TITLE

First Author*, Second Author†and Third Author‡

Abstract

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keywords: first keyword, second keyword, third keyword

* `username@server.domain` first affiliation; second affilitation; third affiliation; acknowledgment for financial support (funding agency, project number);

† first affilitation; second affilitation;

‡ fourth affilitation;

1 Introduction

The phenomenological description of α-decay half-lives uses a simple picture of a preformed α-cluster penetrating through the Coulomb barrier, presented in Refs. [1, 2], with a preformation factor proportional to the fragmentation potential, as shown in Ref. [3]. Simple empirical formulas for the half-lives corresponding to ground-to-ground α-transitions have been given in Ref. [4]. The microscopic description needs a more sophisticated R-matrix theory in terms of the formation amplitude, see for instance Refs. [5, 6, 7, 8, 9].

2 Second section

Let us consider the general α-decay transition

$$P(J_P) \rightarrow D(J) + \alpha(L) \ , \tag{2.1}$$

where J_P denotes the spin/parity of the parent nucleus, J the spin/parity of the daughter nucleus and L the angular momentum of the emitted α-particle. We consider a wave function with a clustered α-daughter ansatz [10] with the total spin of the initial state

$$\Psi_{J_PM_P}(\xi, \mathbf{R}) = \sum_{c=(J,L)} \frac{f_c(R)}{R} \mathcal{Y}^{(c)}_{J_PM_P}(\xi, \hat{R}) \ . \tag{2.2}$$

Here, we introduced the core-angular harmonic

$$\mathcal{Y}^{(c)}_{J_PM_P}(\xi, \hat{R}) = \left[\Phi_J(\xi) \otimes Y_L(\hat{R})\right]_{J_PM_P} \ , \tag{2.3}$$

where $\Phi_{JM_J}(\xi)$ denotes the daughter internal wave function with ξ the daughter degrees of freedom, while $Y_{LM_L}(\hat{R})$ is the standard spherical harmonic describing the angular motion of the α-daughter system. The radial function $f_c(R)$ describes the α-daughter radial motion in the channel $c \equiv (J, L)$. At large distances it has an outgoing asymptotic expression

$$f_c(R) \rightarrow N_c H^{(+)}_L(\kappa_c R, \chi_c) \ , \tag{2.4}$$

in terms of the Coulomb-Hankel spherical wave depending on the reduced radius $\kappa_c R$ and Coulomb parameter

$$\chi_c = \frac{2Z_D Z_\alpha}{\hbar v_c} \sim \frac{2Z_D Z_\alpha}{\sqrt{Q_\alpha - E_c}} \ , \tag{2.5}$$

where Q_α is the Q-value of the decay process. By using the continuity equation one obtains that the total decay width as a sum of partial widths [10]

$$\begin{aligned} \Gamma &= \sum_c \Gamma_c = \sum_c \hbar v_c \lim_{R\to\infty} |f_c(R)|^2 \\ &= \sum_c \hbar v_c |N_c|^2 \ , \end{aligned} \tag{2.6}$$

where $v_c = \hbar\kappa_c/\mu$ is the center of mass velocity at infinity in the α-daughter channel c.

3 Third Section

3.1 First Subsection

The CSM was proposed in Refs. [11] as a tool to describe in a unified way the spectra of vibrational, transitional and rotational nuclei. It treats the surface vibrations of a deformed nucleus by using an exponential superposition of boson operators. The model was later extensively developed for the description of low-lying as well as high spin states in nuclei, including isospin degrees of freedom (for a review, see Ref. [12]).

The wave function of an axially deformed even-even nucleus in its intrinsic system of coordinates is given by a coherent superposition of quadrupole boson operators $b_{2\mu}$ with $\mu = 0$ acting on the vacuum state

$$|\psi_g\rangle = e^{d\left(b^\dagger_{20} - b_{20}\right)}|0\rangle \ , \tag{3.1}$$

in terms of the deformation parameter proportional to the static quadrupole deformation

$$d = \kappa\beta_2 \ . \tag{3.2}$$

Physical states which define the ground band are obtained by projecting out the angular momentum

$$|\varphi^{(g)}_J\rangle = \mathcal{N}^{(g)}_J \hat{P}^J_{M0}|\psi_g\rangle \ , \tag{3.3}$$

in terms of the projection operator

$$\hat{P}^J_{MK} = \sqrt{\frac{2J+1}{8\pi^2}} \int \mathrm{d}\omega D^J_{MK}(\omega)\hat{R}(\omega), \tag{3.4}$$

where $D^J_{MK}(\omega)$ is a Wigner function and $\hat{R}(\omega)$ is a rotation operator, parametrized by the Euler angles ω.

The norm of the wave function is given by

$$\mathcal{N}^{(g)}_J = \left[(2J+1)I^{(0)}_J(d)\right]^{-1/2} e^{d^2/2} , \tag{3.5}$$

in terms of the following integral

$$I^{(0)}_J(d) = \int_0^1 P_J(x)e^{d^2 P_2(x)}dx , \tag{3.6}$$

where $P_J(x)$ are Legendre polynomials. The simplest estimate of the ground band energy spectrum is given by

$$\begin{aligned} E_J(d) &= A_1\left[\langle\varphi^{(g)}_J|\hat{N}|\varphi^{(g)}_J\rangle - \langle\varphi^{(g)}_0|\hat{N}|\varphi^{(g)}_0\rangle\right] \\ &= A_1 d^2\left[\mathcal{I}_J(d) - \mathcal{I}_0(d)\right] , \end{aligned} \tag{3.7}$$

where $\hat{N}$ is the operator for the number of bosons. Here, we defined the following function depending on the deformation parameter

$$\begin{aligned} \mathcal{I}_J(d) &= \frac{I^{(1)}_J(d)}{I^{(0)}_J(d)} \\ I^{(1)}_J(d) &\equiv \frac{dI^{(0)}_J(x)}{dx}, \quad x = d^2 . \end{aligned} \tag{3.8}$$

Notice that for small values of d the energy spectrum has a vibrational character $E_J \sim A_1 J$, while for large values it has a rotational shape $E_J \sim A_1 J(J+1)$ [11]. A one parameter description of the CSM Hamiltonian leads to a universal dependence of the energy ratio on the deformation parameter

$$\frac{E_{J+2}}{E_J} = \frac{\mathcal{I}_{J+2}(d) - \mathcal{I}_0(d)}{\mathcal{I}_J(d) - \mathcal{I}_0(d)} . \tag{3.9}$$

3.2 Second subsection

For an odd-mass nucleus, the state of total angular momentum I and projection M is given by projecting out the product between the coherent state (3.1) and the single particle state ψ_{jm}, where j is a shorthand notation for all of the quantum numbers of the state, that is

$$\Phi_{IM} = P^I_{M0}\left[\psi_j\phi_g\right] . \tag{3.10}$$

A straighforward calculation leads to the following result

$$\Phi_{IM} = \sum_J X_I^{Jj} \left[\varphi_J^{(g)} \otimes \psi_{jm}\right]_{IM}, \tag{3.11}$$

with normalization coefficients X_I^{Jj} given by

$$X_I^{Jj} = \frac{\left(\mathcal{N}_J^{(g)}\right)^{-1} \langle jJ; \Omega 0 | I\Omega\rangle}{\sqrt{\sum_{J'} \left(\mathcal{N}_{J'}^{(g)}\right)^{-2} \left(\langle jJ'\Omega 0|I\Omega\rangle\right)^2}}, \tag{3.12}$$

where Ω is the fixed z-projection of the single-particle angular momentum j.

The states built upon the bandhead $I = j = \Omega$ that follow the sequence $I = \Omega, \Omega+1, \Omega+2, \ldots$ constitute a rotational band. In the Nilsson model, these states are labeled by the set $\Omega^\pi\,[N n_z \Lambda]$, where π is the parity, N is the principal quantum number, n_z the number of nodes of the radial wavefunction in the z direction and Λ the projection of the single-particle orbital angular momentum. The last three numbers act only as labels, as the good quantum numbers are only Ω and π.

The simplest Hamiltonian that can describe such a rotational structure consists of two terms:

$$H = A_1 b_2^\dagger \cdot b_2 - A_2 r^2 \left(b_2^\dagger + \tilde{b}_2\right) \cdot Y_2. \tag{3.13}$$

where by dot we denoted the scalar product. A_1 is a strength parameter required to fit experimental data and A_2 is the strength of the particle-core QQ interaction.

For the description of the rotational band the only relevant parameter is A_1 due to the fact that the particle-core term is common. Instead of solving the eigenvalue problem by a full diagonalization procedure, a simpler approach, involving the analytical expression for the diagonal matrix elements of the Hamiltonian (3.13) in the basis of Eq. (3.11) suffices:

$$\begin{aligned}\langle IM|H|IM\rangle &= A_1 d^2 f_{j\Omega I} - d\left(N + \frac{3}{2}\right) \times \\ &\times \langle j2; \Omega 0|j\Omega\rangle \langle j2; \frac{1}{2}0|j\frac{1}{2}\rangle,\end{aligned} \tag{3.14}$$

with $f_{j\Omega I}$ given by

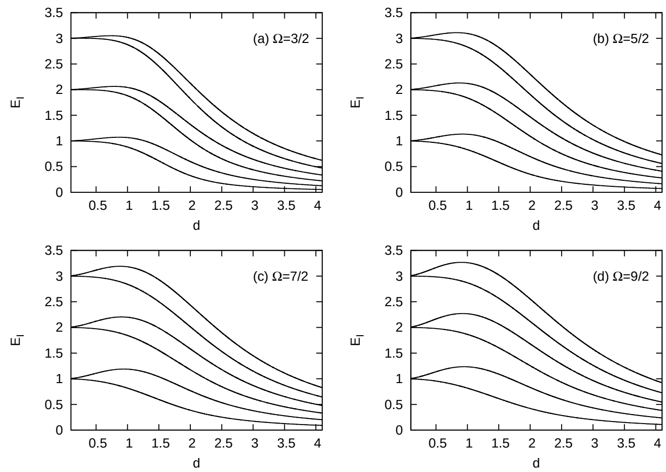

Figure 1: Normalized energy levels E_I as function of deformation d, for different values of the single particle angular momentum projection Ω.

$$f_{j\Omega I} = \frac{\sum\limits_{J} \langle Ij; \Omega - \Omega | J0 \rangle^2 \mathcal{I}_J^{(1)}(d)}{\sum\limits_{J} \langle Ij; \Omega - \Omega | J0 \rangle^2 \mathcal{I}_J^{(0)}(d)} \ . \tag{3.15}$$

The shape of such a spectrum is dependent both on the deformation parameter and on the value of Ω, as can be seen in Fig. 1.

While this approach is adequate, if a greater precision in the description of the nuclear energy spectrum is required, then more terms can be added to the Hamiltonian (3.13). Let us also mention that the development presented here is appropriate for any rotational band built upon an angular momentum projection $\Omega \neq \frac{1}{2}$. The special case $\Omega = \frac{1}{2}$ requires a modification of the formalism.

4 Conclusions

Here include Conclusions.

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