Heterotic – type IIA duality with fluxes

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Work in progress

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Plan of the talk

- Heterotic/ $K3 \times T^2$ with fluxes
- Type IIA dual setup $\longrightarrow$ M-theory on 7d manifolds with $SU(3)$ structure
- Heterotic - type IIA duality – the vector multiplet sector
- Conclusions

Heterotic/$K3 \times T^2$

N=2 supergravity in 4d

n_v vector multiplets $\leftrightarrow T^2$

n_h hypermultiplets $\leftrightarrow K3$

Gauge group in absence of fluxes $U(1)^{n_v+1}$

$$A^0 = g_{\mu 4} \; , \quad A^1 = g_{\mu 5} \; , \quad A^2 = B_{\mu 4} \; , \quad A^3 = B_{\mu 5} \; .$$

Hypermultiplets: $K3$ moduli and gauge bundle moduli

Heterotic/$K3 \times T^2$ with fluxes

Gauge field fluxes on T^2 and various twisting of $T^2 \to$ non-Abelian structure and gaugings in the vector multiplet sector

$$\int_{T^2} F^a = f^a \ ,$$

$$F^0 = dA^0 \ , \quad F^1 = dA^1 \ ,$$

$$F^2 = dA^2 + f^a A^a \wedge A^1 \ , \quad F^3 = dA^3 - f^a A^a \wedge A^0 \ , \qquad F^a = dA^a + f^a A^0 \wedge A^1 \ ,$$

Type IIA dual setup?

✘ Type IIA/CY$_3$ – Abelian vector multiplet sector.

✘ No known fluxes induce gaugings in the vector multiplet sector

Hint: look at the duality in 5d: Heterotic/$K3 \times S^1$ vs M-theory/CY$_3$

Heterotic T^2 fluxes $=$ monodromy of gauge field-scalars around $S^1 \rightarrow$ 4d

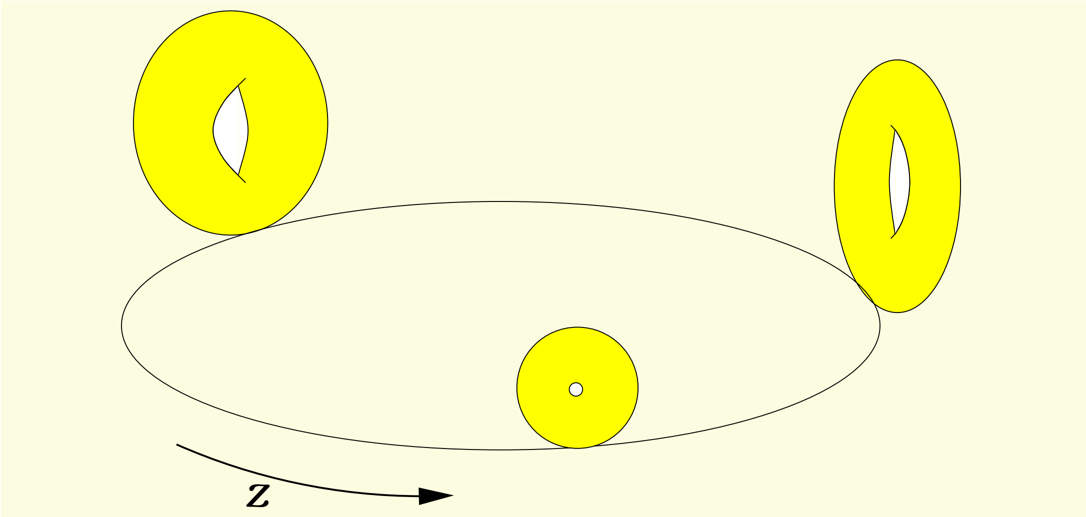
➠ do the same in M-theory

Isometry of the vector moduli space $SO(1,1) \times SO(1, n_v - 2)$

Perform Scherk–Schwarz compactification: twist the vectors multiplets by an element of the isometry group as we go around the circle

Do it in one step $\rightarrow$ M-theory on 7d manifolds with $SU(3)$ structure.

7d manifolds with $SU(3)$ structure



Twist $H^2(CY_3)$ as we go around the circle $\omega_i \longrightarrow \gamma_i^j \omega_j$

Consistency condition: $\gamma_i^j \in$ U-duality group $\Gamma(\mathbf{Z}) = SO(1, n_v - 2, \mathbf{Z})$

7D manifolds with $SU(3)$ structure

Spread monodromy evenly over the circle

$$\omega_i(z+\epsilon) = \omega_i(z) + \epsilon M_i^j \omega_j(z) \ , \quad M \ - \ \text{constant} \ , \quad \gamma_i^j = (e^M)_i^j$$

ω_i - harmonic on CY_3 slices

$$\mathbf{d\omega_i = M_i^j \omega_j \wedge dz}$$

Consistency condition on the CY_3 intersection numbers $\mathcal{K}_{ijk}$

$$0 = \int_{X_7} d(\omega_i \wedge \omega_j \wedge \omega_k) \quad \Leftrightarrow \quad \mathcal{K}_{ijl} M_k^l + \mathcal{K}_{jkl} M_i^l + \mathcal{K}_{kil} M_j^l = 0$$

Duality to Heterotic/$K3 \times T^2$

$CY_3 = K3$ fibered over a $\mathbf{P_1}$ base.

Limit Vol$(\mathbf{P_1}) \sim$ large ➞ non-vanishing intersection numbers

$$\mathcal{K}_{123} = -1 \; , \qquad \mathcal{K}_{1ab} = 2\delta_{ab} \; , \quad a, b = 4, \ldots, h^{1,1} = n_v \; .$$

Solve constraint $M^l_{(i}\mathcal{K}_{jk)l} = 0$:

$$M_2^2 = m_2 \; , \quad M_a^2 = m_a \; , \quad M_3^3 = m_3 \; , \quad M_a^3 = \tilde{m}_a \; , \quad M_a^b = -M_b^a = m_a^b \; ,$$

$$M_2^a = \tfrac{1}{2}\tilde{m}_a \; , \quad M_3^a = \tfrac{1}{2}m_a \; , \quad M_a^a = -\tfrac{1}{2}M_1^1 = \tfrac{1}{2}(m_2 + m_3) \; .$$

Duality to Heterotic/$K3 \times T^2$

$\tilde{m}_a \neq 0 \leftrightarrow$ heterotic gauge field fluxes on T^2 [Aharony, Berkooz, Louis, AM]

Identifies A^0 on the heterotic side with the KK vector along the M-theory circle.

$m_2 + m_3 \neq 0$ – not a valid twist (not in the U-duality group)

m_a T-dual to $\tilde{m}_a \rightarrow$ need sort of T-fold on heterotic side

$m = m_2 = -m_3$ and m_a^b in heterotic?

[Dabholkar, Hull; Reid-Edwards, Spanjaard] $\longrightarrow$ Heterotic/T^d duality twists.

Double twist reduction

Compactification on $S^1 \rightarrow SO(1, n_v - 2)$ symmetry

compactification on the second S^1 with $SO(1, n_v - 2)$ duality twist

Twist matrix

$$N_I{}^J = \begin{pmatrix} f & 0 & M^b \\ 0 & -f & W^b \\ -W_a & -M_a & S_a{}^b \end{pmatrix}$$

Match perfectly the M-theory twists

$$\begin{array}{cccc} f \leftrightarrow m; & M^a \leftrightarrow \tilde{m}_a; & W^a \leftrightarrow m_a; & S_a{}^b \leftrightarrow m_a{}^b \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \text{geom flux} & T^2 - \text{flux} & \text{non} - \text{geom flux} & \text{Cartan torus twist} \end{array}$$

Tempting speculation: Heterotic – F-theory duality

Extension in [Reid-Edwards, Spanjaard] – "R"-fluxes (T-duality along non-isometric directions)

New twist matrix $\tilde{N}_I{}^J$

$$\tilde{N}_I{}^J = \begin{pmatrix} q & 0 & P^b \\ 0 & -q & V^b \\ -V_a & -P_a & \tilde{S}_a{}^b \end{pmatrix}$$

Type IIA/M-theory – need a second circle $\rightarrow$ F-theory/$CY_3 \times S^1 \times S^1$

Twist the $(1,1)$ basis along both directions in $T^2 \rightarrow$ new twist matrix $\tilde{M}_i{}^j$ may match $\tilde{N}_I{}^J$: check algebra, gaugings, constraints...

Comparison

Heterotic	F-theory
Str constants: extrapolations/dualities (no direct way to compute)	?
Gaugings from structure constants – no direct way to compute	?
All vector fields involved in the gaugings type IIA symplectic frame $\longrightarrow$ massive tensors	Tensor multiplets in 6d can get masses from the twist to 4d
Potential from $N = 2$ sugra	Compute potentnial?

Conclusions

✔ Fairly complete map of the fluxes in heterotic – type IIA duality

✔ Vector multiplet sector $\longrightarrow$ clean map; involves M-theory and possibly F-theory

✔ Non-geometric fluxes in heterotic $\leftrightarrow$ geometric fluxes in M-theory

✔ R-type fluxes in heterotic $\leftrightarrow$ geometric fluxes in F-theory

✘ $SU(3) \times SU(3)$ - structure in IIA $\longrightarrow$ magnetic gauge fields $\leftrightarrow$ heterotic version?