SCIENTIFIC REPORT FOR THE PROJECT IDEI nr 160/2021

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The structure of the report is the following. In the first part we describe the scientific activity done this year. Then, after the references, we present the dissemination activities (publications and presentations to conferences). Finally we provide a summary of the main scientific results.

DESCRIPTION OF THE SCIENTIFIC ACTIVITY AND MAIN RESULTS

(a) Estimation of α-like correlations from α transfer spectroscopic factors

In the previous studies we have shown that the ground states of $N = Z$ nuclei can be described by a condensate of α-like quartets [1–6]. To probe experimentally the quartet condensation, the best alternative is to perform α transfer reactions along a chain of $N = Z$ nuclei. Such an experiment has been approved at IPN-Orsay/University Paris-Saclay.

The scope of the present study is to estimate the spectroscopic factors for the α transfer in the framework of quartet condensation model (QCM) [1–6]. In order to achieve that, we have first evaluated the spectroscopic factors for the transfer of a generic quartet operator between $N = Z$ nuclei. More precisely, we calculated the quartet-transfer amplitude (QTA) defined by

$$T_k = |\langle QCM(n_q + 1)|q_k^+|QCM(n_q)\rangle| \tag{1}$$

where

$$q_k^+ = [[a^+a^+]^{JT}[a^+a^+]^{JT}]_k^{J=0,T=0} \tag{2}$$

are all the non-collective quartets with J=0 and T=0 which can be formed out of the non-collective pairs with T=0,1 and angular momentum J. The QTA is calculated considering that the ground state of $N = Z$ nuclei are described by the QCM state [4]

$$|QCM(n_q)\rangle = (Q^+)^{n_q}|-\rangle, \tag{3}$$

where n_q is the number of quartets built with the valence nucleons. The quartet Q^+ is defined by

$$Q^+ = \sum_{i,i',k,k',J,T} x_{ii'kk';J,T}[\mathcal{A}^{+JT}(ii')\mathcal{A}^{+JT}(k,k')]^{(0,0)}. \tag{4}$$

where $\mathcal{A}^{+JT}_{J_z,T_z}(i,i') = [a_i^+a_{i'}^+]^{J,T}_{J_z,T_z}/\sqrt{(1+\delta(i,i'))}$.

Having in mind the proposed α transfer experiment at ALTO/IPN-Orsay/University Paris-Saclay, we have calculated the QTA for the chain of sd shell nuclei with $N = Z$. The calculations are done with the two-body interaction USDB [22], which provides a realistic description of the sd shell nuclei. In the sd shell there are 62 quartets which can be formed from 28 pairs. As an example, in Fig. 1 we show the QTA for two transfer reactions. Form

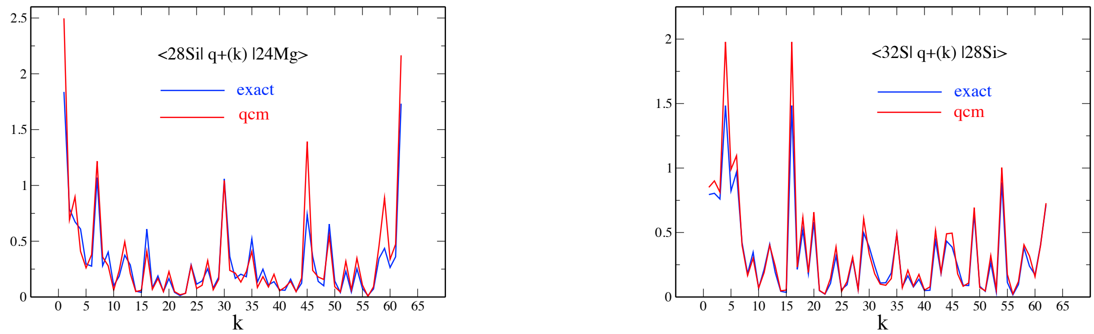

FIG. 1: Quartet-transfer amplitudes (1) for $\langle^{28}Si|q_k^+|^{24}Mg\rangle$ (left) and $\langle^{32}S|q_k^+|^{28}Si\rangle$ (right).

a detailed analysis of the results shown in Fig. 1 one concludes that the highest peaks for T_k are associated to the quartets built on anti-aligned and maximum-aligned pairs. That means that the quartet-pair transfer is driven mainly by those α-like quartets which enclose the maximum amount of pairing-type correlations. From Fig. 1 can be also seen that there is a different pattern of QTA before and after the middle of the sd shell. In particular, by analysing the QTA for the transition $^{32}S|q_k^+|^{28}Si\rangle$ it can be seen that the largest peaks correspond in fact to the same configuration. This is a consequence of the fact that the 62 quartets are not linearly intependent.

In Fig. 1 are shown also the T_k results obtained when in (1) a shell model wave function is used instead of the QCM state. One can notice that the T_k calculated with the QCM state are significantly larger than the ones calculated with the SM state. This means that the quartet condensate assumption for the ground states of $N = Z$ nuclei is expected to have significant consequences for the α transfer, a fact which could be tested by the α transfer experiments.

The calculated quartet-transfer amplitudes (1) are further employed in order to evaluate the spectroscopic factors for the α-transfer. For that one needs to add the contribution of the wave function which describes the α particle. The latter is approximated by two neutrons and two protons sitting in the lowest level of a mean-field which simulates the α particle properties. Then, to calculate the α transfer spectroscopic factor, the α particle wave function is expanded in the single-particle representation we have used for calculating the quartet transfer amplitude. This is a work in progress [7].

(b) The effect of proton-neutron pairing and α-like correlations on the superallowed α-decay in self-conjugate nuclei above ^{100}Sn

This study is related to the superallowed α decays ^{108}Xe $\rightarrow$ ^{104}Te $\rightarrow$ ^{100}Sn, recently observed at Argonne National Laboratory,[8]. These alpha decays, which involve for the first time heavy N = Z nuclei, are the fastest in the entire nuclear chart, much faster than the most known decay ^{212}Po $\rightarrow$ ^{208}Pb.

The fact that the decay width is much larger in ^{104}Te than in ^{212}Po is most probably related to the proton-neutron (pn) pairing and α-like correlations in N = Z nuclei, which are stronger than in the N > Z nuclei. One alternative to investigate this issue is by using the superfluid tunneling model (STM) [9]. This model has been employed recently to describe the influence of pairing on superallowed α decay, but without taking into account the effect of pn pairing [10]. Below we introduce shortly the standard STM model and then we present two alternatives for including in STM the pn pairing and α-like correlations [12].

In STM the decaying nucleus is supposed to deform progressively, up to the final decaying phase, by passing through various configurations which differ from each other by the crossing of two single-particle orbits. The transition matrix element (TME), which connects two successive configurations, is supposed to depend on the pairing interaction. The TME employed in the previous calculations includes only the like-particle pairing, which is treated in the BCS approach. Hence, for a state-independent interaction of strength G, which acts between neutrons and between protons, the TME is approximated by the expression [9]

$$v = -\frac{\Delta_n^2 + \Delta_p^2}{4G} \tag{5}$$

where Δ_n and Δ_p are the gaps for the neutron-neutron (nn) and proton-proton (pp) pairing.

To introduce the effect of pn pairing in the TME we propose two alternatives. The simplest one is to use the framework of HFB approach. In order to derive the TME corresponding to HFB, we follow the approximation scheme applied in Ref. [9]. More precisely: (a) the single-particle energies which characterize the system are devided in two parts, i.e., the levels which increase/descrease with the deformation. At each level crossing a nn, pp or pn pair is moved from an upsloping to a downsloping level, keeping the nucleus in its ground state; (b) we expand the HFB state in terms of the wave functions Ψ_+ and Ψ_- associated to the upsloping and downsloping orbits:

$$|HFB\rangle = (\sum_{n_2} c_{n_2}|\Psi_+(n_2)\rangle)(\sum_{n_1} d_{n_1}|\Psi_-(n_1)\rangle) \tag{6}$$

(c) we express $\langle HFB|V_P|HFB\rangle$, where V_P is the pairing interaction, in term of the functions $|\Psi_\pm\rangle$ and we make the assumption that $\sum_n c_{n+1}c_n = \sum_n d_{n+1}d_n \approx 1$.

With the approximations mentioned above we finally obtained for the transition matrix the expression

$$v = \frac{1}{4}Tr\left(\frac{1}{2}\Delta t^+\right) \tag{7}$$

where Δ and t are the matrices for the gap and the pairing tensor given in [11].

The HFB approach does not conserve exactly the particle number and the isospin. This is a major drawback when applied to $N = Z$ nuclei. A better alternative, which conserves exactly these quantites, is the QCM approach we have introduced on Ref. [2]. In this approach the ground state of N=Z nuclei is described by

$$|QCM\rangle = (Q^\dagger_{T=1} + \Delta_0^{\dagger 2})^{n_q}|0\rangle, \tag{8}$$

where $n_q = (N + Z)/2$. The operator $Q^\dagger_{T=1}$ is the isovector quartet built by two isovector pairs coupled to the total isospin $T = 0$ while Δ_0 is the collective isoscalar pair.

Since QCM conserves the particle number, to derive the corresponding TME one cannot apply the same approximation employed for the HFB approach. However, one can deduce the expression of TME for QCM from the the general features of the BCS and HFB results (5,7). Thus, on one hand, it can be noticed that in both cases the TME is equal, up to the factor 1/4, to the pairing energy. On the other hand, it can be easily shown that the factor 1/4 is not specific to the BCS or HFB approximations. In fact, this factor takes into account that only 1/4 of the pairing force contributes to TME. This can be seen by expressing the pairing force operator in terms of the pair operators which scatter pairs on upsloping (u) and downsloping (d) states, namely

$$P^+P = P^+_d P_u + P^+_u P_d + P^+_u P_u + P^+_d P_d \tag{9}$$

All the terms in the equation above have equal contributions to the pairing energy. However, from all of them, only the first one, which scatters a pair from the upsloping to the downsloping level, is contributing to TME. Hence, based on the two considerations mentioned above, one concludes that the TME corresponding to the QCM approach can be written as

$$v = \frac{1}{4}\Big(\sum_{ij,\tau=\pm1,0} V_{ij}^{T=1} < QCM|P^+_{i\tau}P_{j\tau}|QCM >' + \sum_{ij} V_{ij}^{T=0} < QCM|D^+_{i0}D_{j0}|QCM >'\Big) \tag{10}$$

In the equation above the notation $\langle QCM|...|QCM\rangle'$ indicates that the average of the pairing forces does not include the contribution of the self-energy terms.

To estimate the contribution of pn pairing to the transition matrix element (10) for the α decay of ^{104}Te and ^{108}Xe, we follow the Skyrme+QCM calculation scheme we have used previously in Ref. [6]. For the Skyrme functional we use the parametrisation UNE1 [21] while for the pairing interaction we employ a zero range force. In order to make the connection with the experimental data on superallowed α decay, the isovector force is chosen to reproduce, in QCM, the pairing energies employed in Ref. [10]. The effect of the isovector pn pairing on TME is estimated by comparing the QCM results with the ones predicted by the PBCS calculations in which it is considered only the nn and pp pairing. Finally, we switch on the isoscalar pairing interaction and estimate its effect on the transition matrix element (10). This is a work in progress [12].

(c) Intrinsic states of deformed $N = Z$ nuclei in a quartet formalism

In a recent article [13] we have shown that the band-like structure of $N = Z$ nuclei can be obtained by making a configuration mixing calculations in terms of quartets extracted from various trial states. In the present study we show that these trial states act as intrinsic states, from which the band-like structures can be generated directly, through angular momentum projection [15].

The intrinsic states considered in this study are defined by [15]

$$|\Theta_g\rangle = \mathcal{N}_g (Q_g^+)^n |0\rangle, \tag{11}$$

$$|\Theta_k\rangle = \mathcal{N}_k Q_k^\dagger (Q_g^\dagger)^{(n-1)} |0\rangle, \tag{12}$$

where $Q_g^+ = \sum_J \alpha_J^{(g)} (q_g^+)_{J0}$ and $Q_k^\dagger = \sum_J \alpha_J^{(k)} (q_k^\dagger)_{Jk}$ are linear superpositions of the most general collective quartets $(q_k^+)_{Jk}$ which can be formed with the valence nucleons. As can be seen, the intrinsic ground state (11) is a quartet condensate while the " excited " intrinsic states (12) are obtained by breaking a quartet from the intrinsic ground state and replacing it with an "excited" quartet. The structure of the quartets are fixed variationally from the minimisation of the average of a two-body Hamiltonian of shell-model type.

From the intrinsic states (11,12) are generated, through angular momentum projection, various band-like structures, analogous to the ground, beta and gamma bands commonly introduced for $N > Z$ nuclei.

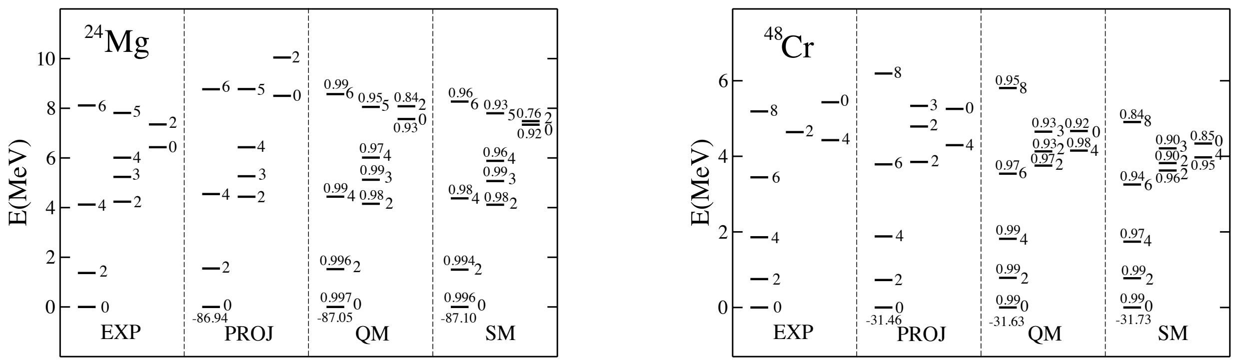

FIG. 2: The spectra for ^{24}Mg (left) and ^{48}Cr (right). See the text for details.

This formalism has been applied to the *sd* and *pf* shell nuclei. The calculations are done with the realistic shell-model forces USDB [22] and KB3G [23]. As an example, in Fig. 2 we show the results for ^{24}Mg and ^{28}Cr. The present results, indicated by PROJ, are compared to the experimental levels, the previous QM results [13], obtained by diagonalisation in the quartet basis, and to the shell model (SM) predictions. The numbers on the top of SM levels indicate the overlap between the SM states and the PROJ states. An overall good agreement is found between the PROJ states and the experimental spectra. One can also notice that the PROJ states can be easily traced back to the SM states. By using this correspondence, in Fig. 2 the SM spectrum was split in band-like structures. It is worth mentioning that SM, without additional calculations, provides just a sequence of levels, without any information about the relations between them.

The fact that the band-like structure of $N = Z$ nuclei can be described by intrinsic states based on quartets indicates that the quartets are the fundamental degrees of freedom both for the ground and low-lying excited states of even-even $N = Z$ nuclei.

(d) **Coexistence of quartets and pairs in $N > Z$ nuclei**

In previous studies we have shown that the ground states of $N > Z$ systems interacting by proton-neutron pairing forces have a simple structure: a condensate of quartets to which it is attached a pair condensate formed by the excess neutrons [16, 17]. The scope of this study is to investigate whether similar quartet-pair structures can be identified in realistic calculations of $N > Z$ nuclei based on general two-body interactions of shell-model type [18]

Following the suggestion of pairing calculations mentioned above, we suppose that the ground state of $N > Z$ nuclei can be described by the trial state [18]

$$|\Omega(m,n)\rangle = (\Gamma^{+})^{m}|\Psi_0(n)\rangle, \tag{13}$$

where $|\Psi_0(n)\rangle$ is the state which describes the N=Z subsystem. In the first approximation, $|\Psi_0(n)\rangle$ is taken as a condensate of intrinsic quartets. Then, with the quartets found variationally by minimizing the quartet condensate, we make a configuration mixing calculations for the ground state of the $N = Z$ subsystem. The extra neutrons are described by the pair condensate built by the collective pair $\Gamma^{+} = \sum_i \alpha_i [a_i^{+} a_i^{+}]^{J=0,T=1}_{M_T=1}$, which is determined variationally.

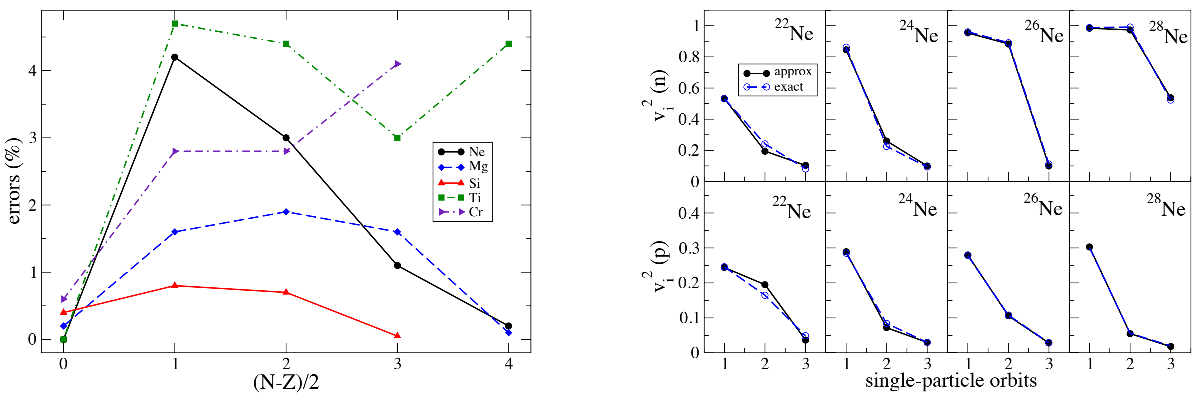

FIG. 3: (Left) Errors for the ground state energies predicted by the approx (6). (Right) Occupation probabilities of the single-particle orbits $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ for Ne isotopes.

We have applied the ansatz (13) for the ground state of sd and pf nuclei . As example, in Fig. 3 are given the errors in the binding energies, relative to the shell model values, and the occupation probabilities of single-particle orbits for ^{24}Mg, provided by the approximation (13). It can be seen that the errors for the binding energies are small, below 5%. One can also see that the occupation probabilities for the single-particle orbits follows closely the exact values. These results validate the ansatz (13) for the ground states of $N > Z$. We can thus conclude that the ground states of $N > Z$ nuclei have indeed a simple structure : a product between a state describing the $N = Z$ subsystem, expressed in terms of quartets, and a pair condensate formed by the extra neutrons.

(e) Excited states of zero seniority based on a pair condensate

Recent studies have shown that the excited states of a proton-neutron pairing Hamiltonian can be obtained by breaking a quartet from the quartet condensate, which describes the ground state, and replacing it with an "excited" quartet [19]. It was also shown recently that in N=Z nuclei there is an excited state of low energy which have the structure of a quartet condensate [4]. In this study we show that similar excited states can be found in $N > Z$ systems in which the neutrons and the protons move in different major shells [20].

We consider a system of neutrons or protons interacting by a two-body force. As shown in previous studies, the ground state of this system can be described by a PBCS condensate of collective pairs. Starting from this condensate we construct two types of excited states of zero seniority. One type is obtained by breaking a pair from the ground state condensate and replacing it by "excited" collective pairs built on time-reversed single-particle orbits. These states have the form [20]

$$|N; 1_k >= \tilde{\Gamma}_k^\dagger (\bar{\Gamma}^\dagger)^{N-1}|0> \tag{14}$$

where $\Gamma^\dagger$ and $\hat{\Gamma}^\dagger$ are collective pairs determined variationally.

The second types of excitations are obtained by breaking all the pairs from the ground state condensate and replacing them by a unique excited collective pair. This excited pair condensate (EPC) has formally the same structure as the ground state condensate, i.e.

$$|EPC(k)\rangle = (\hat{\Gamma}_k^\dagger)^N|0> . \tag{15}$$

The excited states mentioned above have been analysed for various systems and two-body forces. As an example, in Fig. 4 are shown the results for ^{108}Sn obtained with a state-dependent pairing interaction derived from the G-matrix.

The results for the states (14) and the EPC states (15) are shown in the middle column. On the right are shown the experimentally known levels of spin $J = 0$. One can notice that

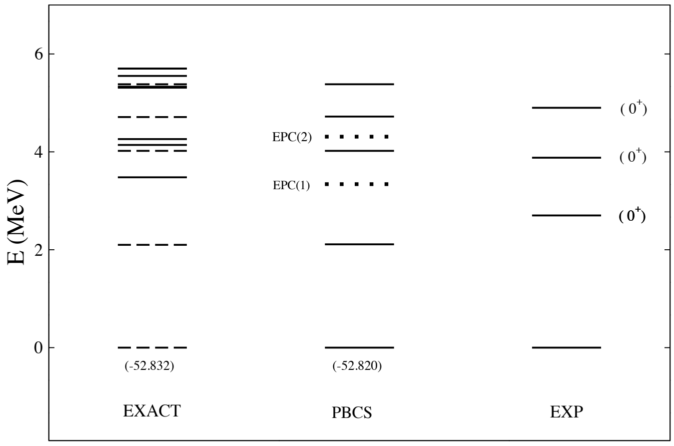

FIG. 4: The excitation energies corresponding to the states (14) and to the EPC states, compared to the experimental $J = 0$ levels of ^{108}Sn and with exact calculations. By dashed lines are shown those exact levels which corresponds to the one-broken-pair states.

the energies of these levels are well described by the states (14). In turn, these states are also very close to the exact states indicated by dashed lines. Consequently, we conclude that the lowest three known $J = 0$ states in ^{108}Sn have a simple physical interpretation: they are states of one-broken-pair type [20].

In Fig. 4 are shown also the energies of the lowest two EPC states (15). They correspond the the first two minima obtained variationally. The lowest EPC state is practically built on the first two single-particle orbitals, $g_{7/2}$ and $d_{5/2}$. On the other hand, the second EPC state is spread out on all the orbits. In the limit of small pairing strength the two EPC states correspond to three pairs promoted from the orbit $f_{7/2}$ to the orbit $d_{5/2}$ and, respectively, to one pair promoted from $f_{7/2}$ to $3s_{1/2}$.

As can be seen the EPC states are surprisingly low in energy, between the 1st and the 3rd one-broken pair states. Whether one could excite nuclei in such a particular low energy state, described by a pair condensate, is an interesting question which deserves further theoretically and experimental studies.

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SUMMARY OF DISSEMINATION ACTIVITIES

Publications

As it was planned, in the first two years we have completed 6 papers, 3 in 2021 and 3 in 2022. Two of the papers submitted for publications in 2021 appeared in 2022. They are:

Band-like structures and quartets in deformed $N = Z$ nuclei

M. Sambataro and N. Sandulescu, Phys. Lett. B **827**, 136987 (2022)

Proton-neutron pairing and binding energies of nuclei close to N=Z line

D. Negrea, N. Sandulescu, D. Gambacurta, Phys. Rev. C **105**, 034325 (2022).

The papers completed in 2022 are :

Intrinsic states of deformed $N = Z$ nuclei in a quartet formalism

M. Sambataro and N. Sandulescu, submitted

Coexistence of quartets and pairs in $N > Z$ nuclei

M. Sambataro, N. Sandulescu, D. Gambacurta, submitted

Excited states of zero seniority based on a pair condensate

Th. Popa, N. Sandulescu and M. Sambataro, submitted

Presentations to conferences

This year the director of the project presented two talks at the following conferences:

Proton-neutron pairing and quartet correlations in nuclei

Frontiers Nuclear Structure Theory (Stocholm, 23-25.05, 2022)

Proton-neutron pairing and quartetting in ground/excited states of N=Z nuclei

European Nuclear Physics Conference (Santiago de Compostela, 24-28/10/ 2022)

In conclusion, the scientific and the dissemination activities are in the agreement with the ones originally planned.

SUMMARY OF THE SCIENTIFIC ACTIVITY

This year the activity was focused on the following subjects:

(a) We have calculated the spectroscopic factors for the transfer of α-like quartets between even-even N=Z nuclei with the valence nucleons moving in the *sd* shell. The calculations have been done in the quartet condensation model, which takes into account the proton-neutron pairing and α-like correlations.

(b) We have extended the superfluid tuneling model (STM) in order to take into account the effect of proton-neutron pairing on superallowed α decay in even-even N=Z nuclei above A=100. The extension of STM was done first in the Hartree - Fock - Bogoliubov approach and then in the framework of quartet condensation model.

(c) We have shown that the low-lying excitations of even-even N=Z nuclei can be described by angular momentum projection from intrinsic states based on α-like quartets. More precisely: the ground state band was generated from a quartet condensate while the β and γ-like bands have been projected out from intrinsic states obtained by breaking a quartet from the quartet condensate and replacing it with an " excited" quartet.

(d) We have studied the competition between the quartets and pairs in the ground state of $N > Z$ nuclei with the valence nucleons moving in the same major shell. We have shown that the ground state of these nuclei can be approximated by a product between a state which describes the $N = Z$ subsystem, expressed in terms of quartets, and a pair condensate formed with the excess neutrons.

(e) We have studied the state of zero seniority for like-particle systems and we have identified a new class of excited states which have the structure of a pair condensate. We have shown that in ^{108}Sn these states appear at a surprising low energies.

Project coordinator

Dr. N. Sandulescu