Moduli Stabilisation in Heterotic Models with Matter Fields

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Work in progress

Tel Aviv, 22 September, 2009

Motivation

Moduli stabilisation – fair understanding in type II theories

Realistic string models – open problem

Questions:

- do we need to fix al moduli? if not which should be unfixed
- what is the connection between the matter sector and the stabilisation of moduli?
- are there realistic models with stabilised moduli?

Plan of the talk

- Introduction: Fluxes and manifolds with $SU(3)$ structure
- Basics of heterotic compactifications on manifolds with $SU(3)$ structure
- Moduli stabilisation in heterotic models
- Conclusions

Introduction: Fluxes and manifolds with $SU(3)$ structure

General setup: string compactifications on Calabi–Yau manifolds ($SU(3)$ holonomy)

Type II theories – many possible fluxes: NS-NS and RR

– there are instances where all moduli are stabilised

This is not the full story $\rightarrow$ use dualities to see if we are missing something

Manifolds with $SU(3)$ structure naturally fit into the scheme - torsion $=$ geometric fluxes.

In type II – need to add a visible sector $\rightarrow$ open string moduli...

Heterotic String Models

- easy to get realistic models
- less possibilities to fix moduli
- NS-NS fluxes can fix complex structure moduli
- dilaton and Kähler moduli are unfixed.
- use manifolds with $SU(3)$ structure

Manifolds with $SU(3)$ structure

$SU(3)$ structure – structure group of the frame bundle $= SU(3)$.

$\exists$ a connection compatible with the structure – has torsion ∇^T

$SU(3)$ invariant forms J and $\Omega \to \nabla^T J = \nabla^T \Omega = 0$.

$dJ \neq 0$ and $d\Omega \neq 0$ specify the structure.

Calabi–Yau compactifications – need harmonic 2, 3 and 4-forms

$$\int_{X^6} \omega_i \wedge \tilde{\omega}^j = \delta_i^j \; , \qquad \int_{X^6} \alpha_A \wedge \beta^B = \delta_A^B \; .$$

$SU(3)$ structure:

$$\begin{aligned} d\omega_i &= q_i^A \alpha_A - p_{iA} \beta^A \; , \\ d\alpha_A &= p_{iA} \tilde{\omega}^i \; , \qquad d\beta^A = q_i^A \tilde{\omega}^i \\ \text{constraint :} & \qquad p_{iA} q_j^A - p_{jA} q_i^A = 0 \; . \end{aligned}$$

dJ and $d\Omega$ – given implicitely by the expansion of J and Ω.

Heterotic string on manifolds with $SU(3)$ structure

Need to solve the Bianchi identity

$$dH = trF \wedge F - tr\tilde{R} \wedge \tilde{R}$$

Connection relevant for anomaly cancellation: $\tilde{w} = w - H/2$.

Use "standard" embedding;

$\tilde{w}$ – SO(6) holonomy $\rightarrow$ breaks gauge group to $SO(10)$

Split $\tilde{w} = \tilde{w}^{\|} + \tilde{w}^{\perp}$

$$\mathbf{15} = \underbrace{\mathbf{8}}_{su(3)^{\|}} + \underbrace{\mathbf{1} + \mathbf{3} + \bar{\mathbf{3}}}_{su(3)^{\perp}}$$

$\tilde{w}^{\perp} \sim \mathbf{3} + \bar{\mathbf{3}} \rightarrow$ can be absorbed into the 4d matetrfields.

Results

- E_6 gauge group
- $h^{1,1}$ fields in $\overline{\mathbf{27}}$, C^i
- $h^{2,1}$ fields in $\mathbf{27}$, D^a
- $h^{1,1}$ Kähler moduli, T^i
- $h^{2,1}$ complex structure moduli, Z^a
- axio-dilaton, S
- bundle moduli

Kähler potential

$$K(S,T,Z,C,D) = K_0(S,T,Z) + \alpha' K_1(T,Z,C,D)\ ,$$

$$\begin{aligned} K_0 &= -\log\left(S+\bar{S}\right) - \log\tfrac{1}{6}[\mathcal{K}_{ijk}(T^i+\bar{T}^i)(T^j+\bar{T}^j)(T^k+\bar{T}^k)] \\ &\quad -\log\tfrac{1}{6}[\tilde{\mathcal{K}}_{abc}(Z^a+\bar{Z}^a)(Z^b+\bar{Z}^b)(Z^c+\bar{Z}^c)] \\ K_1 &= 4e^{(K_{cs}-K_K)/3} g_{ij} C^{i\bar{P}} \bar{C}^{j}{}_{\bar{P}} + e^{(K_K-K_{cs})/3} g_{a\bar{b}} D^{aP} \bar{D}^{\bar{b}}_{P} - 2\left(K_i K_a C^i_P D^{aP} + c.c.\right)\ , \end{aligned}$$

Superpotential

$$W(T,Z,C,D) = W_0(T,Z) + \alpha' W_1(Z,C,D) \ ,$$

where

$$\begin{aligned} W_0 &= i\left(\xi + i e_i T^i\right) + \left(\epsilon_a + i p_{ia} T^i\right) Z^a + \tfrac{i}{2}\left(\mu^a + i q_i^a T^i\right) \tilde{\mathcal{K}}_{abc} Z^b Z^c \\ &\quad + \tfrac{1}{6}\left(\rho + i r_i T^i\right) \tilde{\mathcal{K}}_{abc} Z^a Z^b Z^c \ , \\ W_1 &= 2\left[p_{ia} - (r_i Z^a + q_i^a)\tilde{\mathcal{K}}_{abc} Z^b\right] C^i D^c \\ &\quad - \tfrac{1}{3}\left[\mathcal{K}_{ijk} j_{\bar{P}\bar{R}\bar{S}} C^{i\bar{P}} C^{j\bar{R}} C^{k\bar{S}} + \tilde{\mathcal{K}}_{abc} j_{PRS} D^{aP} D^{bR} D^{cS}\right] \ . \end{aligned}$$

Gauge kinetic function $f_{AB} = S\delta_{AB}$

Moduli stabilisation in heterotic models

The dilaton does not appear in $W \to$ consider stabilisation by gaugino condensate in the hidden sector $\to$ need a small W.

There exist superpotential couplings CD $(\mathbf{27}, \overline{\mathbf{27}})$

Similar couplings exist for pairs (T, Z).

In the limit $W \ll 1$ (susy preserving vacuum) the masses for the pairs (T, Z) and (C, D) are related $\to$ integrate out these fields

2 cases

I. $h^{1,1} > h^{2,1} \to$ effective model with moduli T and matter fields in $\overline{\mathbf{27}}$, C

II. $h^{2,1} > h^{1,1} \to$ effective model with moduli Z and matter fields in $\mathbf{27}$, D

Case I

Effective theory: Supergravity + super Yang-Mills theory E_6 gauge group + one chiral superfield in $\overline{\mathbf{27}}$ C^A + one chiral singlet superfield T $\left(h^{1,1} = 1,\ h^{2,1} = 0\right)$

$$K = -3\ln(T + \bar{T}) + \frac{3}{T + \bar{T}} C^A \bar{C}_A$$

$$W = ieT + \frac{1}{3} j_{ABC} C^A C^B C^C \ .$$

Supersymmetric solutions

$\mathbf{C = 0}$**:** $D_T W = ie - 3/(T + \bar{T})(ieT) = 0 \Rightarrow e = 0$ not good.

$\mathbf{C \neq 0}$ what changes?

a. $E_6 \supset SO(10) \times U(1)$ $\overline{\mathbf{27}} = \mathbf{10}^{-2} \oplus \overline{\mathbf{16}}^{1} \oplus \mathbf{1}^{4}$

b. $E_6 \supset SU(3) \times SU(3) \times SU(3)$ $\overline{\mathbf{27}} = (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) \oplus (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{3})$

a

$< \mathbf{1} > \neq 0, \quad < \mathbf{10} > = < \mathbf{16} > = 0 \;\longrightarrow\; E_6 \to SO(10)$

No $\mathbf{1}^3$ coupling in W

$$D_1 W = 0 + \frac{3\bar{C}_1}{T+\bar{T}} W = 0 \quad \Longrightarrow W = 0 \,,$$

$$D_T W = e = 0 \quad \text{not} \quad \text{good}$$

b

$< (\mathbf{1},\mathbf{3},\mathbf{3}) >\neq 0 \ \longrightarrow \ E_6 \to SU(3)\times SU(2)\times SU(2)$

There exist $(\mathbf{1},\mathbf{3},\mathbf{3})^3 \equiv B^3$ coupling in W

$$D_B W = B\cdot B + \bar{B}\cdot W = 0$$

B – small fluctuations $\Rightarrow B \ll 1 \ \Rightarrow \ W = eT \sim B \ll 1$

but e is cuantised and $T+\bar{T} \gg 1$ for the supergravity approximation

Case II

Effective theory: Supergravity + super Yang-Mills theory E_6 gauge group + one chiral superfield in $\mathbf{27}$, D^A + one chiral singlet superfield Z ($h^{2,1} = 1,\ h^{1,1} = 0$)

$$
\begin{aligned}
K &= -3\ln(Z+\bar{Z}) + \frac{3}{Z+\bar{Z}} D^A \bar{D}_A \\
W &= \xi + i\epsilon Z + \frac{i}{2}\mu Z^2 + \frac{\rho}{6} Z^3 + \frac{1}{3} j_{ABC} D^A D^B D^C \ .
\end{aligned}
$$

This system has susy solutions, but $W \sim 1$.

Breaking E_6

Charged fields – small fluctuatios around the background

$\longrightarrow$ matter superpotential naturally small

Look for solutions with $W_{flux} = 0$

Generate a small non-vanishing W by breaking E_6.

Tractable system: one T and one Z plus corresponding matter fields C and D

Find Minkowski solutions for

$$W = i(\xi + ieT) + (\epsilon + ipT)Z + \frac{i}{2}(\mu + iqT)Z^2 + \frac{1}{6}(\rho + irT)Z^3$$

ie

$$\partial_T W = 0\ , \qquad \partial_Z W = 0\ , \qquad W = 0\ .$$

plus constraint (coming from BI $dH = 0$)

$$\xi r - \epsilon q + \mu p - \rho e = 0\ .$$

$r \neq 0$

$$\partial_T W = -e + ipZ - \frac{q}{2}Z^2 + \frac{i}{6}rZ^3 \ ,$$

has one purely imaginary solution iz_0

shift Z by iz_0 $\rightarrow$ end up with the same system with redefined Z and flux parameters which has $e = 0$

Solution for Z with $Re\ Z \neq 0 \rightarrow p \neq 0$

Solve $\partial_Z W = 0$, impose $Re\ W = 0$ and the constraint and obtain

$$Im\, W = \frac{4p}{3}tz \neq 0 \ .$$

$r = 0$

Solve $\partial_T W = 0$ and $\partial_Z W = 0$ imposing the constraint

$\rightarrow Re\, W = \frac{2}{3} q t \neq 0$

Note: $Im\, W$ can be set to zero by tunning the flux ξ which is unconstrained.

Conclusions

- Heterotic string compactifications on manifolds with $SU(3)$ structure
- by integrating out massive fields $\rightarrow$ can consider simple systems which have either $h^{2,1}=0$ or $h^{1,1}=0$
- None of these systems has satisfactory solutions (ie have $W \ll 1$)
- Minkowski solutions with $W_{flux}=0$ do not exist
- seems heterotic on manifolds with $SU(3)$ structure and standard embedding do not have reasonable solutions with fixed moduli