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On the effect of the magnetic interaction on the gamma disintegration of the orthopositronium

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Abstract

Magnetic interactions of positronium with matter are introduced in order to account for the damped oscillations, diminished lifetime and oscillations amplitude of the quantal beats exhibit by the three-gamma disintegration of the orthopositronium in magnetic field. The magnetic interactions give a dependence on concentration of the characteristics of the damped oscillations. These damped oscillations can give an estimation of the magnetic interaction experienced by positrons in magnetic substance.

Positrons in matter can form two varieties of bound-states with the electron: orthopositronium (triplet spin state) and parapositronium (singlet spin state). The former undergoes a three-gamma disintegration with lifetime $\tau_t = \gamma_t^{-1} \simeq 10^{-7}s$, while the latter decays by emitting two gamma quanta with lifetime $\tau_s = \gamma_s^{-1} \simeq 10^{-10}s$, much below the detection resolution ($\sim 10^{-9}s$). In moderate magnetic fields there occurs a mixing of ortho- and parapositronium states, which gives rise to quantal beats in the three-gamma decay. Their frequency is given by $\Omega = 4\mu^2 H^2/\hbar\Delta W$, where μ is the Bohr magneton, H is the magnetic field and $\Delta W \simeq 8.3 \cdot 10^{-4}eV$ is the hyperfine splitting between the ortho- and parapositronium ground-state energies.[1] For $H = 500Gs$ the beat frequency is $\Omega \simeq 6.5 \cdot 10^7 s^{-1}$.

Specifically, the decaying population of orthopositronium is governed by the kinetic equation

$$\partial N/\partial t = -\gamma_t([1 + (p/4)\sin\Omega t]N \ , \tag{1}$$

where p is the polarization of the positron beam; the solution of equation (1) is given by

$$N = N_0 e^{-\gamma_t t}[1 - (\gamma_t p/2\Omega)\sin^2 \Omega t/2] \ , \tag{2}$$

for $\gamma_t p/\Omega \ll 1$.

In the presence of magnetic matter, positronium experiences magnetic interactions. These magnetic interactions are mainly of dipolar character, involving the magnetic moment μ_p of the positronium and the magnetic moment μ_m of the substance, as for paramagnetic matter. Positronium has no definite magnetic moment. Its magnetic moment is given $\mu(\sigma_{z+} - \sigma_{z-})$, where $\sigma_\pm$ are Pauli matrices for positron (+), and, respectively, electron (−). This magnetic moment has an average value $\mu_p = \mu(4\mu H/\Delta W)$ on positronium states, which amounts to cca $\mu_p = 0.014\mu$ for

[1]See Antiphys. Rev. **uapr2** (2004) for a review, and references therein.

$H = 500Gs$. The effective dipolar interaction reads $V \sim \mu_p \mu_m / a_m^3$, where a_m is the mean spacing of the magnetic molecules; we can also write it as $V \sim (\mu_p \mu_m / a^3)c$, where c is the relative concentration of the magnetic substance in the host substance whose molecules are separated by a mean spacing a. Actually, such an effective interaction can be much enhanced, in comparison with usual magnetic interaction in magnetic resonance, as a consequence of the positron penetration in molecular orbitals of the magnetic substance. Consequently, we write it as $V = (\mu h)c$, where $h \sim \mu_m / a^3$ stands for an effective magnetic field experienced by positronium in magnetic substance. A similar estimation in magnetic resonance in solids or liquids gives h of the order of a few Gs' , typically. For instance, a proton with magnetic momentum $\mu_{proton} = 1.4 \cdot 10^{-23} erg/Gs$ (or $Gs \cdot cm^3$), gives a field $h = \mu_{proton}/a^3 \simeq 2Gs$ at distance $a = 2$Å. Positronium can experience a much higher field in magnetic matter, and we can employ the effective field h as a measure of such an interaction.

The magnetic interaction produces in general transitions between quantal states, acting as a residual interaction not affecting the quantal scheme of energy levels. The rate of such transitions is given by $g \simeq V^2/\hbar\delta E$, where $1/\delta E$ stands for the density of states. We may take $\delta E \sim V$, and get $g \simeq V/\hbar = g_0 c$, where $g_0 = \mu h/\hbar$. Such a transition rate have two effects: first, it takes the ortopositronium into parapositronium, so a term $-gN$ must be added to the *rhs* of equation (1);[2] second, the positrons get depolarized, so that a kinetic decay $p = p_0 e^{-\gamma t}$ must be included in equation (1) for the polarization. It is not obvious that the new parameter γ is numerically equal to g, in view of the uncertainties in determining the effective magnetic interaction and transition rates, though it has the same origin, and the same dependence $\gamma = \gamma_0 c$ on concentration, where γ_0 is a convenient parameter. For generality, we preserve the two parameters $g_0$and γ_0 distinct. The kinetic equation for the decaying orthopositronium reads now

$$\partial N/\partial t = -(\gamma_t + g)N - [(\gamma_t p_0/4)e^{-\gamma t} \sin \Omega t]N \ . \tag{3}$$

We left aside the kinetics of parapositronium, in view of its high disintegration rate; actually, quantal beats are induced in its disintegration through the same g-coupling, with the orthopositronium longer lifetime γ_t^{-1}, but their amplitude seems to be extremely small (of the order g/γ_s).[3]

Equation (3) has solution

$$N = N_0 e^{-(\gamma_t + g)t} \left\{ 1 + \frac{\gamma_t p_0}{4} \left[e^{-\gamma t} \frac{\Omega \cos \Omega t + \gamma \sin \Omega t}{\Omega^2 + \gamma^2} - \frac{\Omega}{\Omega^2 + \gamma^2} \right] \right\} \ . \tag{4}$$

First, it is worth noting that the ortopositronium lifetime is diminished according to

$$\tilde{\tau}_t = \frac{\tau_t}{1 + g_0 \tau_t c} \ . \tag{5}$$

Usually, the measurements are being made for two different positions of the detectors, so the oscillating terms in (4) have opposite signs. The sum to difference of the two contributions is defined then as a function R; it is therefore given by

$$R = \frac{\gamma_t p_0 / 4}{(\Omega^2 + \gamma^2) - \gamma_t p_0 \Omega / 4} \cdot e^{-\gamma t} (\Omega \cos \Omega t + \gamma \sin \Omega t) \ , \tag{6}$$

[2] We note that the rate of collisions with the molecules of the substance $\omega_{coll} \simeq v/a$, where $v \simeq 10^7 cm/s$ is comparable with the uncertainty time $\hbar/\Delta W$ of the ortho-parapositronium ($\sim 10^{-12} - 10^{-13}s$), so the transitions are possible, indeed. (a is about 30Å)

[3] They were analyzed in Antiphys. Rev. **uapr2** (2004).

which tells that the oscillations have an amplitude

$$A=\frac{(\gamma_t p_0/4)\sqrt{\Omega^2+\gamma^2}}{(\Omega^2+\gamma^2)-\gamma_t p_0\Omega/4}\ , \tag{7}$$

a relaxation time

$$\tau_r=\gamma^{-1}=\tau_0/c\ , \tag{8}$$

where $\tau_0=1/\gamma_0$, and a phase α given by $\tan\alpha=\gamma/\Omega$; they read, therefore,

$$R=Ae^{-\gamma t}\cos(\Omega t-\alpha)\ . \tag{9}$$

Equations (5), (7) and (8) can be used to fit the dependence on concentration of the ortopositronium lifetime, amplitude and, respectively, relaxation time of the oscillations, by employing two parameters: g_0 and γ_0 (or τ_0). These parameters may serve to characterize the magnetic interactions experienced by orthopositronium in magnetic substance, via an effective magnetic field h.

We note a certain particularity of the amplitude A given by (7). For $\Omega\gg\gamma$ it becomes $A=\gamma_t p_0/\Omega$, as in (2), and it does not depend on concentration, as expected. In the opposite case $\Omega\ll\gamma$, we get $A=\gamma_t p_0/\gamma=(\gamma_t p_0/\gamma_0)/c$, and it goes inversely proportional to the concentration.

Preliminary data [4] indicate $g_0=0.1ns^{-1}$ and $\gamma_0=0.09ns^{-1}(\tau_0\simeq 10ns)$ from fiting the lifetime (5) and the relaxation time (8), which support indeed the common origin of the two parameters g and γ as arising from dipolar magnetic interactions. They correspond to an effective internal magnetic field $h\simeq 10Gs$ ($g_0=\mu h/\hbar$). Making use of these two parameters we get also a satisfactory fit to the amplitude data byusing (7). It is also worth noting that the phase α has a dependence on concentration, according to $\tan\alpha=\gamma/\Omega=(\gamma_0/\Omega)c\simeq 5c$ (for $\Omega=1/50ns^{-1}$).

[4] I. Vatza, private communication.