Scientific Report (2011-2016)

Project PN-II-ID-PCE-2011-3-0083 (contract 53/05.10.2011)

It is believed that quantum information processing and quantum communication have the potential to revolutionize many areas of science and technology. These emerging research fields exploit fundamentally new modes of computation and communication, because they are based on the physical laws of quantum mechanics instead of classical physics. They hold the promise of huge computing power beyond the capabilities of any classical computer, they guarantee absolutely secure communication, and they are directly linked to emerging quantum technologies, such as, for example, quantum based sensors and quantum metrology. From the point of view of the physicist, the efforts towards the understanding of the mentioned physical processes opens two main research directions:

(1) The intrinsic features of the physical information: The physical information is always connected to specific physical representations, its generation is related to the existence of an underlying support, while its transportation is secured by a carrying vector. The increase of the bit densities and of their access speeds are straightforward consequences of the decrease of the dimensions of the physical systems involved in the information processing.

(2) Carrier vectors for the information transmission: An important achievement during the past decade was represented by the discovery of means of distortionless transmission of the information at large distances and with high data transmission speeds. At present, the most efficient way for the information transmission at large distances is through monomode optical fibers, where temporal optical solitons, as carriers of information, can propagate in a distorsionless, robust way. Intensive fundamental studies on different kind of optical solitons (temporal, spatial and spatiotemporal ones) by

various research groups all-over the world, including one of the participants to this research project, followed by advanced material studies, have resulted in a rapid progress of this research area.

The project "Fundamental studies in physics of quantum information and in nonlinear optics of few-cycle solitons" *PN-II-ID-PCE-2011-3-0083 (contract 53/05.10.2011), financed by the Funding Agency UEFISCDI, Ideas Program,* had the following objective in the period October 2011-December 2011:

Study of new polarization effects of few-optical-cycle solitons by considering the vectorial nature of the optical field.

It is well known that ultrashort optical pulses with duration of merely a few femtoseconds find diverse applications in the area of light-matter interactions, high-order harmonic generation, extreme nonlinear optics [M. Wegener, *Extreme Nonlinear Optics* (Springer, Berlin, 2005)], single-cycle nonlinear optics [E. Goulielmakis et al., *Science* **320**, 1614 (2008)], and attosecond physics [A. Scrinzi et al., *J. Phys. B* **39**, R1 (2006)]. The slowly varying envelope approximation (SVEA) is no longer valid under these special conditions. Although some generalizations of the common nonlinear Schroedinger equation have been proposed, we believe that a completely different approach to the study of few-cycle pulses which completely abandons the SVEA is desirable.

One of the aims of this project was to develop a semiclassical theory based on Heisenberg-Maxwell-Bloch equations in order to investigate the effects coming from the intrinsic vectorial structure of the electric field.

In this general situation, the complex nonlinear partial differential equations that describe the evolution of the two field components U and V are as follows:

$$U_Z - U_{TTT} - [(U^2 + V^2)U]_T = 0\,, \qquad (1)$$

$$V_Z - V_{TTT} - [(U^2 + V^2)V]_T = 0\,. \qquad (2)$$

In the above equations (the modified Korteweg-de Vries-type equations), Z is the normalized distance and T is the retarded time, which is proportional to (t-z/V_f), where V_f is the phase velocity.

If we will make the transformation f=U+iV, the above equations can be written in a compact form:

$$f_Z - f_{TTT} - (|f|^2 f)_T = 0 \qquad (3)$$

Thus Eq. (3) is the complex modified Korteweg-de Vries equation (of type I) that is not completely integrable from the strict mathematical point of view. An approximate solution has the form:

$$f(T,Z) = \sqrt{6}\, b\, \mathrm{sech}\left[b\left(T - 3\mu^2 Z\right)\right] e^{i\mu\left[T-\left(\mu^2-3b^2\right)Z\right]} \qquad (4)$$

We mention that this solution is valid for relatively long pulses, i.e. for b « ω.

In Figs. 1 si 2 we show the stable evolution (the robust propagation) of the optical field in the relevant case b = 1 and ω = 2 (in the left panel we show the robust evolution of the x-component of the vectorial optical field, whereas in the right panel we illustrate the evolution of the norm of the optical field).

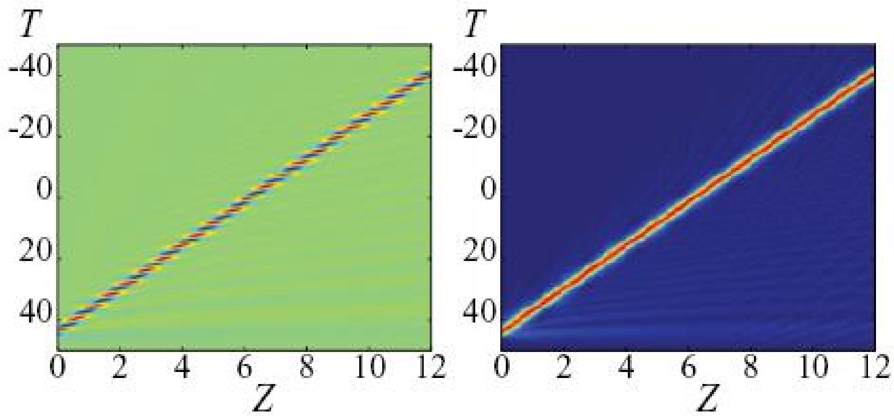

Fig. 1. Robust evolution of a few-cycle optical field.

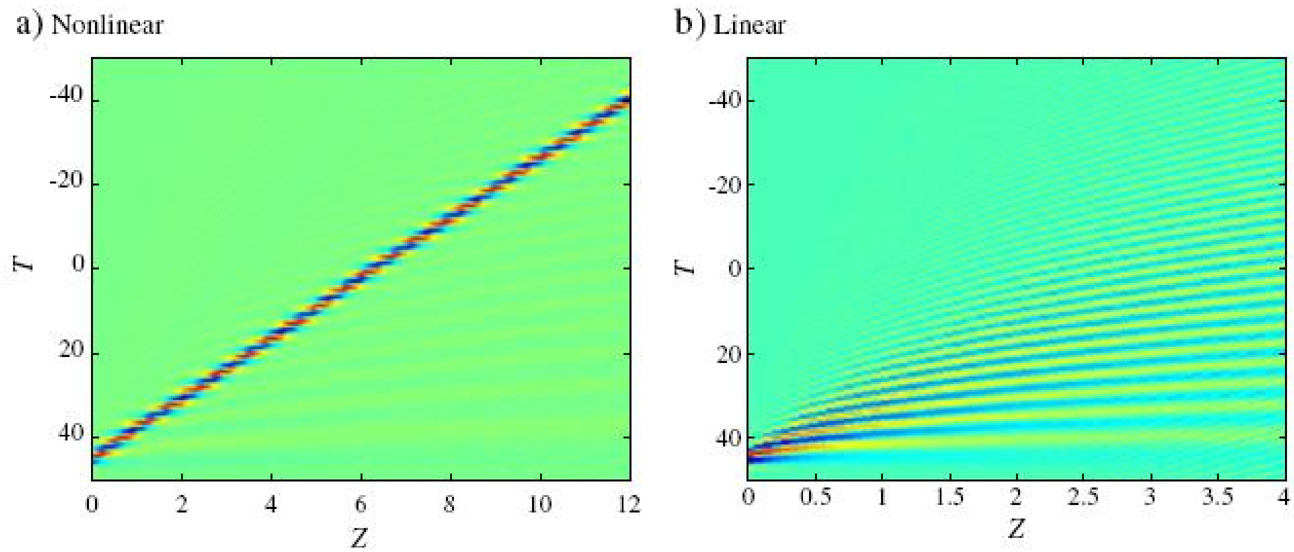

Fig. 2. A comparison between linear (right panel) and nonlinear (left panel) evolution of the x-component of the vectorial optical field (that is circularly polarized), for b = 1 and ω = 2.

The main results have been published in:

1. H. Leblond, H. Triki, F. Sanchez, D. Mihalache, Opt. Commun. **285**, 356-363 (2012).

One of the objectives of the project during the year 2012 was:

Study of existence and robustness of vectorial ultrashort optical solitons.

We have demonstrated the formation of two or even three ultrashort solitons with the duration of merely 0.5 cycles from an input containing 2-3 optical cycles, see Fig. 3. Additionally, we have studied the existence and robustness of spatiotemporal ultrashort optical solitons that can be formed in carbon nanotubes, by using the short-wave approximation. We have demonstrated the robust propagation of such light bullets over a few mm, i.e. over a few 1000 wavelengths, a result that is crucial for possible applications of such ultrashort light bullets in the transmission and processing of information with very high data rates.

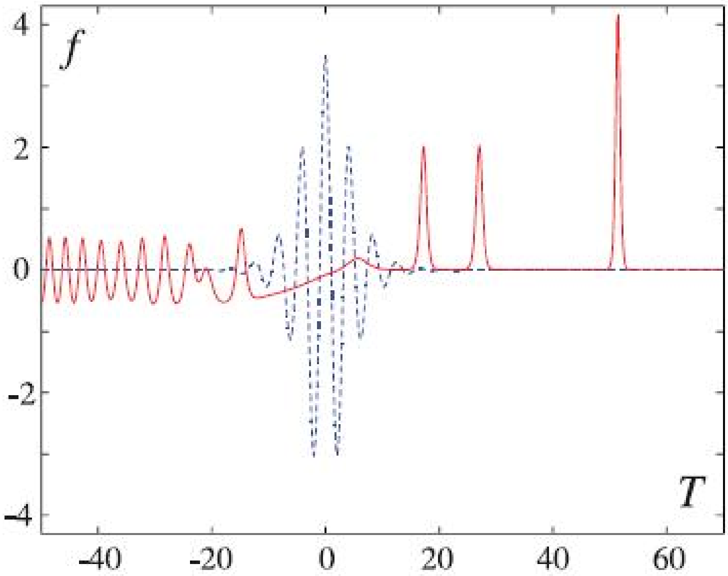

Fig. 3. Generation of three ultrashort solitons of unequal amplitudes with duration of only 0.5 optical cycles from an input with duration of a few cycles.

The main results of these studies were published in the following two papers:

H. Leblond, H. Triki, D. Mihalache, Phys. Rev. A **85**, 053826 (2012).
H. Leblond, D. Mihalache, Phys. Rev. A **86**, 043832 (2012).

One of the objectives of the project during the year 2013 was:

Study of the influence of quantum fluctuations and dissipation on quantum correlations (entanglement and discord) for bimodal Gaussian states.

In recent years, there has been an increasing interest in using the non-classical entangled states of continuous variable systems in the applications of quantum information processing, communication and computation. In this respect, Gaussian states, in particular, two-mode Gaussian states, play a key role since they can be easily created and controlled experimentally. Due to the unavoidable interaction with the environment, in order to describe realistically quantum information processes it is necessary to take decoherence and dissipation into consideration. In the framework of the theory of open systems based on completely positive quantum dynamical semigroups, we gave a description of continuous variable quantum entanglement and quantum discord for a system consisting of two non-interacting non-resonant bosonic modes embedded in a thermal environment, for the case when the asymptotic state is a Gibbs state corresponding to two independent quantum harmonic oscillators in thermal equilibrium. We described the evolution of entanglement in terms of the covariance matrix for Gaussian input states and the evolution under the dynamical semigroup ensures the preservation in time of the Gaussian form of the states. Namely, we studied the time evolution of logarithmic negativity, which characterizes the degree of entanglement. The dynamics of the quantum entanglement strongly depends on the initial states and the parameters characterizing the environment (the dissipation coefficient and temperature). For all values of the temperature of the thermal reservoir, an initial separable squeezed thermal state remains separable for all times. In the case of an entangled initial squeezed thermal state, entanglement suppression (entanglement sudden death) takes place for all the temperatures of the environment, including zero temperature. The time when the entanglement is suppressed decreases with increasing the temperature and dissipation. We analyzed the time evolution of the Gaussian quantum discord, which is a measure of all the quantum correlations in the bipartite state, including entanglement, and showed that discord decays asymptotically in time under the effect of the thermal bath. This is in contrast to the sudden death of entanglement. The time evolution of quantum discord is very similar to that of entanglement before the sudden suppression of the entanglement. Quantum discord is decreasing with increasing the temperature. After the sudden death of entanglement the non-zero values of discord manifest the existence of quantum correlations for separable mixed states. We described also the time evolution of classical correlations and quantum mutual information, which measures the total correlations of the quantum system.

The main results were published in the following papers:

A. Isar, Physica Scripta **T147**, 014015 (2012).

A. Isar, Physica Scripta **T153**, 014035 (2013).

A. Isar, Physica Scripta **87**, 038108 (2013).

The objective of the project during the year 2014 was:

Study of quantum correlations in Gaussian open systems in the two reservoir model.

In the framework of the theory of open systems based on completely positive quantum dynamical semigroups, we investigated the Markovian dynamics of the quantum entanglement for a system composed of two non-interacting modes, each one embedded in its own thermal bath. By using the Peres–Simon necessary and sufficient condition for separability of two-mode Gaussian states, we described the evolution of entanglement in terms of the covariance matrix for Gaussian input states. For an entangled initial squeezed thermal state, in particular a squeezed vacuum state, entanglement sudden death takes place. The system evolves in the limit of large times to an equilibrium state which is always separable. We calculated the asymptotic logarithmic negativity, which characterizes the degree of entanglement of the quantum state. It depends only on temperatures and does not depend on the initial Gaussian state. It takes negative values, confirming the fact that the asymptotic state is separable.

Within the framework of the same theory of open quantum systems, we investigated the Markovian dynamics of the Gaussian quantum discord for a subsystem composed of two bosonic modes, each one embedded in its own thermal bath. We have presented and discussed the influence of the reservoirs on the dynamics of quantum discord in terms of the covariance matrix for squeezed thermal initial states. We assumed that the asymptotic state of the considered open system is the Gibbs state corresponding to two independent quantum harmonic oscillators, each one in thermal equilibrium with its thermal bath.

The Gaussian discord has nonzero values for all finite times, and its dynamics strongly depends on the parameters characterizing the system (squeezing parameter and damping parameter) and the coefficients describing the interaction of the system with both reservoirs (temperatures and dissipation constants).

The values of the Gaussian discord asymptotically decrease to zero for large times. We described also the time evolution of classical correlations.

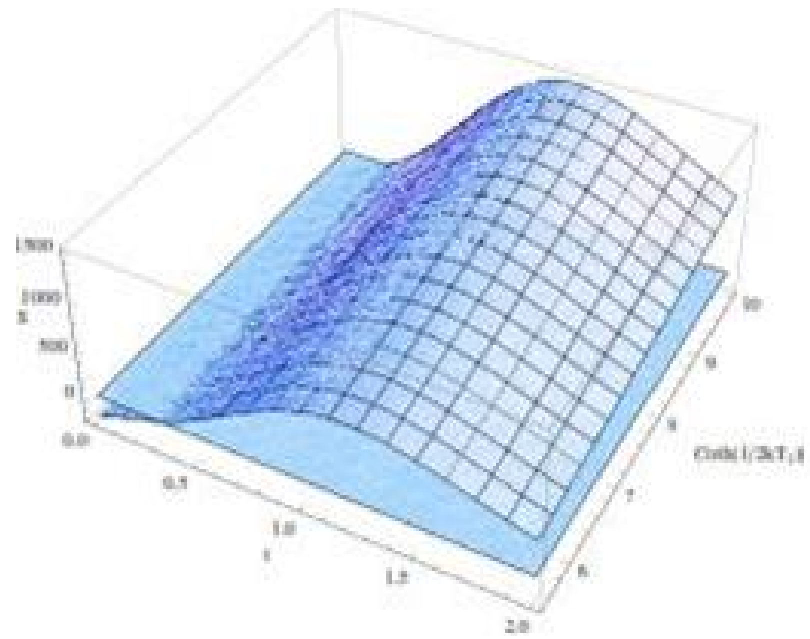

Fig. 4. Separability function S versus time t and environment temperature T_2 for an entangled initial squeezed thermal state with squeezing parameter r = 1.5, n_1 = 1, n_2 = 1, μ = 0.6 and coth(1/2kT_1) = 5.5. We take m = k = hbar = 1.

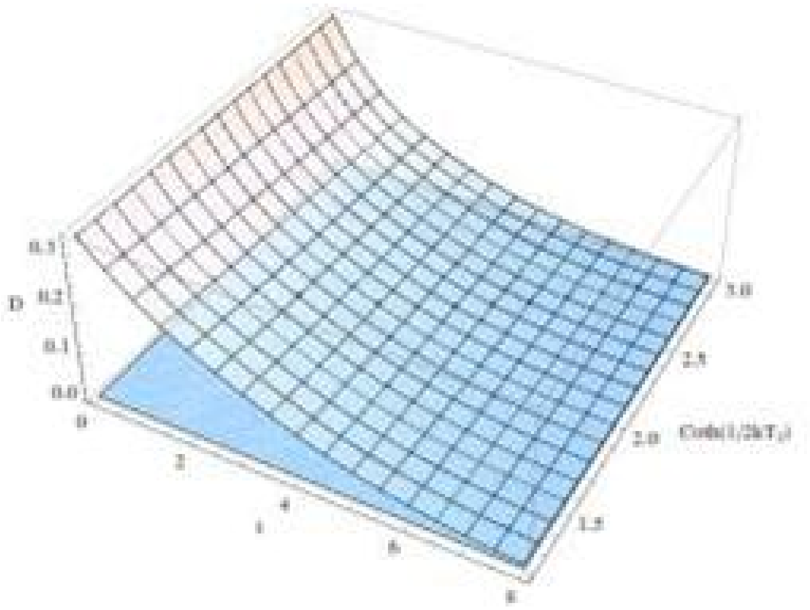

Fig. 5. Gaussian quantum discord D versus time t and temperature T_2 (via coth 1/2kT_2) for an initial squeezed thermal state with r = 0.8, λ_1 = 0.3, λ_2 = 0.2, μ = 0.1, n_1 = 1, n_2 = 3, coth 1/2kT_1 = 1.1.

Presently there is a large debate relative to the physical interpretation of quantum correlations – quantum entanglement and quantum discord. The present results, in particular, the existence of quantum discord and the possibility of maintaining it in thermal environments for long times, might be useful in controlling quantum correlations in open quantum systems and also for applications in the field of quantum information processing and communication.

The main results were published in the following papers:

A. Isar, Phys. Scripta **T 160**, 014015 (2014).

A. Isar, J. Russ. Laser Res. **35**, 62 (2014).

The objective of the project during the year 2015 was:

Application of the quantum Chernoff bound for evaluation of the degree of polarization of the bimodal Werner state.

Entanglement is an important resource in quantum information processing, for example for quantum teleportation and its generalizations, quantum key distribution required in cryptography, superdense coding, quantum computation. During the last decades a lot of attention was paid in analyzing the entanglement of bipartite or multipartite states. Separable states do not violate any Bell's inequalities and, at the same time, they cannot be used as resources in protocols of quantum information theory that involve observers situated at different locations.

Werner has defined an interesting set of states that remain invariant under local unitary transformations [1]. Among these, there is a subset of states that are inseparable and admit a hidden variable model. This means that there is a non-equivalence between inseparability and the property of violating Bell's inequalities. Popescu [2] proposed a hidden variable model for the Werner state of two qubits and proved that this state is useful for quantum teleportation [2].

We have focused on the study of the quantum degree of polarization of the Werner state. We employ the quantum Chernoff bound as a measure of polarization [3,4].

i) We present the hidden variable model proposed by Popescu and emphasize that the result obtained using this model coincides with the one given by quantum mechanics.

ii) We also describe the analytical method of evaluation of the quantum degree of polarization based on the quantum Chernoff bound.

The Werner – Popescu state is a mixed state, which does not violate any Bell type inequality, and at the same time it is useful for quantum teleportation.

The interest in the study of the entanglement of the Werner – Popescu state and the more general one, Werner state, has been increased during the last years. we investigate another important property of the Werner state: the quantum degree of polarization. We use the quantum Chernoff bound as a measure of polarization of a two-mode state of the radiation field.

This bound is a recently introduced measure that enables the discrimination of two quantum states [5,6]. The initial classical problem was formulated and solved by Chernoff in 1952 and consists in finding the minimal error distribution for discriminating two probability distributions in the asymptotic limit.

The behavior of entanglement of the Werner state has widely been studied in the literature. We have focused on a different quantity that characterizes a Werner state, namely the quantum degree of polarization. We derived the analytical expression of the quantum Chernoff degree of polarization [7].

In the following figure it is represented the dependence of the quantum Chernoff degree of polarization of Werner states on the parameter *a* which defines the state [7].

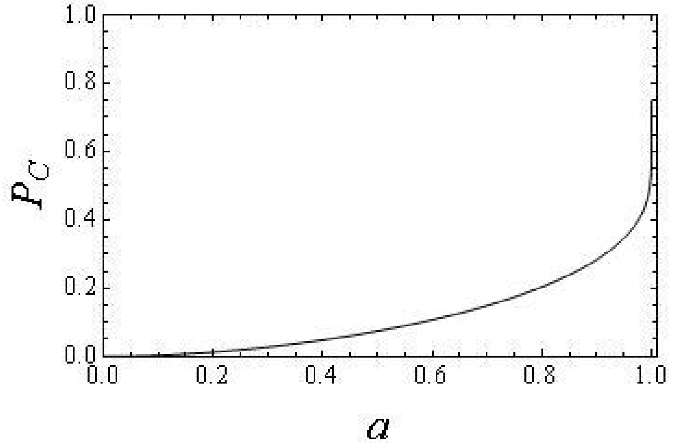

Fig. 6. Dependence of the quantum Chernoff degree of polarization of Werner states on the parameter which defines the state.

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The objective of the project during the year 2016 was:

Application of the quantum Chernoff bound for evaluation of the degree of polarization of the bimodal Werner state.

Entanglement plays an important role in quantum information theory, in particular for quantum information protocols and tasks like quantum teleportation [1] and its generalizations [2–5], quantum cryptography [6], superdense coding [7], and quantum computation [8].

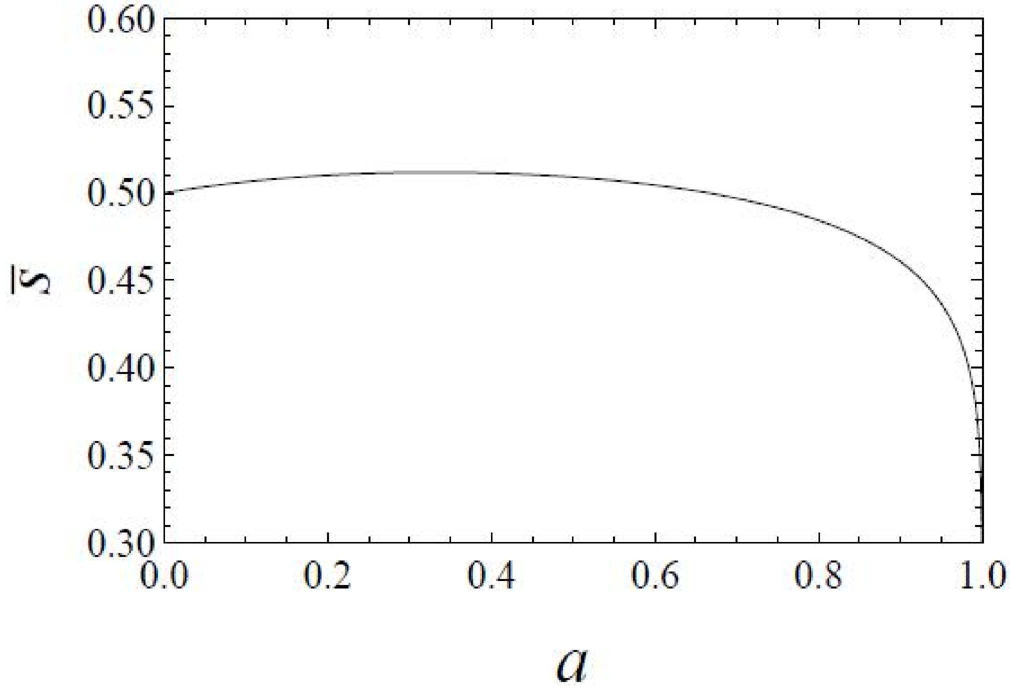
The set of states that remain invariant under local unitary transformations are called Werner states [9]. Popescu [10] proved that the Werner state of two qubits is useful for quantum teleportation. The Werner states have attracted a lot of attention of the quantum information community. Recently we have investigated the quantum degree of polarization based on the quantum Chernoff bound [11]. The quantum Chernoff bound provides the minimal error probability of discriminating between two quantum states when many identical copies are available [12].

We employed the quantum Chernoff bound as a measure of polarization of the Werner state. We find the analytical expression of the parameter that minimizes the function which is required in the evaluation of the polarization. This gives the exact expression of the Chernoff degree of polarization of the Werner state.

The quantum Chernoff bound represents a generalization of a classical problem formulated and solved by Chernoff in 1952 [13], namely one has to find the minimal error distribution for discriminating two probability distributions in the asymptotic limit.

The quantum Chernoff bound was recently used for defining the quantum degree of polarization of a two-mode state of the quantum radiation field [14], [15].

In the following figure it is represented the dependence of the parameter for which one can obtain the minimum of the quantum Chernoff on the parameter *a* which defines the Werner state [16].



We have found the exact expression of the quantum degree of polarization based on the Chernoff bound for the Werner state. The investigation of this topic, which started in Ref. [11], consisted in a numerical study and in a detail presentation of how one can obtain the expression of the parameter that minimizes the function which determines the qquantum Chernoff bound. After getting its formula, it was used for computing the exact expression of the polarization of the Werner state.

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