COLLECTIVE MODES IN ELASTIC NUCLEAR MATTER

Ş. MIŞICU

Theoretical Physics Division, NIPNE-HH,
Bucharest-Magurele, POB MG-6, ROMANIA
**E-mail: misicu@theory.nipne.ro*
http://www.theory.nipne.ro/ misicu/

S. BASTRUKOV

Laboratory of Informational Technologies,
Joint Institute for Nuclear Research,
141980 Dubna, Moscow Region, Russia
E-mail: bast@jinr.ru

We review shortly the continuum mechanical approach to the excitation of collective modes in homogenous nuclear matter. The main point of this treatment is that nuclear matter around the saturation density displays characteristics analogous to a linear elastic continuum. Giant resonances are discussed as typical cases of elastic disturbances in nuclear matter. Within such a macroscopic approach it is possible to understand the gross properties of isoscalar electric and magnetic resonances. We sketch also how this framework can be extended to the case of inhomogenous nuclear matter.

Keywords: Giant Resonances; elasticity; nuclear equation of state; neutron stars.

1. Introduction

Regardless of what type of continuum is assumed, fluid or elastic, an essential ingredient in solving the nuclear dynamical macroscopic equations, is provided by the nuclear equation of state. This fundamental relation, that in principle should be traced back to the effective nucleon-nucleon interaction, provides, for a transient process, a non-linear relation between the pressure(stress) of nuclear matter and the deviations of the density from the equilibrium value. In continuum mechanics the corresponding relation is customarily known under the name of *material law* or behavior law[1] . In continuum mechanics a material law sets constraints on the forces and/or the motions. In the case of *ideal materials* this law consists in particular relations between the stress tensor $\boldsymbol{T}$ and the movements, described by a collective vector field (displacement) $\boldsymbol{u}$ of the body under investigation. For linear elastic bodies this relation reads

$$\boldsymbol{T} = \lambda \mathcal{I} \mathrm{Tr}(\boldsymbol{\epsilon}) + 2\mu\boldsymbol{\epsilon}; \qquad \boldsymbol{\epsilon} = \frac{1}{2}\left(\boldsymbol{\nabla u} + \boldsymbol{u}\nabla\right) \tag{1}$$

The material law for nuclear matter near the saturation point is similar to the one discussed above for an elastic body. This is due to the distortions of the Fermi surface that induce non-diagonal components in the stress tensor[2] . The coefficients controlling the linear proportionality can be derived in the case of nuclear matter, regardless it is homogenous or not, from the corresponding equation of state. Considering a Skyrme like energy density functional[3] and using the Thomas-Fermi approximation for the kinetic energy density, the elastic moduli (Lame constants) are identified to be :

$$\lambda = \left(\frac{K}{9} - \frac{4}{15}\epsilon_F\right) n_0, \quad \mu = \frac{2}{5} n_0 \epsilon_F, \quad n_0 = \frac{\rho_0}{m} \tag{2}$$

In the above formula ε_F is the Fermi energy, which depends on the Fermi momentum (density ρ_0 at saturation) and the effective mass, and K is the nuclear incompressibility and is completely determined by the nuclear equation of state via the definition

$$K = 9\frac{\partial}{\partial\rho}\left(\rho^2 \frac{\partial\epsilon(\rho)}{\partial\rho}\right)_{\rho=\rho_0} \tag{3}$$

Note that the quasi-static approximation that is used frequently in the literature[4] amounts to the substitution $\mathrm{Tr}\boldsymbol{\epsilon} = 0$ in the material law (1). The equation of motion of a nuclear elastic body can de derived by varying the strain energy[3]

$$\rho_0 \ddot{\boldsymbol{u}} = (\lambda + 2\mu)\nabla(\nabla\cdot\boldsymbol{u}) + \mu\nabla\times\nabla\times\boldsymbol{u} \tag{4}$$

2. Giant Resonances in Spherical Nuclei

The credit for recognizing the elastic behavior of nuclear matter, as displayed in the giant resonant response, goes back to the seminal work[5] . In this paper it was pointed out that the experimentally established smooth dependence of the energy of the isoscalar giant quadrupole resonance can be understood as a manifestation of the irrotational and divergenceless oscillatory mode of an elastic solid whose shear modulus equals the pressure of Fermi nucleonic gas: $\mu = p_F = 2n_0\varepsilon_F/5$. Nowadays we have quite compelling arguments showing that both the giant electric and magnetic resonances can be treated on equal footing as a manifestation of spheroidal

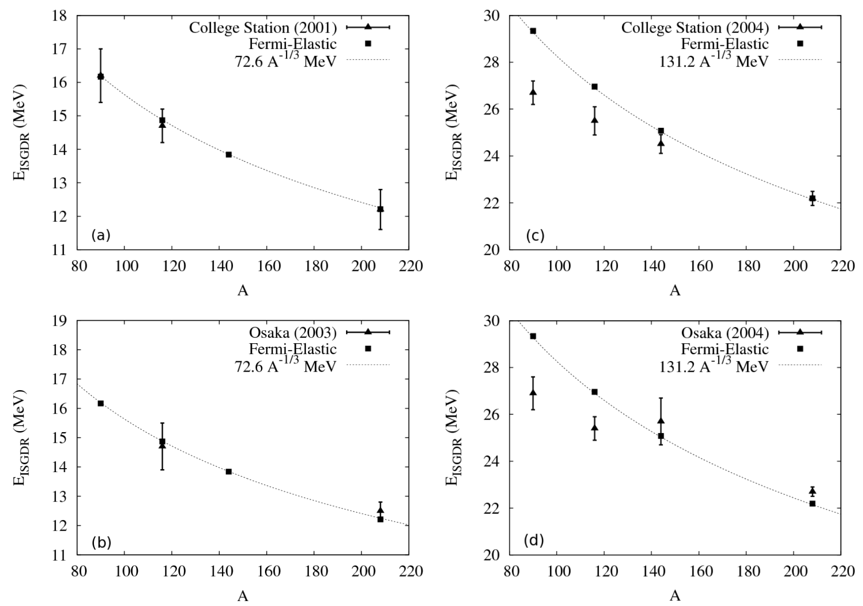

Fig. 1. Energy centroids of the low-lying and high-lying ISGDR as reported in[6] (College Station (2001)),[7] (College Station (2004)) and[8] (Osaka 2003) are compared with our calculations.

(electric) and torsional (magnetic) oscillations of an elastic sphere of nuclear matter[2] .

A first example that we want to discuss is the so-called Isoscalar Giant Dipole Resonance (ISGDR). In ref.[6] experimental results on this type of mode were reported for the strengths in three proton magical nuclei (^{90}Zr, ^{116}Sn and ^{208}Pb) using inelastic scattering of α particles at small angles and it was concluded that the isoscalar $E1$ strength distribution in each nucleus is shared mainly between two components, one located at low energy and and another one at higher energy. In a subsequent publication this group presented new data on the ISGDR.[7] For ^{116}Sn, ^{144}Sm and ^{208}Pb the low-energy peak fall in the interval $(1.71-1.92)\hbar\omega$ whereas the high-energy peak lays between $3\hbar\omega$ and $3.2\hbar\omega$. The upper component covers approximately 3 times more of the energy-weighted sum rule compared to the lower component. Similar values for the two peaks for ^{208}Pb are given in[8] : $1.80\hbar\omega$ and $3.25\hbar\omega$. The data for the energy centroids is ploted in Fig.1. For electric dipole modes one have to make sure, before deriving the oscillations eigenfrequencies spectrum, that the position of the center-of-mass $R_{\rm c.m.}$ is left

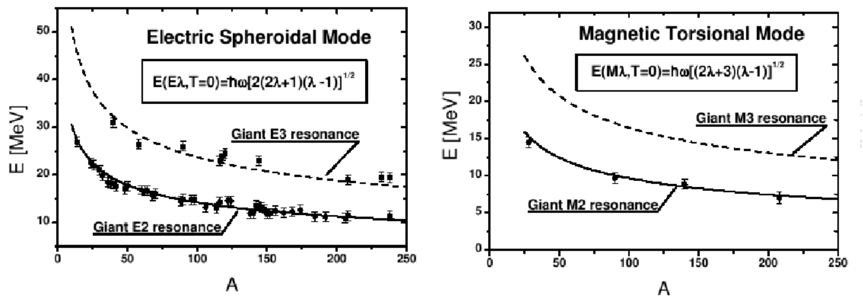

Fig. 2. Energy centroids of isoscalar giant quadrupole and octupole electric resonaces (upper panel), reported in[10] , and magnetic resonances (lower panel), reported in[11] , are compared with the calculation of the continuum mechanics approach.

undisturbed during the perturbation. Quantitatively this constraint can be written as

$$\delta \boldsymbol{R}_{\text{c.m.}} = \frac{\int \, d\boldsymbol{r} \rho(\boldsymbol{r}) \boldsymbol{u}(\boldsymbol{r}, t)}{\int \, d\boldsymbol{r} \rho(\boldsymbol{r})} \tag{5}$$

Assuming a harmonic variation in time, with frequency Ω, of the fluctuating parts of the density and the displacement field, the divergence and the curl of the displacement field are satisfying the scalar and vector Helmholz equation respectively (HE)

$$\left(\Delta + \begin{Bmatrix} k_L^2 \\ k_T^2 \end{Bmatrix} \right) \begin{Bmatrix} \nabla \cdot \boldsymbol{u} \\ \nabla \times \boldsymbol{u} \end{Bmatrix} = 0 \tag{6}$$

corresponding to the wave-numbers $k_{L,T} = \Omega / c_{L,T}$, where $c_L = \sqrt{\lambda + 2\mu/\rho_0}$ and $c_T = \sqrt{\mu/\rho_0}$. The eigenvalues are obtained by imposing boundary conditions on a free surface. In[9] we reported calculations of the energy centroids of the ISGDR that are supporting the experimental conclusions on the existence of a low-lying and a high-lying component of this collective mode (see Fig.2).

The predictive power of the continuum mechanical approach, that we present in this work, is demonstrated also by the fairly accurate account of experimental systematics on the electric octupole ($E3$) and the magnetic quadrupole ($M2$) giant resonances which is attained with no use of any adjustable constants (see Fig.2). The giant magnetic resonances exhibiting torsional oscillations of the nucleus is the mode of giant-resonance nuclear response that can be understood only on the basis of the solid globe model, not a liquid drop. Note that the in calculating these resonances we made a further simplification, i.e. take the quasi-static limit mentioned earlier.

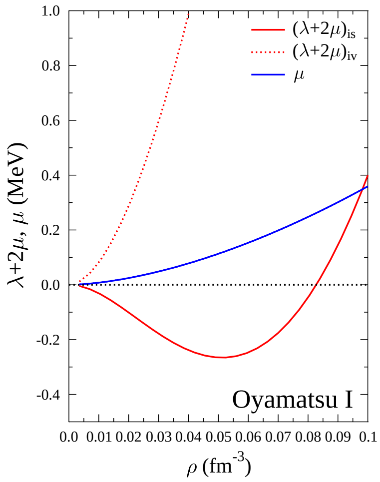

Fig. 3. Effective elastic constants of isoscalar and isovector resonances for dilute nuclear matter found in the inner crust of neutron stars. We used an equation of state designed for rich neutron matter.[13]

3. Giant Resonances in Inhomogenous Nuclear Matter

The resonances discussed above have a pronounced bulk effect and consequently we achieved a good agreement to the experimental data using a continuum model with a constant density, that corresponds grossly to the density of homogenous nuclear matter. In such circumstances resonances such as pygmy resonances, involving a collective displacement of the neutrons in the skin region of the nucleus relative to the core, cannot be described. Another case that involves the propagation of a collective perturbation in a inhomogenous media is found in the inner crust matter of neutron stars where, putatively, nuclear clusters are immersed in a gas of neutrons. Understanding the characteristics of these nuclear collective excitations could be very important in the determination of the specific heat of baryonic inner crust matter of these exotic astrophysical objects[12] .

Thus, the validity of the nuclear matter continuum mechanics framework must be extended to the case of highly inhomogenous nuclear matter. To deal with such cases we propose to generalize the material law (1) to the case of a composite continuum consisting in a number of homogenous subdomains, such that instead of using a single set of constants and variables (Lamé constants, stress and strain tensors) characterizing each homogenous region, we consider a single effective set of effective quantities. The material law for the composite material can written as the generalization of the corresponding law for homogenous matter

$$\langle \boldsymbol{T} \rangle = \boldsymbol{C}^* \langle \boldsymbol{\epsilon} \rangle \tag{7}$$

where the symbol $\langle \ldots \rangle$ denotes averaged quantities and the matrix elements of $\boldsymbol{C}^*$ provide the effective elastic moduli. Evidently, they have to be evaluated for densities far away from the saturation point. In the case of a composite layered medium (two alternating layers, one with dilute pure neutron matter, and another one, more dense and with non-vanishing proton content) the behavior of the effective elastic constants of isoscalar and isovector resonances on a typical range of nuclear matter densities is displayed in Fig.3.

The composite approach to continuum nuclear matter can be applied to a large number of exotic, inhomogenous structures, found in the inner crust of neutron stars or excitations in nuclei with large $n-p$ asymmetries.

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