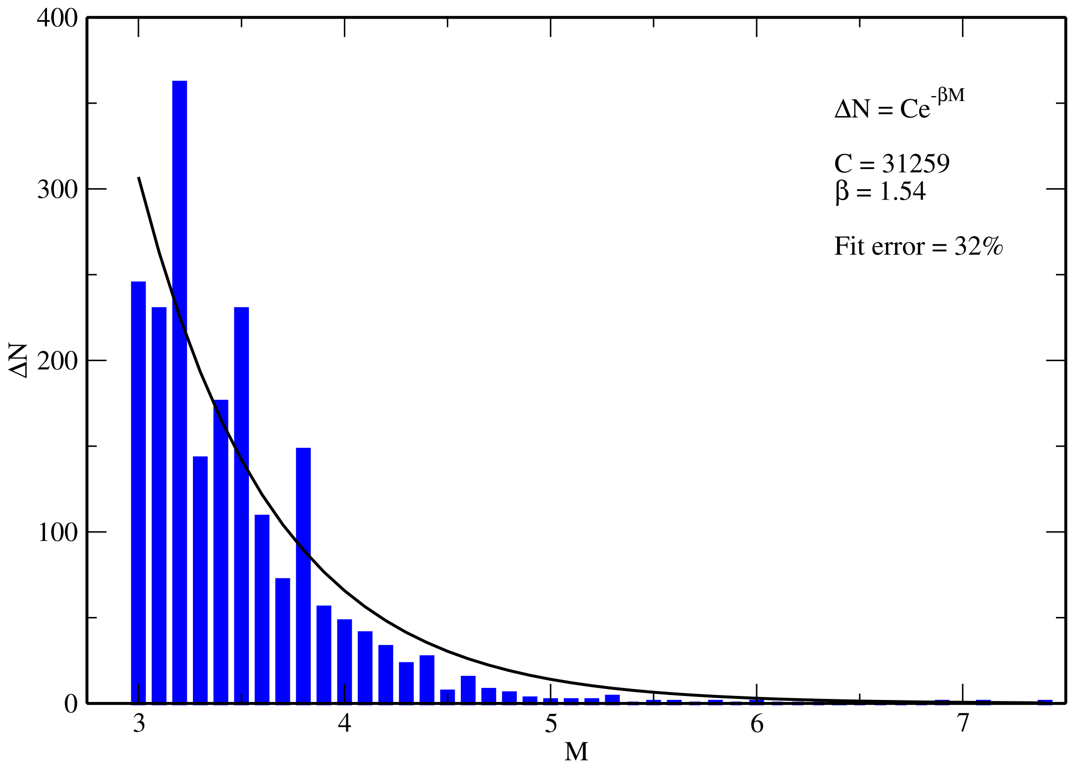
Three fits



Recurrence law $\ln N_{ex} = \ln N_0 - \beta M$ fitted to cumulative distribution



Logarithmic distribution $\ln \Delta N = \ln C - \beta M$ fitted to data



Exponential distribution $\Delta N = C\exp(-\beta M)$ fitted to data

Conclusions:

-quality *vs* accuracy of the fits (as many data as possible, slow variation, little scattering)

-average fit parameters

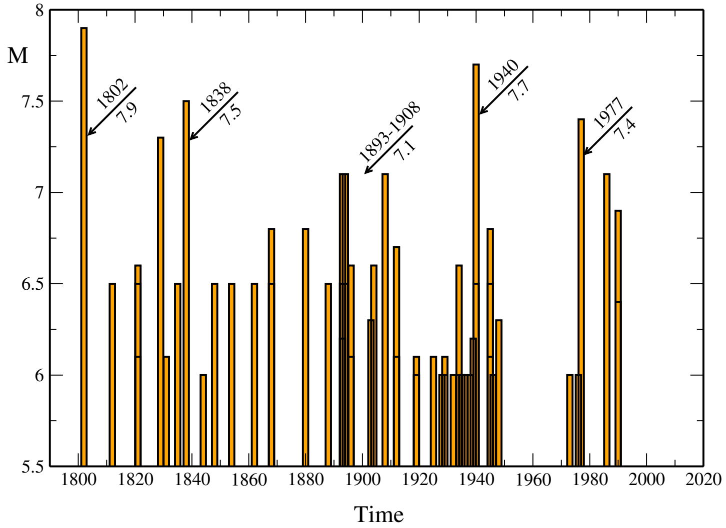
$$-\ln t_0 = 9.68 \ , \ \beta = 1.89 \ (B = \beta/2.3 = 0.82)$$

(18%)

-$r = 0.54$ ($1/r = 1.85$) $\sim$ effective dimension 2 for Vrancea focal fault?

-t_r 34.9years (41%$\sim$14.3years) $M > 7$

-logarithmic oscillations? (critical-point theory?)



Vrancea earthquakes with (moment) magnitude $M > 6$ in the last two centuries (Romanian Earthquakes Catalogue, 2005)

1802, M=7.9 1829, M=7.3 1838, M=7.5 1893, M=7.1, 1894, M=7.1 1908, M=7.1 1940, M=7.7 1977, M=7.4 1986, M=7.

Accompanying seismic activity. Omori's law

-regular earthquakes *vs* foreshocks and aftershocks; main shock, $|\tau|$

-generating distribution $p(x)$, any x (time. magnitude, energy)

-self-replication

$$P(x) = p(x) + r(x)P(x) = p(x) + \frac{p(x)}{p_0}P(x)$$

$$P(x) = \frac{p(x)}{1 - p(x)/p_0}$$

Euler's transform; singular at origin ($p_0 = p(x = 0)$); generalized Omori's law

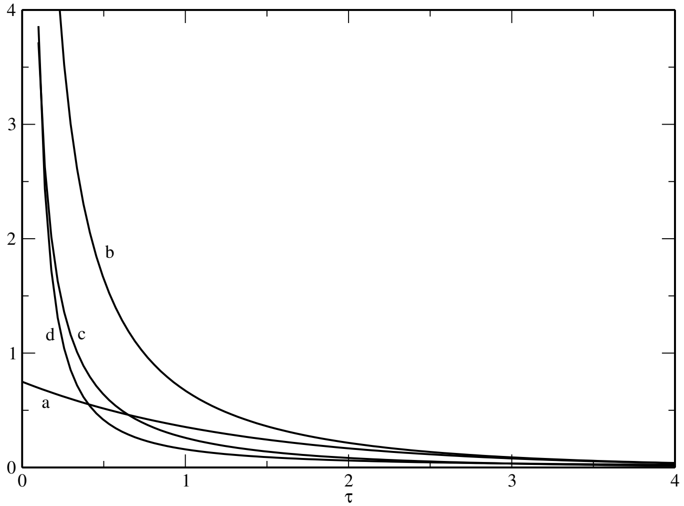
-self-consistency $p(x+y) = p(x)r(y)$, exponential $p(x) = \alpha\,\mathrm{exp}(-\alpha)$ (generating distribution)

-time distribution ($\alpha = 1/t'_c = 2/t_r$, from $|\tau| \rightarrow 0$)

$$P(\tau) = \frac{\alpha}{\mathrm{exp}(\alpha\tau) - 1}$$

-$P(\tau) = \tau_c^{\gamma-1}/\,|\tau|^{\gamma}$, $\gamma = 1^{+}$; Omori's law

-sudden fall, long tail



Exponential distribution $\alpha \exp(-\alpha\tau)$ for $\alpha = 0.75$ (curve *a*), self-replication distribution $\alpha/[(\exp(\alpha\tau) - 1]$ for $\tau > \tau_c = 1$ (curve *b*), normalized self-replication distribution (curve *c*) and normalized Omori's law $(\gamma - 1)\tau_c^{\gamma-1}/\tau^{\gamma}$ for $\gamma = 1 - 1/\ln(\alpha\tau_c)$ (curve *d*). The normalized distributions exhibit a sudden fall, and a possible long tail for small α.

Bath's law

$$P(m) = \beta e^{-\beta|m|}\ ,\ \bar{m} = 0\ ,\ \delta m = (\overline{m^2})^{1/2} = \sqrt{2}/\beta$$

-average greatest aftershock by $\sqrt{2}/\beta \simeq 1.2$ lower in magnitude ($\beta = 1.17$)

-distribution in "energy"

$$P(E) = (rE_0^r/E^{1+r})(1+E_0/E)^{-1-r} \rightarrow E_o(1+r)e^{-(1+r)E_0 x}\ ,\ x = 1/E = 1/\varepsilon$$

-rate of released energy

$$dn/d\tau \sim 1/|\tau|\ ,\ d\varepsilon/d|\tau| = -(1+r)E_0 t_c'/\tau^2$$

Critical-point Theory

$$\frac{r}{E_0} \cdot \frac{dE}{(1 + E/E_0)^{1+r}} = Ah(\tau)dt \ , \ \tau = t_c - t$$

$$h(\tau) = \frac{1-m}{\tau_0}(\tau/\tau_0)^{-m} \ , \ 0 < m < 1$$

$$\ln(E/E_0) \simeq -\frac{1-m}{r}\ln(\tau/\tau_0)$$

$$\ln(R/R_0) \simeq -\frac{1-m}{3r}\ln(\tau/\tau_0)$$

self-similarity $h(\alpha\tau) = \beta h(\tau)$; logs oscillations $\sim \cos(const \cdot \ln\tau)$

Critical exponent m

$$E \sim \tau^{(m-1)/r} \longleftrightarrow d\varepsilon/d\,|\tau| \sim 1/\tau^2$$

$$m = 1 - 2r = 1/3?; = 0?$$

Amplification factors

1 Local seismic effects. Forced linear harmonic oscillator with friction

$$m\ddot{x} + kx + \alpha x \doteq f,\ \omega^2 = k/m,\ \lambda = \alpha/2m\omega,\ \omega' = \omega(1-\lambda^2)^{1/2}$$

$$x = \frac{1}{\omega'}\int_0^t d\tau (f/m) e^{-\lambda\omega(t-\tau)} \sin\omega'(t-\tau)$$

2 **Resonance**, $f = f_0 \cos \omega_0 t$

$$x = \frac{f_0}{2m\lambda\omega^2}(1 - e^{-\lambda\omega t}) \sin \omega t$$

Maximum displacement $d_{max} = 2f_0/m\omega_0^2$ (from $ac_{max} = f_0/m$)

Displacement amplification factor

$F_d = |x|_{max}/d_{max} \cong \frac{1}{4\lambda}(1-e^{-\lambda(2k+1)\pi/2}),\ \ \omega t \cong (2k+1)\pi/2,\ \ k = 0, 1, 2...$

Typical values: $F_d \simeq 1.18,\ 1.96,\ 2.75,\ 3.53$; on increasing time

Similarly for velocities (highest amplification) and accelerations

3 Shocks

$$f = -f_0 \Delta t e^{-\Delta^2 t^2/2}$$

Amplification

$$F_d = 1/\sqrt{2\pi e}, \;\; F_v = 1, \;\; F_a = 2.28,$$

concentrated in acceleration.

4 Local effects

-amplification factors different for different locations

-amplification decreases with increasing magnitude (empir, ?)$\rightarrow$non-linear efects

-extended local efects (propagation of seismic waves)

Non-linear elasticity

1 Axially anisotropic solid

Density of elastic energy

$$\mathcal{E}_{el} = \lambda u_{ii}^2 + \mu u_{ij}^2 + \tau u_{i3}^2 + \sigma u_{33}^2 + \nu u_{33} u_{ii}, \;\; i,j = 1,2$$

Elastic waves

$$\omega_1^2(\mathbf{q}) = \frac{1}{2\rho}\left(2\mu q_\perp^2 + \tau q_3^2\right)$$

$$\omega_{2,3}^2(\mathbf{q}) = \tfrac{1}{4\rho}([4(\lambda+\mu)+\tau]\, q_\perp^2 + (\tau+4\sigma)\, q_3^2 \pm$$
$$\pm\left[\left[[4(\lambda+\mu)-\tau]\, q_\perp^2 + (\tau-4\sigma)\, q_3^2\right]^2 + 4(\tau+2\nu)^2 q_3^2 q_\perp^2\right]^{\frac{1}{2}})$$

-highly-non-linear dispersion (quartic, mode coupling)

-mixed polarizations

-extension to including non-linearities

2 Non-linear diffusion

-diffusion in complex-structure bodies, statistic fluid model, non-equilibrium; localization of seismic waves in heterogeneous structures; en diff like particles?

-non-linear diffusion equations

$$\frac{1}{S}\frac{\partial n}{\partial t} = \triangle n \pm A(\mathsf{grad} n)^2$$

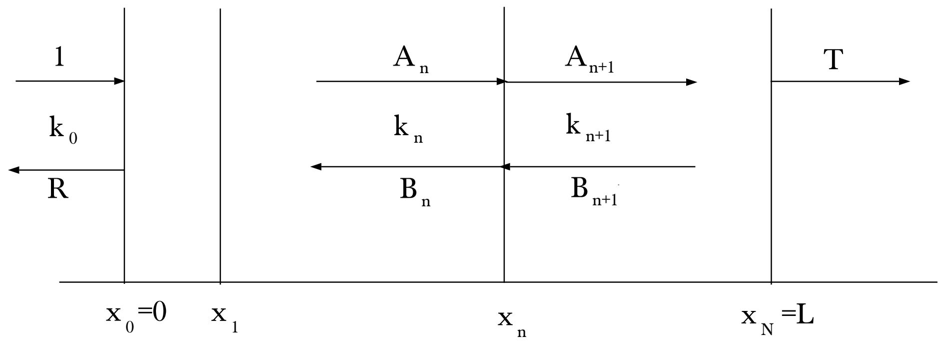
-solution (2-dimensional)

$$n \sim \frac{1}{A} \ln \left| \ln \left(r^2/4tS \right) \right| \quad , \quad r^2/4tS \ll 1$$
$$n \sim -\ln \left| (r - r_0)/2\sqrt{tS} \right| \quad , \quad \left| (r - r_0)/2\sqrt{tS} \right| \ll 1$$

-spatial auto-organization (disk, rings), slow diffusion fronts

3 Geometric rays approximation

-quasi-classical approximation, geometric optics, rays



$$\psi_n(x) = A_n e^{ik_n x} + B_n e^{-ik_n x}\ ,\ x_{n-1} < x < x_n,\ n = 0, 1, ...N + 1$$

$$Te^{ikx} = M_N M_{N-1} ... M_1 M_0 \begin{pmatrix} e^{ik_0 x} \\ Re^{-ik_0 x} \end{pmatrix}$$

$$\psi(x) = \frac{C}{\sqrt{k(x)}} e^{\pm i \int_{x_0}^{x} k \cdot dx}$$

-refraction, advanced dispersion, defects in inhomogeneous structures

4 A non-linear equation of elastic waves

-longitudinal deformations, cubic anharmonicitis

$$\partial^2 u/\partial t^2 = (\partial^2 u/\partial x^2)[v_l^2 + v^2(\partial u/\partial x)]$$

-solution

$$u(t,x) = g(t-t_0)\int_0^x dx f(x-x_0) - (v_l/v)^2 x + c$$

$$g(t) = |s|\,[\sqrt{3}\frac{1 - cn(\sqrt{\sqrt{3}\,|s|}\,|\omega t|)}{1 + cn(\sqrt{\sqrt{3}\,|s|}\,|\omega t|)} - 1]sgn(\omega^2),\ g(0) = -s,\ \dot{g}(0) = 0$$

$$f \sim |h|\, sgn(\omega/v)^2 + (\omega/2v)^2 x^2 \ , \ x \sim 0; \ \ f \sim (\omega/2v)^2 x^2 \ , \ x \to \pm\infty$$

-singularity of the solution

$$t_n \ = \ (4K/\,|\omega|\,\sqrt{\sqrt{3}\,|s|})(n + 1/2), \ \ (K \simeq 4)$$

$$g \sim 1/\omega^2 (t - T)^2$$

-unbound energy, finite-time ruptures in structure, dislocations at spatial limits

-quasi-plane waves, asymptotic series

$$u = a\cos(\omega t - kx) + \frac{1}{16}\varepsilon a^2 k^2 (x + v_l t)\cos[2(\omega t - kx)] +$$

$$+\frac{1}{128}\varepsilon^2 a^3 k^4 (x + v_l t)^2 [\cos[3(\omega t - kx)] - \cos(\omega t - kx)] + ...,$$

-perturbational parameters: $\varepsilon = (v/v_l)^2$, $ak \sim a/\lambda$, $lk \sim (x + v_l t)/\lambda$

-finite-distance, finite-time non-linear effects, for small amplitudes

-amplification factors $F = 1 + \varepsilon al/4\lambda^2$

-non-linear coupling, longitudinal (u) and transverse (v) deformations

$$u_0 = a\cos(\omega_1 t - k_1 x),\ v_0 = b\cos(\omega_2 t - k_2 x);\ v_1 = B\sin(\Omega t - Kx),$$
$$\Omega = \omega_1 \pm \omega_2,\ K = k_1 \pm k_2$$

-resonance

$$B = \frac{1}{2}abk_2\frac{v_l^2 k_1 k_2}{\Omega^2 - v_t^2 K^2},\ \ \omega_2 = \omega_1(1 + v_t/v_l)/2$$

-attenuation, different-directions coupling (directivity effects)