Tests of Heterotic – F-theory duality with fluxes

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Plan of the talk

- Heterotic – type IIA duality in four dimensions
- Heterotic fluxes and type IIA (M-theory) dual setup
- Heterotic compactifications with duality twists and R-fluxes
- F-theory dual setup
- Conclusions

Heterotic – type IIA duality in 4d

Heterotic$/K3 \times T^2 \leftrightarrow$ type IIA/CY$_3$.

N=2 supergravity in 4d coupled to n_v vector multiplets and n_h hypermultiplets

Heterotic $\rightarrow$ Coulomb branch $\rightarrow U(1)^{n_v+1}$

KK vectors, B-field on T^2 and 10d gauge fields

$$A^0 = g_{\mu 4}\ , \quad A^1 = g_{\mu 5}\ , \quad A^2 = B_{\mu 4}\ , \quad A^3 = B_{\mu 5}\ , \quad A^a\ .$$

Scalar fields

$$\begin{aligned} u &\ - \ \text{complex structure of } T^2\ ; & t &\ - \ \text{complexified volume of } T^2\ ; \\ n^a &= A_5^a + uA_4^a\ ; & s &\ - \ \text{axio} - \text{dilaton} \end{aligned}$$

Type IIA vector multiplet sector $\leftrightarrow \omega_i \in H^{1,1}(CY_3)$

Gauge group $U(1)^{h^{1,1}+1} \to h^{1,1} = n_v$

Gauge fields comming from RR 3-form potential C_3 expanded in the CY$_3$ harmonic $(1,1)$ forms and the 10d graviphoton A^0

$$\hat{C}_3 = \ldots + A^i \omega_i + \ldots$$

Scalars in the vector multiplets – complexified Kähler moduli

$$B + iJ = (b^i + iv^i)\omega_i$$

Heterotic/$K3 \times T^2$ with fluxes

Gauge field fluxes on T^2 and various twisting of $T^2 \to$ non-Abelian structure and gaugings in the vector multiplet sector

$$\int_{T^2} F^a = f^a \ ,$$

$$F^0 = dA^0 \ , \quad F^1 = dA^1 \ ,$$

$$F^2 = dA^2 + f^a A^a \wedge A^1 \ , \quad F^3 = dA^3 - f^a A^a \wedge A^0 \ , \qquad F^a = dA^a + f^a A^0 \wedge A^1 \ ,$$

$$Dt = \partial t + \sqrt{2} f^a n^a A^1 + f^a A^a \ ; \quad Dn^a = \partial n^a - \frac{1}{\sqrt{2}} f^a (A^3 + uA^1) \ .$$

Type IIA dual setup?

✘ Type IIA/CY$_3$ – Abelian vector multiplet sector.

✘ No known fluxes induce gaugings in the vector multiplet sector

Hint: look at the duality in 5d: Heterotic/$K3 \times S^1$ vs M-theory/CY$_3$

Heterotic T^2 fluxes $=$ monodromy of gauge field-scalars around $S^1 \rightarrow$ 4d

➠ do the same in M-theory

Isometry of the vector moduli space $SO(1,1) \times SO(1, n_v - 2)$

Perform Scherk–Schwarz compactification: twist the vectors multiplets by an element of the isometry group as we go around the circle

Do it in one step $\rightarrow$ M-theory on 7d manifolds with $SU(3)$ structure.

7d manifolds with $SU(3)$ structure

Spread monodromy evenly over the circle

$$\omega_i(z+\epsilon) = \omega_i(z) + \epsilon M_i^j \omega_j(z)\ , \quad M \ - \ \text{constant}\ , \quad \gamma_i^j = (e^M)_i^j$$

ω_i - harmonic on CY_3 slices

$$\mathbf{d\omega_i = M_i^j \omega_j \wedge dz}$$

Consistency condition on the CY_3 intersection numbers $\mathcal{K}_{ijk}$

$$0 = \int_{X_7} d(\omega_i \wedge \omega_j \wedge \omega_k) \quad \Leftrightarrow \quad \mathcal{K}_{ijl} M_k^l + \mathcal{K}_{jkl} M_i^l + \mathcal{K}_{kil} M_j^l = 0$$

Duality to Heterotic/$K3 \times T^2$

$CY_3 = K3$ fibered over a $\mathbf{P_1}$ base.

Limit Vol$(\mathbf{P_1}) \sim$ large ➞ non-vanishing intersection numbers

$$\mathcal{K}_{123} = -1 \; , \qquad \mathcal{K}_{1ab} = 2\delta_{ab} \; , \quad a, b = 4, \ldots, h^{1,1} = n_v \; .$$

Solve constraint $M^l_{(i}\mathcal{K}_{jk)l} = 0$:

$$M^2_2 = m_2 \; , \quad M^2_a = m_a \; , \quad M^3_3 = m_3 \; , \quad M^3_a = \tilde{m}_a \; , \quad M^b_a = -M^a_b = m^b_a \; ,$$

$$M^a_2 = \tfrac{1}{2}\tilde{m}_a \; , \quad M^a_3 = \tfrac{1}{2}m_a \; , \quad M^a_a = -\tfrac{1}{2}M^1_1 = \tfrac{1}{2}(m_2 + m_3) \; .$$

Duality to Heterotic/$K3 \times T^2$

$\tilde{m}_a \neq 0 \leftrightarrow$ heterotic gauge field fluxes on T^2 [Aharony, Berkooz, Louis, AM]

Identifies A^0 on the heterotic side with the KK vector along the M-theory circle.

$m_2 + m_3 \neq 0$ – not a valid twist (not in the U-duality group)

m_a T-dual to $\tilde{m}_a \rightarrow$ need sort of T-fold on heterotic side

$m = m_2 = -m_3$ and m_a^b in heterotic?

[Dabholkar, Hull; Reid-Edwards, Spanjaard] $\longrightarrow$ Heterotic/T^d duality twists.

Double twist reduction

Compactification on $S^1 \to SO(1, n_v - 2)$ symmetry

compactification on the second S^1 with $SO(1, n_v - 2)$ duality twist

Twist matrix

$$N_I{}^J = \begin{pmatrix} f & 0 & M^b \\ 0 & -f & W^b \\ -W_a & -M_a & S_a{}^b \end{pmatrix}$$

Match perfectly the M-theory twists

$$\begin{array}{cccc} f \leftrightarrow m; & M^a \leftrightarrow \tilde{m}_a; & W^a \leftrightarrow m_a; & S_a{}^b \leftrightarrow m_a{}^b \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \text{geom flux} & T^2 - \text{flux} & \text{non} - \text{geom flux} & \text{Cartan torus twist} \end{array}$$

Heterotic R-fluxes

Extension in [Reid-Edwards, Spanjaard] – "R"-fluxes (T-duality along non-isometric directions) twist with the full 4d duality group $SO(2, n_v - 1)$

New twist matrix $\tilde{N}_I{}^J$

$$\tilde{N}_I{}^J = \begin{pmatrix} q & 0 & U^b \\ 0 & -q & V^b \\ -V_a & -U_a & G_a{}^b \end{pmatrix} \; ; \qquad [N, \tilde{N}] = 0$$

No direct explicit compactification; twist matrices $\rightarrow$ structure constants in the underlying $N = 4$ theory $\rightarrow$ determines gaugings and structure constants in the $N = 2$ theory.

No direct method of computing the potential; can only be determined from $N = 2$ sugra relations in 4d

Killing vectors [14]

$$k_1 = -(fu + \sqrt{2}W^a n^a)\partial_u + (ft + \sqrt{2}M^a n^a)\partial_t + \tfrac{1}{\sqrt{2}}(M^a u - W^a t + \sqrt{2}S_{ab}n^b)\partial_a \qquad (1)$$

$$k_2 = -(qu + \sqrt{2}V^a n^a)\partial_u + (qt + \sqrt{2}U^a n^a)\partial_t + \tfrac{1}{\sqrt{2}}(U^a u - V^a t + \sqrt{2}G_{ab}n^b)\partial_a$$

$$k_3 = -\big(f - qn^a n^a - \sqrt{2}U^a n^a u)\partial_u + (qt^2 + \sqrt{2}U^a n^a t)\partial_t$$

$$\quad +(qtn^a + \sqrt{2}U^b n^b n^a - \tfrac{1}{\sqrt{2}}U^a(ut - n^b n^b) - \tfrac{1}{\sqrt{2}}M^a\big)\partial_a$$

$$k_4 = \ldots \text{similar to } k_3$$

$$k_b = -(V^b n^a n^a - U^b u^2 - \sqrt{2}G_b{}^a n^a u - W^b)\partial_u + (U^b n^a n^a - V^b t^2 - \sqrt{2}G_b{}^a n^a t - M^b)\partial_t$$

$$\quad + \left[\big(U^b u - V^b t + \sqrt{2}G_{bc}n^c\big)n^a - \tfrac{1}{\sqrt{2}}G_{ba}(ut - n^c n^c) + \tfrac{1}{\sqrt{2}}S_{ab}\right]\partial_a$$

F-theory dual

1$^{\text{st}}$ twist matrix – need 1 KK gauge field $\longrightarrow$ M-theory/CY$_3 \times S^1$

2$^{\text{nd}}$ twist matrix – need 2 KK gauge fields $\longrightarrow$ F-theory/CY$_3 \times T^2$

Twist CY$_3$ over T^2 as in the M-theory case $\longrightarrow$ eight-dimensional manifold with SU(3) structure.

$$\mathbf{d}\omega^{\mathbf{a}} = (\mathbf{M_i})^{\mathbf{a}}{}_{\mathbf{b}}\omega^{\mathbf{b}} \wedge \mathbf{dz^i}$$

Consistency: $d^2\omega^a = 0 \longrightarrow [M_1, M_2] = 0$

M_i – antisymmetric $\longrightarrow$ number of parameters coresponds to that on the heterotic side

Compactification

Compare heterotic result 12 to F-theory compactifications

No 12d effective acion for F-theory $\rightarrow$ 2 step compactification:

- compactify F-theory on CY_3 – known from dualities
- Sherk-Schwarz compactification on T^2 – fields which come from the expansion in ω^a vary on T^2.

$$\left.\begin{array}{ccc} v^a & - & CY_3 \text{ Kahler moduli} \\ B^a & - & \text{from expansion of } C_4 \end{array}\right\} \text{anti} - \text{self} - \text{dual tensor multiplet in 6d}$$

supergravity multiplet: g_{MN} and B_{MN}-self-dual tensor field

No action for 6d sugra with arbitrary number of tensor multiplets $\rightarrow$ ignore the self-duality condition in 6d and impose it in 4d.

Start from 6d kinetic term

$$-\frac{1}{2} g_{\alpha\beta} \hat{d}\hat{B}^\alpha \wedge *\hat{d}\hat{B}^\beta$$

4d degrees of freedom

$$\begin{aligned} \hat{B}^\alpha &= B^\alpha + A_i^\alpha \wedge dz^i + b^\alpha dz^1 \wedge dz^2 \\ dB^\alpha &\sim *db^\alpha \,, \qquad dA_1^\alpha \sim *dA_2^\alpha \end{aligned}$$

4d vector multiplet: $(A_1^\alpha, b^\alpha, v^\alpha)$ or vector-tensor multiplet: $(A_1^\alpha, B^\alpha, v^\alpha)$

F-theory compactification on 8d manifold with SU(3) structure $\Leftrightarrow$ allow 6d tensor multiplets dependence on T^2

$$\partial_i \hat{B}^\alpha = (M_i)^\alpha{}_\beta \hat{B}^\beta \,, \qquad \partial_i v^\alpha = (M_i)^\alpha{}_\beta v^\beta$$

6d field strengths

$$\hat{d}\hat{B}^\alpha = dB^\alpha + dA_i^\alpha \wedge dz^i + db^\alpha \wedge dz^1 \wedge dz^2 + (M_i)^\alpha{}_\beta (B^\beta + A_j^\beta \wedge dz^j + b^\beta dz^1 \wedge dz^2) \wedge dz^i$$

4d field strengths

$$
\begin{aligned}
H^\alpha &= dB^\alpha + F_1^\alpha \wedge V^1 + (dA_2^\alpha + M_2{}^\alpha{}_\beta) \wedge V^2 \\
F_1^\alpha &= dA_1^\alpha + M_1{}^\alpha{}_\beta B^\beta - Db^\alpha \wedge V^2 \\
F_2^\alpha &= dA_2^\alpha + M_2{}^\alpha{}_\beta B^\beta + Db^\alpha \wedge V^1 \\
Db^\alpha &= db^\alpha - M_1{}^\alpha{}_\beta A_2^\beta + M_2{}^\alpha{}_\beta A_1^\beta
\end{aligned}
$$

$V^{1,2}$ – KK vectors on T^2

4d action

$$
-\frac{1}{2}\left(g_{\alpha\beta} H^\alpha \wedge *H^\beta + g^{ij} g_{\alpha\beta} F_i^\alpha \wedge *F_j^\beta + g_{\alpha\beta} Db^\alpha \wedge *DB^\beta\right)
$$

Self-duality conditions

$$\eta_{\alpha\beta} Db^{\beta} = g_{\alpha\beta} * H^{\beta} \; ; \qquad \eta_{\alpha\beta} F_i^{\beta} = \epsilon_{ij} g^{jk} g_{\alpha\beta} * F_k^{\beta} \; .$$

Can be obtained by adding suitable total derivative terms to the action

$$S_t = -\eta_{\alpha\beta} H^{\alpha} \wedge Db^{\beta} - \eta_{\alpha\beta} F_1^{\alpha} \wedge F_2^{\beta} + 2\eta_{\alpha\beta} M_2^{\beta}{}_{\gamma} dA_1^{\alpha} \wedge B^{\gamma} + \eta_{\alpha\beta} M_1^{\alpha}{}_{\delta} M_2^{\beta}{}_{\gamma} B^{\delta} \wedge B^{\gamma}$$

Can not directly eliminate magnetic dof. (1103.4813, Andrianopoli et al)

Magnetic gaugings in Db^{α} can be mapped after electric-magnetic duality and field redefinitions to electric gaugings quadratic in the scalars.

Need to perform electric magnetic duality for V^1 or V^2, but they both participate in gaugings.

One vanishing twist matrix $\rightarrow$ result identical to M-theory compactifications on 7d mf with SU(3) structure

$M_1 \sim M_2$ same results as in the heterotic case except magnetic gaugings

Conclusions

- Heterotic R-fluxes $\leftrightarrow$ F-theory on 8d mf with SU(3) structure
- Heterotic – can determine gaugings
- Expect massive tensors after electric magnetic duality
- F-theory/CY$_3$ + Sherk-Schwarz compactification on T^2
- Massive tensors present
- No explicit relation between the two sides when both twist matrices $\neq 0$
- Need a better understanding of N=2 gauged sugra with vector-tensor multiplets