# Optical spatiotemporal solitons: Past, present, and future

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## Abstract

A brief overview of the field of optical spatiotemporal solitons ("light bullets") is given.

Solitons, i.e., self-trapped light beams or pulses that are supported by a balance between diffraction and/or dispersion and nonlinearity, are prominent objects in nonlinear optics [1]. Optical spatiotemporal solitons (STS) [2], alias superspikes [3] or *light bullets* [4], were predicted in many works [2] - [16]. They result from the simultaneous balance of diffraction and group-velocity dispersion (GVD) by self-focusing. Although they cannot be stable in the uniform self-focusing Kerr ($\chi^{(3)}$) medium [8], stability can be achieved in saturable [3,6,10], quadratically nonlinear ($\chi^{(2)}$) [2,12–14], and graded-index Kerr media [15]. STS can also be found in off-resonance two-level systems [17], in self-induced-transparency media [18], as well as in engineered tandem structures made with quadratically nonlinear pieces [19].

One of major goals in the field of optical solitons is the formation of light fields that are *localized* in all three dimensions of space as well as time, which we will refer to as three-dimensional (3D) STS. These 3D localized objects hold promise for potential applications in future ultrafast all-optical logic devices, where each STS may represent an elementary *bit of information*, provided that stable STS can be formed from pulses at reasonable energy levels in available optical materials.

While a fully localized "light bullet" in three dimensions has not yet been found in an experiment, two-dimensional (2D) STS in bulk $\chi^{(2)}$ media, such as $LiIO_3$ and barium metaborate (BBO), were observed in Ref. [20]. That work reported the formation of pulses in quadratic media, which overcome diffraction in one transverse spatial dimension and GVD in the longitudinal direction. However, such experiments were performed using tilted-pulse techniques, thus employed highly elliptical beams; therefore, diffraction is negligible in the remaining transverse spatial dimension.

Optical vortex solitons constitute another class of self-supporting objects, that have attracted much attention because of possible applications for the all-optical processing of information, or in guiding and trapping of atoms. The concepts of a multidimensional optical soliton and of an optical vortex may be combined, giving rise to *spinning* (vortex) solitons. Starting with the seminal works [21], both delocalized ("dark") and localized ("bright") optical vortices were investigated in various 2D environments [22–25]. In the 3D case the bright spinning solitons take the shape of a torus ("doughnut") [26,27].

For bright vortex solitons, stability is a major concern, as, unlike their zero-spin counterparts, the spinning solitons are apt to be destabilized by azimuthal perturbations. For 2D models with $\chi^{(2)}$ nonlinearities, an azimuthal instability was discovered by simulations [28] and observed experimentally [29]. As a result, a soliton with spin 1 splits into three or two fragments in the form of separating zero-spin soliton. Numerical simulations of the 3D spinning STS in the $\chi^{(2)}$ model also demonstrates splitting into moving zero-spin solitons [27].

Nevertheless, the $\chi^{(2)}$ nonlinearity acting in combination with the self-*defocusing* Kerr [$\chi_{-}^{(3)}$, where we use the subscript "minus" to stress the self-repulsion] nonlinearity gives rise to the first examples of stable spinning (ring-shaped) 2D solitons with spin $s = 1$ and 2 [23]. Models of this type for spatial [(2+1)-dimensional] solitons are well known [30,31]). The stability of the spinning solitons in the $\chi^{(2)} - \chi_{-}^{(3)}$ model may be realized as a result of competition between the self-focusing and self-defocusing nonlinearities. This understanding is further supported by the fact that stable spinning solitons of the same type have also been found in another optical model displaying both focusing and defocusing nonlinearities, viz., the one based on the cubic-quintic (CQ) nonlinear Schrödinger (NLS) equation. It addition to optics, the same model finds applications, e.g., to the description of Bose-Einstein condensates (BECs) [32] and Langmuir waves in plasmas [33] (however, in the former case, the quintic nonlinearity arises from three-body interactions, which also give rise to losses by recombination of BEC constituents into different species, thus making the quintic nonlinear coefficient a complex one).

In the first direct simulations of 2D solitons with spin $s = 1$ in the CQ model, reported in the pioneer work [34], it was found that they are robust, provided that their energy is not too small [34]. Later analysis, based on the computation of linear-stability eigenvalues, demonstrated that some of the spinning 2D solitons considered in Ref. [34] are subject to a weak azimuthal instability. Nonetheless, in another part of their existence region, where they have a very large energy, the solitons with spin $s = 1$ and $s = 2$ were confirmed to be stable in the 2D CQ model [35] (see also Ref. [36] for the stability calculations of the solitons with spin $s = 1$). Stable 2D vortex solitons in the CQ model can self-trap from Gaussian inputs with an embedded vorticity [37]. Notice that all the solitons with $s \geq 3$ have been demonstrated to be unstable [35].

Recently we have shown the formation of stable two-dimensional spinning solitons in a bimodal system described by coupled cubic-quintic nonlinear Schrödinger equations [38]. The cubic part of the model includes the self-phase modulation, cross-phase modulation, and four-wave mixing. Thresholds for the formation of both spinning and non-spinning solitons were found. Instability growth rates of perturbation eigenmodes with different azimuthal indices were calculated as functions of the solitons' propagation constant. As a result, existence and stability domains were identified for the solitons with vorticity $s = 0$, 1, and 2 in the model's parameter plane. These *vectorial vortex solitons* were found to be stable if their energy flux exceeds a certain critical value, so that, in typical cases, the stability domain of the $s = 1$ solitons occupies about 18% of their existence region, whereas that of the $s = 2$ solitons occupies 10% of the corresponding existence region [38].

The composite solitons (or in other words, the vectorial solitons) bifurcate from single-component (scalar) soliton solutions, the difference between them being that the two-component solitons provide for a smaller value of the Hamiltonian for the same energy flux, i.e., they are, globally, *more stable* than their single-component counterparts. Direct simulations of the full nonlinear system were found to be in perfect agreement with the linear stability analysis: stable solitons easily self-trap from arbitrary initial pulses with embedded vorticity, while unstable vortex solitons split into a set of separating zero-spin fragments whose number is exactly equal to the azimuthal index of the strongest unstable perturbation eigenmode [38].

However, a challenging issue is the search for physically relevant models in which *stable* 3D spinning solitons exist. In fact, the only previously known model which could support stable 3D vortex solitons was the Skyrme model (see reviews [39]). Very recently, we have found stable 3D spinning STS in the CQ model, which could again be construed as a result of the competition between the self-focusing and self-defocusing [40]. Direct simulations of the 3D CQ model [41] proves that 3D spinning solitons with moderate energies are unstable against azimuthal perturbations, while the ones with very large energies, i.e., broad "doughnuts" with a small hole in the center, were robust under propagation. However, a consistent stability analysis makes it necessary to compute eigenvalues of small perturbations. By calculating the instability growth rates it was shown in [40] that sufficiently broad STS with spin $s = 1$ are stable, the stability region occupying $\approx 20\%$ of their existence region, while all the STS with $s \geq 2$ are unstable. We infer that the existence of stable spinning 3D solitons is a more generic fact, which is not limited to the CQ model considered in Ref. [40]. Thus the spinning STS solitons, of sufficiently large energy, may be also stable in the 3D version of the above-mentioned $\chi^{(2)} - \chi^{(3)}_{-}$ model with the self-defocusing cubic term [42]. On the contrary to the above-mentioned results for the 2D spinning spatial solitons (2D vortex solitons), spinning light bullets may only be stable if the topological charge ("spin") $s = 1$, while in the case $s = 2$ they are always unstable to azimuthal perturbations. A general conclusion is that the spinning solitons can be stable, provided that they are broad enough (so that the soliton's energy exceeds a certain critical value, or, in other words, the size of the internal hole is essentially smaller than the overall size of the soliton). The results obtained recently in Refs. [40,42] suggest the conclusion that stable vortex solitons are generic objects, provided that the medium's nonlinearity contains competing elements and the soliton's energy is large enough (in all the known models lacking the nonlinear competition, the bright vortex solitons are subject to a strong azimuthal instability).

One might assume that, very generally speaking, the bright 2D vortex solitons and the bright 3D spinning solitons are not absolutely stable objects, but rather metastable ones. Indeed, the energy flux of the spinning soliton is, generally, larger than that of its zero-spin counterpart, hence it could be possible that a very strong initial perturbation would provoke its rearrangement into a zero-spin soliton, the angular momentum being carried away with emitted radiation. Thus the stability of the 2D vortex solitons with $s = 1$ and $s = 2$ against small perturbations is provided for by potential barriers separating them from the solitons with $s = 0$. In terms of this consideration, further numerical results demonstrate that the spinning and non-spinning solitons are separated by extremely high barriers, which makes the transition process $s \neq 0 \rightarrow s = 0$ practically impossible. To illustrate this point, we have calculated the distributions of the intensity and phase inside a strongly perturbed initial soliton with $s = 1$ (the random perturbation is $\simeq$ 30% of the soliton's amplitude) and in a finally established one. We have found that the soliton was able to completely heal the damage, remaining a truly stable object. The above results apply also to the case of bright 3D spinning solitons with spin $s = 1$ which form in media with competing nonlinearities.

Parallel to the theoretical activity in the area of optical spatiotemporal solitons, progress has been recently made also in experimental studies of optical bullets. For the first time, a 2D STS was observed in a second-harmonic generation (SHG) setting [20]. These experiments employed the clever technique of achromatic phase-matching or titled-pulse wave fronts [43–45] that was used earlier in the first recorded experimental evidence of temporal soliton formation in a quadratic crystal [46]. The experimental set-up used in [20] was soon improved, and more detailed and accurate experimental results are now available [47]. It has also been showed experimentally that STS's of this type may find application to the design of ultrafast all-optical logic gates [48]. There is a strong hope that, eventually, a fully localized 3D STS will be experimentally observed in SHG optical crystals. One promising approach to this is to use layered periodic strucures [19] or Bragg gratings in order to create an environment that will support three-dimensional spatiotemporal optical solitons.

The field of optical spatiotemporal solitons is still in its infancy, however given its rapid growth over the last five years, one can expect many new and exciting developments over the next years. Many frontiers in this area remain to be explored.

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## REFERENCES


[1] G. I. Stegeman, D. N. Christodoulides, and M. Segev, IEEE J. Select. Top. Quant. Electron. **6**, 1419 (2000).

[2] A. A. Kanashov and A. M. Rubenchik, Physica D **4**, 122 (1981).

[3] J. T. Manassah, P. L. Baldeck, and R. R. Alfano, Opt. Lett. **13**, 1090 (1988); J. T. Manassah, *ibid.* **16**, 563 (1991).

[4] Y. Silberberg, Opt. Lett. **15**, 1282 (1990).

[5] A. B. Blagoeva *et al.*, IEEE J. Quant. Electron. **QE-27**, 2060 (1991).

[6] N. Akhmediev and J. M. Soto-Crespo, Phys. Rev. A **47**, 1358 (1993).

[7] Y. Chen and J. Atai, Opt. Lett. **20**, 133 (1995).

[8] L. Bergé, Phys. Rep. **303**, 260 (1998).

[9] R. H. Enns *et al.*, Opt. Quant. Electron. **24**, S1295 (1992); R. McLeod, K. Wagner, and S. Blair, Phys. Rev. A **52**, 3254 (1995); J. T. Manassah and B. Gross, Laser Phys. **7**, 9 (1997).

[10] D. E. Edmundson, Phys. Rev. E **55**, 7636 (1997).

[11] V. Skarka, V. I. Berezhiani, and R. Miklaszewski, Phys. Rev. E **56**, 1080 (1997).

[12] K. Hayata and M. Koshiba, Phys. Rev. Lett. **71**, 3275 (1993); B. A. Malomed *et al.*, Phys. Rev. E **56**, 4725 (1997); D. V. Skryabin and W. J. Firth, Opt. Commun. **148**, 79 (1998); D. Mihalache, D. Mazilu, B. A. Malomed, and L. Torner, *ibid.* **152**, 365 (1998).

[13] D. Mihalache, D. Mazilu, J. Dörring, and L. Torner, Opt. Commun. **159**, 129 (1999).

[14] D. Mihalache, D. Mazilu, B. A. Malomed, and L. Torner, Opt. Commun. **169**, 341 (1999); D. Mihalache, D. Mazilu, L.-C. Crasovan, L. Torner, B. A. Malomed, and F. Lederer, Phys. Rev. E **62**, 7340 (2000).

[15] S. Raghavan and G. P. Agrawal, Opt. Commun. **180**, 377 (2000).

[16] H. E. Nistazakis, D. J. Frantzeskakis, and B. A. Malomed, Phys. Rev. E **64**, 026604 (2001); C. Polymilis, D. J. Frantzeskakis, A. N. Yannacopoulos, K. Hizanidis, and G. Rowlands, J. Opt. Soc. Am. B **18** , 75 (2001).

[17] I. V. Mel'nikov, D. Mihalache, and N.-C. Panoiu, Opt. Commun. **181**, 345 (2000).

[18] M. Blaauboer, B. A. Malomed, and G. Kurizki, Phys. Rev. Lett. **84**, 1906 (2000).

[19] L. Torner, S. Carrasco, J. P. Torres, L.-C. Crasovan, and D. Mihalache, Opt. Commun. **199**, 277 (2001).

[20] X. Liu, L. J. Qian, and F. W. Wise, Phys. Rev. Lett. **82** , 4631 (1999); X. Liu, K. Beckwitt, and F. Wise, Phys. Rev. E **62**, 1328 (2000).

[21] G. A. Swartzlander, Jr., and C. T. Law, Phys. Rev. Lett. **69**, 2503 (1992); A. W. Snyder, L. Poladian, and D. J. Mitchell, Opt. Lett. **17**, 789 (1992).

[22] P. Di Trapani *et al.*, Phys. Rev. Lett. **84**, 3843 (2000).

[23] I. Towers, A.V. Buryak, R.A. Sammut, and B.A. Malomed, Phys. Rev. E **63**, 055601(R) (2001).

[24] A. S. Desyatnikov and Yu. S. Kivshar, Phys. Rev. Lett. **87**, 033901 (2001).

[25] Z. H. Musslimani, M. Soljacic, M. Segev, and D. N. Christodoulides, Phys. Rev. E **63**, 066608 (2001).

[26] A. Desyatnikov, A. Maimistov, and B. Malomed, Phys. Rev. E **61**, 3107 (2000).

[27] D. Mihalache, D. Mazilu, L.-C. Crasovan, B. A. Malomed, and F. Lederer, Phys. Rev. E **62**, R1505 (2000).

[28] L. Torner and D. V. Petrov, Electr. Lett. **33**, 608 (1997); W. J. Firth and D. V. Skryabin, Phys. Rev. Lett. **79**, 2450 (1997); D. V. Skryabin and W. J. Firth, Phys. Rev. E **58**,

3916 (1998).

[29] D. V. Petrov *et al.*, Opt. Lett. **23**, 1444 (1998).

[30] A. V. Buryak, Yu. S. Kivshar, and S. Trillo, Opt. Lett. **20**, 1961 (1995), M. A. Karpierz, Opt. Lett. **20**, 1677 (1995).

[31] O. Bang, J. Opt. Soc. Am. B **14**, 51 (1997); O. Bang, Yu. S. Kivshar, and A. V. Buryak, Opt. Lett. **22**, 1680 (1997).

[32] F. Kh. Abdullaev, A. Gammal, L. Tomio, and T. Frederico, Phys. Rev. A **63**, 043604 (2001); A. Gammal, T. Frederico, L. Tomio, and F. Kh. Abdullaev, Phys. Lett. A **267**, 305 (2001).

[33] C. T. Zhou and X. T. He, Physica Scripta, **50**, 415 (1994).

[34] M. Quiroga-Teixeiro and H. Michinel, J. Opt. Soc. Am. B **14** , 2004 (1997).

[35] I. Towers *et al.*, Phys. Lett. A **288**, 292 (2001); B. A. Malomed, L.-C. Crasovan, D. Mihalache, Physica D **161**, 187 (2002).

[36] V. I. Berezhiani, V. Skarka, and N. B. Aleksic, Phys. Rev. E **64**, 057601 (2001).

[37] H. Michinel *et al.* J. Opt. B: Quantum Semiclass. Opt. **3**, 314 (2001).

[38] D. Mihalache, D. Mazilu, I. Towers, B. A. Malomed, and F. Lederer, J. Opt. A: Pure Appl. Opt. **4**, 615 (2002).

[39] D. O. Riska, Adv. Nucl. Phys. **22**, 1 (1996); T. Gisiger and M. B. Paranjape, Phys. Rep. **306**, 110 (1998).

[40] D. Mihalache, D. Mazilu, L.-C. Crasovan, I. Towers, A. V. Buryak, B. A. Malomed, L. Torner, J. P. Torres, and F. Lederer, Phys. Rev. Lett. **88**, 073902 (2002).

[41] D. Mihalache, D. Mazilu, L.-C. Crasovan, B. A. Malomed, and F. Lederer, Phys. Rev. E **61**, 7142 (2000).

[42] D. Mihalache *et al.*, Phys. Rev. E **66**, 016613 (2002).

[43] V.D. Volosov, S.G. Karpenko, N.E. Kornienko, and V.L. Strizhevskii, Sov. J. Quantum Electron. **4**, 1090 (1975).

[44] O.E. Martinez, IEEE J. Quantum Electron. **25**, 2464 (1989).

[45] G. Szabo and Z. Bor, Appl. Phys. B **50**, 51 (1990); **58**, 237 (1994).

[46] P. Di Trapani, D. Caironi, G. Valiulis, R. Danielius, and A. Piskarskas, Phys. Rev. Lett., **81**, 570 (1998).

[47] X. Liu, K. Beckwitt, and F. Wise, Phys. Rev. Lett. **85**, 1871 (2000).

[48] X. Liu, K. Beckwitt, and F. Wise, Phys. Rev. E **61**, R4722 (2000).