

SM in Weyl conformal geometry

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Based on: [arXiv: 2203.05381](#); [2104.15118 \(EPJ C\)](#)

See also: [arXiv: 1812.08613 \(JHEP\)](#), [2007.14733 \(EPJ C\)](#);
[1904.06596 \(PRD\)](#); [1906.11572 \(JHEP\)](#); [2003.08516 \(EPJC\)](#)

- Outline:

- Scale symmetry: discrete, global, quantum, local, **gauged**.
- Weyl geometry.
- Weyl quadratic gravity: spontaneous breaking to Einstein gravity [no matter]
- SM in Weyl geometry: natural minimal embedding, gauge theory of SM & gravity
- Applications: inflation, Higgs physics, matter creation from geometry, etc.

- Symmetries beyond SM and GR...

- SM + its gauge symmetry and Higgs mechanism confirmed experimentally (LHC); **origin** of EW scale.
- Gravity? **origin of Planck mass (M_P)? of mass?** Geometric? Go **beyond** Einstein gravity and SM.
Need a **quantum** theory of gravity (various options: supergravity, string theory,..).

- Why scale symmetry:

- SM with $m_{\text{Higgs}}=0$ scale invariant; Early Universe or at short distances: EFTs are scale invariant.
- can be: discrete, global, quantum, local, **gauged=Weyl gauge symmetry (WGS)**

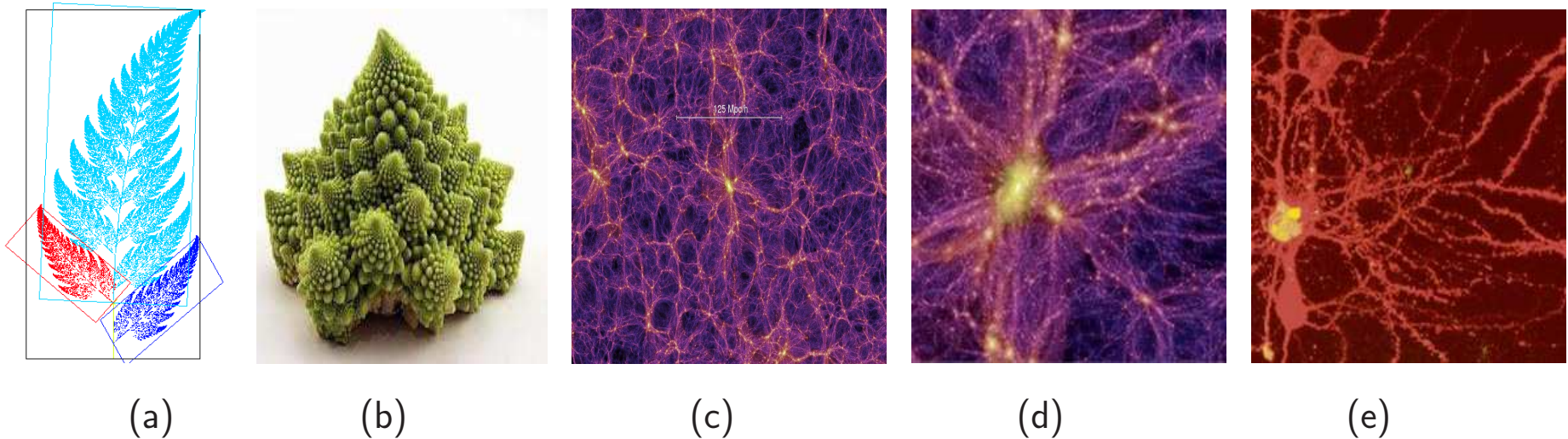
- **History:** Weyl geometry (WG) has WGS built in! first gauge theory [of scale invariance] 1918!

- WG \Rightarrow Weyl \tilde{R}^2 gravity. [Weyl thought it describes Gravity “+” Electromagnetism - soon disregarded]
- **Critique** by Einstein: WG **non-metric** $\nabla_\mu g_{\alpha\beta} \neq 0$. Critique assumes **massless** Weyl gauge field ω_μ !

- **Goal:** non-metricity not a problem but an advantage: **the origin of mass!**

WGS: spontaneous breaking, ω_μ is **massive**; mass scales ($M_P, m_\omega, \Lambda, \dots$) from geometry, Einstein gravity below m_ω .

- **Discrete scale symmetry in Nature:** : self-similarity/self-affinity across different scales [fractals]

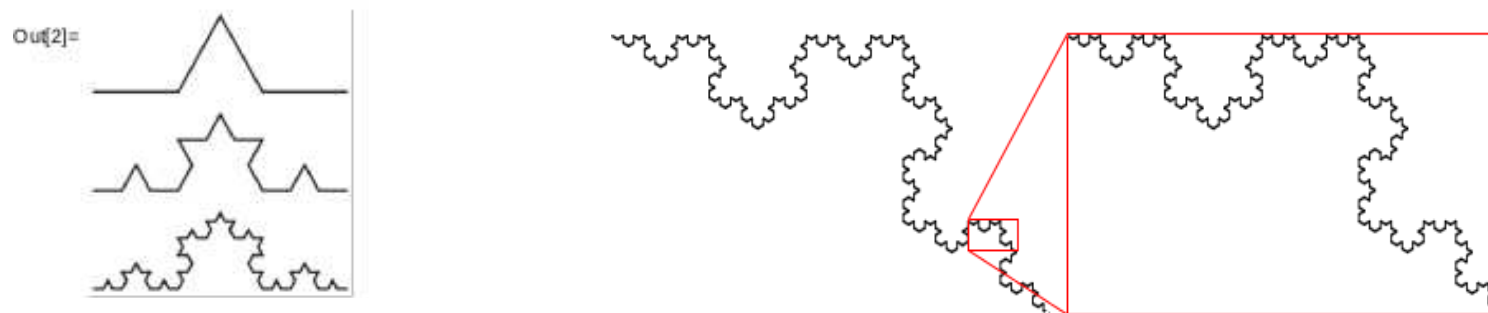


(c), (d): cluster of galaxies, stars, DM. astro-ph/0504097

<https://wwwmpa.mpa-garching.mpg.de/galform/millennium/>

(e): neurons synapses in a mouse brain, μm size: [Mark Miller, Brandeis U 2006]

(f): `In[2]:= Column[Table[Graphics[KochCurve[n]], {n, 1, 3}]]`



- self-repeating patterns at **large and small** scales, new structure revealed. **Koch curve/snowflake**

- Global scale symmetry

$$x'_\mu = \rho x_\mu; \quad \phi'(\rho x) = (1/\rho) \phi(x), \quad \text{forbids } \int d^4x m^2 \phi^2$$

- SM with Higgs ϕ of mass $m_\phi = 0$ is scale invariant [Bardeen 1995]
- no dim-ful couplings; scales generated from vev's, e.g. $M_P \sim \langle \sigma \rangle$. Broken by **quantum** corrections.
- Global symmetries **broken** by BH physics

[Kallosh, Linde, Susskind, hep-th/9502069]

- Quantum scale symmetry (QSS):

- replace DR scale $\mu \rightarrow \sigma$ (dilaton) to keep scale invariance in $d=4 - 2\epsilon$; **extra field! Different theory!**
- QSS broken spontaneously; if $\langle \sigma \rangle \rightarrow \infty$ decouples, usual results (breaking by DR) recovered.
- at 1-loop [Englert et al 1976], Shaposhnikov 0809.3406; D.G. 1508.00595] and in SM [D.G., Z. Lalak, P. Olszewski, 1612.09120]
- at 2-loops [D.G., Z. Lalak, P. Olszewski, 1608.05336]; 3-loops in SM [D.G. 1712.06024, Gretschev, Monin 1308.3863]
- **protects a classical hierarchy** $\phi \ll \sigma$; c-terms: $\phi^6/\sigma^2, \phi^8/\sigma^4 \dots$ [D.G. 1508.00595, 1712.06024]
- Higgs-Gravity coupling: $\xi \phi^2 R \rightarrow$ tuning higgs selfcoupling $\beta_\lambda \sim \lambda(\dots) + \xi(\dots)$.

• **Local scale symmetry:** L invariant under : $\hat{g}_{\mu\nu} = \Omega(x)^2 g_{\mu\nu}$, $\hat{\phi} = \frac{\phi}{\Omega(x)}$, $\hat{\psi} = \frac{\psi}{\Omega(x)^{3/2}}$

- Include gravity (ϕ real):

$$L_0 = -\frac{1}{2} \sqrt{g} \left[\frac{1}{6} \phi^2 R + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right], \quad \Leftrightarrow \quad L_0 = -\frac{1}{2} \sqrt{\hat{g}} M_P^2 \hat{R} \quad \text{generated spontaneously}$$

$$\Omega^2 = \frac{\phi^2}{\langle \phi \rangle^2}, \quad M_P^2 \equiv \frac{1}{6} \langle \hat{\phi} \rangle^2, \quad \text{"gauge fixing"}$$

Einstein frame: $M_p \sim \langle \phi \rangle$ then ϕ decouples \Rightarrow Conformal SM:

[t'Hooft 1104.4543; 1410.6675; Bars, Steinhardt, Turok 1307.1848, Englert et al 1976]

But:

- a) - has a **negative** kinetic term for ϕ
- b) - Fake conformal symmetry? **vanishing current**. [Jackiw, Pi 2015],
- c) - It **assumes** $\langle \phi \rangle \neq 0$. How does this happen?
- d) - ϕ : compensator, **added "ad-hoc"** to enforce symmetry; ϕ and $M_p \sim \langle \phi \rangle$: **not** a geometric origin.
- e) - L_0 has a symmetry that its underlying geometry (connection) does **not!** consistent? Γ not invariant.

\Rightarrow We want to avoid these issues: \Rightarrow **gauged** scale invariance.

• **Gauged scale invariance:** $\hat{\omega}_\mu(x) = \omega_\mu(x) - \frac{1}{\alpha} \partial_\mu \ln \Omega(x)^2, \quad \hat{g}_{\mu\nu}(x) = \Omega(x)^2 g_{\mu\nu}(x), \quad \hat{\phi}(x) = \frac{\phi(x)}{\Omega(x)} \quad (*)$

• **Weyl geometry:** $(g_{\mu\nu}, \omega_\mu): \quad \tilde{\nabla}_\mu g_{\alpha\beta} = -\alpha \omega_\mu g_{\alpha\beta},$

Riemannian geometry:

“metric” form: $\tilde{\nabla}'_\lambda g_{\mu\nu} = 0, \quad \partial_\lambda \rightarrow \partial_\lambda + \text{charge} \times \alpha \times \omega_\lambda$

$$\nabla_\mu g_{\alpha\beta} = 0$$

$$\Rightarrow \tilde{\Gamma}_{\mu\nu}^\rho = \Gamma_{\mu\nu}^\rho + (\alpha/2) (\delta_\mu^\rho \omega_\nu + \delta_\nu^\rho \omega_\mu - g_{\mu\nu} \omega^\rho) \quad \text{inv of } (*);$$

$$\Gamma_{\mu\nu}^\rho = \text{Levi-Civita}; \quad \nabla_\mu \text{ with } \Gamma$$

$$\Rightarrow \tilde{R} = R - 3\alpha \nabla_\mu \omega^\mu - 3/2 \alpha^2 \omega^\mu \omega_\mu; \quad \hat{R} = \frac{\tilde{R}}{\Omega^2} \quad (!)$$

$$\Rightarrow \tilde{D}_\mu \phi = (\partial_\mu - \alpha/2 \omega_\mu) \phi \Rightarrow \hat{D}_\mu \hat{\phi} = (1/\Omega) \tilde{D}_\mu \phi;$$

$$\Rightarrow F_{\mu\nu} = \tilde{\nabla}_\mu \omega_\nu - \tilde{\nabla}_\nu \omega_\mu = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \quad \text{inv } (*). \quad \text{Also } \alpha \omega_\mu \sim \tilde{\Gamma}_\mu - \Gamma_\mu \text{ deviation from Levi-Civita.}$$

$$\Rightarrow \text{if } \omega_\mu \rightarrow 0: \quad \tilde{\Gamma} \rightarrow \Gamma, \quad \text{Weyl geometry} \rightarrow \text{Riemannian}; \quad \tilde{R} \rightarrow R, \quad \text{Weyl tensor } \tilde{C}_{\mu\nu\rho\sigma} \rightarrow C_{\mu\nu\rho\sigma}$$

$$\Rightarrow \text{All invariants of } (*) \text{ [no matter]:} \quad \sqrt{g} \tilde{R}^2, \quad \sqrt{g} F_{\mu\nu}^2, \quad \sqrt{g} \tilde{C}_{\mu\nu\alpha\beta}^2; \quad \text{no higher dim ops (no scale!)}$$

• **Weyl quadratic action** \Rightarrow Einstein gravity + massive ω_μ

[D.G. arXiv:2203.05381, 2104.15118, 1812.08613]

$$\mathcal{L}_0 = \sqrt{g} \left[\frac{1}{4!} \frac{1}{\xi^2} \tilde{R}^2 - \frac{1}{4} F_{\mu\nu}^2 \right] = \sqrt{g} \left[\frac{1}{4!} \frac{1}{\xi^2} (-2\phi_0^2 \tilde{R} - \phi_0^4) - \frac{1}{4} F_{\mu\nu}^2 \right], \quad \text{eq. motion } \phi_0^2 = -\tilde{R}.$$

$$\text{Riemannian notation: } \mathcal{L}_0 = \sqrt{g} \left\{ \frac{-1}{2\xi^2} \left[\frac{1}{6} \phi_0^2 R + (\partial_\mu \phi_0)^2 \right] - \frac{\phi_0^4}{4! \xi^2} + \frac{\alpha^2}{8\xi^2} \phi_0^2 \left[\omega_\mu - \frac{1}{\alpha} \partial_\mu \ln \phi_0^2 \right]^2 - \frac{1}{4} F_{\mu\nu}^2 \right\}.$$

If $\langle \phi_0 \rangle \neq 0$, “gauge fixing”: $\Omega^2 = \phi_0^2 / \langle \hat{\phi}_0 \rangle^2 \Rightarrow \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$, $\langle \hat{\phi}_0 \rangle^2 = 6 \xi^2 M_p^2$, $\hat{\omega}_\mu = \omega_\mu - \frac{1}{\alpha} \partial_\mu \ln \phi_0^2$

$$\mathcal{L}_0 = \sqrt{\hat{g}} \left[-\frac{1}{2} M_p^2 \hat{R} + \frac{3}{4} M_p^2 \alpha^2 \hat{\omega}_\mu \hat{\omega}^\mu - \frac{1}{4} \hat{F}_{\mu\nu}^2 - \Lambda M_p^2 \right], \quad M_p^2 \equiv \frac{\langle \phi_0^2 \rangle}{6 \xi^2}, \quad \Lambda \equiv \frac{1}{4} \langle \phi \rangle^2.$$

\Rightarrow Einstein-Proca action & M_p , Λ , via **Stueckelberg** mechanism. ω_μ massive, can decouple!

$\Rightarrow \tilde{\Gamma} \rightarrow \Gamma$ then **WG** \rightarrow **Riemannian**. Spontaneous breaking, non-trivial.

\Rightarrow **Einstein gravity=broken phase of Weyl action**. Metricity restored below $m_\omega \sim M_p$! $\alpha \ll 1$? **No ghost!**

\Rightarrow what if no $F_{\mu\nu}^2$: Weyl integrable theory = metric. Symmetry has vanishing current! not interesting....

• **Weyl quadratic gravity** \Rightarrow Einstein gravity + massive ω_μ

\Rightarrow we have: $\nabla^\mu J_\mu = 0$; $J_\mu = \alpha/(2\xi^2) \phi_0 [\partial_\mu - (\alpha/2) \omega_\mu] \phi_0$; if ϕ_0 constant $\nabla_\mu \omega^\mu = 0$ gauge fixing.

\Rightarrow if ω_μ not dynamical: $\omega_\mu \sim (1/\alpha) \partial_\mu (\ln \phi_0^2)$, $J_\mu = 0$ (metric case).

- Eq. motion: $\square \phi_0^2 = 0$; FRW metric: $\phi_0(t)^2 = c_1 \int^t d\tau / a(\tau)^3 + c_2 \rightarrow$ constant. [GG Ross et al 1801.07676]

- ϕ_0 is part of \tilde{R}^2 : then $M_p^2 \sim \langle \phi_0 \rangle^2 / \xi^2$, $\Lambda \sim \langle \phi_0 \rangle^2$, $m_\omega^2 \sim \alpha^2 M_p^2$: have **(non-metric) geometric origin**.

\Rightarrow **Non-metric geometry as the origin of mass!**

- Other terms: L_C , due to Weyl tensor ($\tilde{C}_{\mu\nu\rho\sigma}$) does not change the result:

[Mannheim]

$$L_C = \frac{\sqrt{g}}{\eta} \tilde{C}_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma} = \frac{\sqrt{g}}{\eta} \left[C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{3}{2} \alpha^2 F_{\mu\nu}^2 \right]$$

Weyl geometry

Riemannian geometry

\Rightarrow Weyl's theory: **gauge theory of scale inv**, an embedding of Einstein gravity. Renormalizable? [Stelle 1979]

• Weyl quadratic gravity: Adding matter (SM Higgs)

$$\begin{aligned} \mathcal{L} &= \sqrt{g} \left\{ \frac{1}{4! \xi^2} \tilde{R}^2 - \frac{1}{4} F_{\mu\nu}^2 - \frac{1}{12} \xi_1 h^2 \tilde{R} + \frac{1}{2} (\tilde{D}_\mu h)^2 - \frac{\lambda}{4!} h^4 \right\}, & \tilde{R}^2 &\rightarrow -2\phi_0^2 \tilde{R} - \phi_0^4 \\ &= \sqrt{g} \left\{ -\frac{1}{12} \underbrace{\left[\frac{1}{\xi^2} \phi_0^2 + \xi_1 h^2 \right]}_{= 6\rho^2} \tilde{R} - \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (\tilde{D}_\mu h)^2 - \frac{1}{4!} \left[\lambda h^4 + \frac{1}{\xi^2} \phi_0^4 \right] \right\}, & \tilde{D}_\mu h &= (\partial_\mu - \alpha/2 \omega_\mu) h. \end{aligned}$$

Riemannian notation:

$$\mathcal{L} = \sqrt{g} \left\{ -\frac{1}{2} \left[\rho^2 R + 6 (\partial_\mu \rho)^2 \right] + \frac{3}{4} \rho^2 \left(\omega_\mu - \frac{1}{\alpha^2} \partial_\mu \ln \rho^2 \right)^2 - \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (\tilde{D}_\mu h)^2 - V(h, \rho) \right\}.$$

Stueckelberg: **radial** direction $\ln \rho$ eaten by ω_μ . Next (*): $\Omega = \frac{\rho^2}{\langle \hat{\rho} \rangle}, \langle \hat{\rho} \rangle = M_P, \hat{\omega}_\mu = \omega_\mu - \frac{1}{\alpha^2} \partial_\mu \ln \rho^2$

$$\Rightarrow \text{Einstein-Proca: } (\omega_\mu) : \quad \mathcal{L} = \sqrt{\hat{g}} \left\{ -\frac{1}{2} M_P^2 \hat{R} + \frac{3}{4} \alpha^2 M_P^2 \hat{\omega}_\mu \hat{\omega}^\mu - \frac{1}{4} \hat{F}_{\mu\nu}^2 + \frac{1}{2} (\hat{D}_\mu \hat{h})^2 - V \right\},$$

Unitarity gauge: $\hat{h} \rightarrow M_p \sqrt{6} \sinh \frac{\sigma}{M_p \sqrt{6}}, \quad \hat{\omega}_\mu \rightarrow \hat{\omega}_\mu + \partial_\mu \ln \cosh^2 \frac{\sigma}{M_p \sqrt{6}}$

$$\Rightarrow \mathcal{L} = \sqrt{\hat{g}} \left\{ -\frac{1}{2} M_p^2 \hat{R} + \frac{3}{4} \alpha^2 M_p^2 \hat{\omega}_\mu \hat{\omega}^\mu \cosh^2 \frac{\sigma}{M_p \sqrt{6}} - \frac{1}{4} \hat{F}_{\mu\nu}^2 + \frac{1}{2} (\partial_\mu \sigma)^2 - V(\sigma) \right\},$$

where $V(\sigma) = V_0 \left\{ \xi^2 \left[1 - \xi_1 \sinh^2 \frac{\sigma}{M_p \sqrt{6}} \right]^2 + \lambda \sinh^4 \frac{\sigma}{M_p \sqrt{6}} \right\},$

$$\Rightarrow \text{Higgs coupling to } \omega_\mu: \quad \mathcal{L} \sim \sqrt{\hat{g}} (1/8) \alpha^2 \sigma^2 \hat{\omega}_\mu \hat{\omega}^\mu + \dots$$

- Higgs from Weyl vector fusion: $\omega_\mu \omega_\mu \rightarrow \sigma \sigma \Rightarrow$ Higgs has a (non-metric) geometric origin!.

- Matter (higgs) creation from geometry (ω_μ is “geometric”). Irreversible processes [I. Prigogine et al, 1986]

• Palatini quadratic gravity

[D.G. arxiv:2003.08516; 2007.14733]

- Palatini approach to gravity due to Einstein (1925): $\tilde{\Gamma}$ unknown, fixed by eqs of motion (action).

- $\tilde{\Gamma}$ independent of $g_{\mu\nu} \Rightarrow$ invariant of (*); define $\omega_\mu = (1/2)(\tilde{\Gamma}_\mu - \Gamma_\mu)$. $\tilde{R} = R(\tilde{\Gamma}, g)$.

- same $V(\sigma)$ but $\xi_1 \rightarrow 4\xi_1, \lambda \rightarrow 16\lambda$, (different non-metricity). More operators.

• SM Fermions in WG:

$$L_\psi = \frac{i}{2} \sqrt{g} \bar{\psi} \gamma^a e_a^\mu \underbrace{\left[\partial_\mu + \frac{1}{2} s_\mu^{ab} \sigma_{ab} \right]}_{\nabla_\mu} \psi + \text{h.c.} \quad \text{spin connection (Riemann): } s_\mu^{ab} = -e^{\lambda b} (\partial_\mu e_\lambda^a - \Gamma_{\mu\lambda}^\nu e_\nu^a).$$

- Weyl spin connection: $\tilde{s}_\mu^{ab} = s_\mu^{ab} \Big|_{\partial_\lambda \rightarrow \partial_\lambda + (\text{charge}) \alpha \omega_\lambda}$, $s_\mu^{ab} \rightarrow \tilde{s}_\mu^{ab} = s_\mu^{ab} + (1/2) \alpha (e_\mu^a e^{\nu b} - e_\mu^b e^{\nu a}) \omega_\nu$.

$\Rightarrow \tilde{s}_\mu^{ab}$ is Weyl gauge invariant (like $\tilde{\Gamma}$). Lagrangian: replace $\partial_\lambda \psi \rightarrow \partial_\lambda + d_\psi \alpha \omega_\lambda$. Then ω_μ cancels out:

$$L_\psi = \frac{i}{2} \sqrt{g} \bar{\psi} \gamma^a e_a^\mu \left[\partial_\mu + d_\psi \alpha \omega_\mu + \frac{1}{2} \tilde{s}_\mu^{ab} \sigma_{ab} \right] \psi = \frac{i}{2} \sqrt{g} \bar{\psi} \gamma^a e_a^\mu \left[\partial_\mu + \frac{1}{2} s_\mu^{ab} \sigma_{ab} \right] \psi + \text{h.c.}$$

In SM: $\mathcal{L}_\psi = \frac{i}{2} \sqrt{g} \bar{\psi} \gamma^a e_a^\mu \left[\partial_\mu - ig \vec{T} \vec{A}_\mu - i Y g' \hat{B}_\mu + \frac{1}{2} s_\mu^{ab} \sigma_{ab} \right] \psi + \text{h.c.}$

$\Rightarrow \mathcal{L}_\psi$ as in SM in Riemannian geometry, **no coupling to ω_μ !** Yukawa interactions: invariant [Kugo 1977]

- Note: if $U(1)_Y \times D(1)$ kinetic mixing: $\hat{B}_\mu = B'_\mu - \omega'_\mu \tan \tilde{\chi}$; coupling $\propto Y$. ω_μ **anomaly free & massive!**

- SM Gauge bosons:

$$\mathcal{L}_b = -\frac{1}{4}\sqrt{g} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma},$$

with $F_{\mu\nu} = \tilde{\nabla}_\mu A_\nu - \tilde{\nabla}_\nu A_\mu \dots = \partial_\mu A_\nu - \partial_\nu A_\mu \dots$, symmetric $\tilde{\Gamma}$. \mathcal{L}_b invariant under (*) for $\hat{A}_\mu = A_\mu$.
 \Rightarrow Action similar to (pseudo)Riemannian case.

\Rightarrow SM in Weyl geometry: minimal embedding, no new dof's beyond SM & WG. Higgs coupling to ω_μ .

- $m_\omega \sim \alpha M_p$ can be light for $\alpha \ll 1$. TeV = current bound on non-metricity [Latorre, Y. Lobo]

- Quantum corrections: $\delta m_\sigma^2 \propto m_\omega^2$ ('new physics' scale). Light m_ω : solution to mass hierarchy?.

[D.G. 2203.05381, 2104.15118]

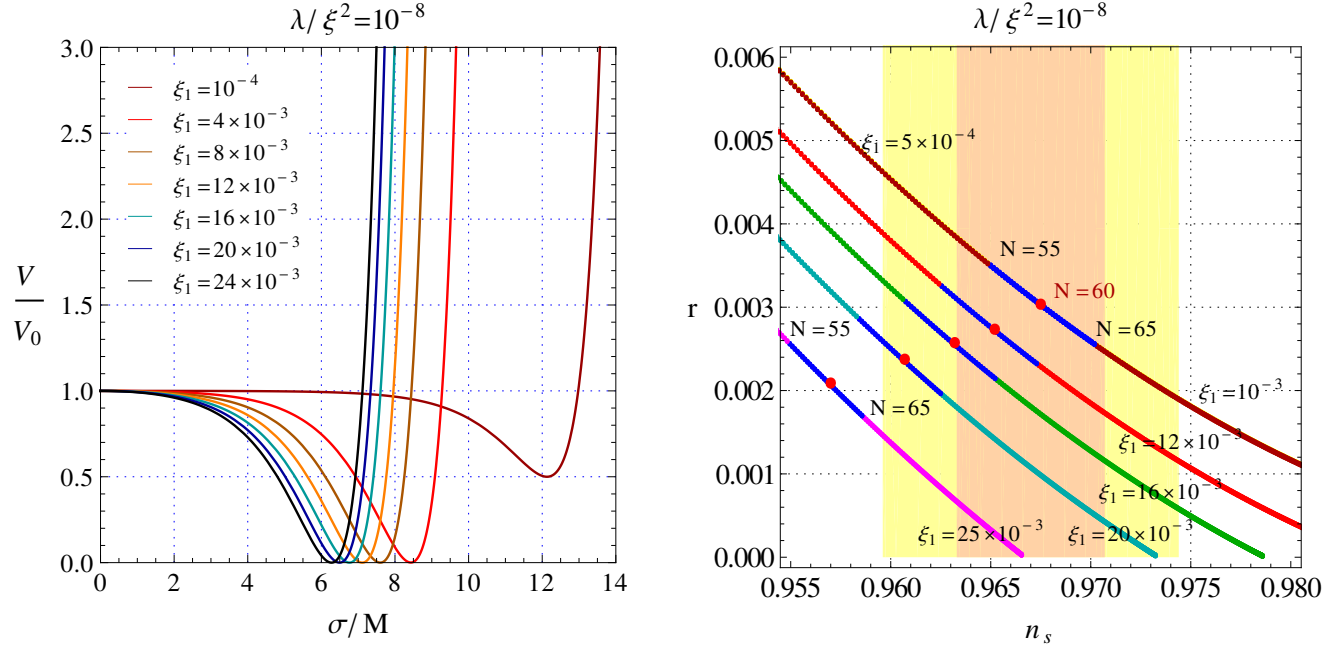
- Non-metricity (solid state physics): d=0 defects: metric anomalies/point defects: missing/extra atoms

- destroy crystalline structure, modify local notion of length, described by non-metric (Weyl) geometry.

[A. Roychowdhury, A. Gupta, 1601.06905]

• Weyl R^2 -inflation

$$V = V_0 \left\{ \left[1 - \xi_1 \sinh^2 \frac{\sigma}{2 M_p \sqrt{6}} \right]^2 + (\lambda/\xi^2) \sinh^4 \frac{\sigma}{2 M_p \sqrt{6}} \right\}$$



$$\lambda/\xi^2 \ll \xi_1^2 \ll 1: \quad \epsilon = \frac{M_p^2 V'^2}{2 V^2} = \frac{\xi_1^2}{3} \sinh^2 \frac{2\sigma}{M_p \sqrt{6}} + \mathcal{O}(\xi_1^3); \quad \eta = M_p^2 \frac{V''}{V} = -\frac{2\xi_1}{3} \cosh \frac{2\sigma}{M_p \sqrt{6}} + \mathcal{O}(\xi_1^2)$$

$$\Rightarrow \quad r = 3(1 - n_s)^2 - (16/3) \xi_1^2 + \mathcal{O}(\xi_1^3)$$

$0.002567 \leq r \leq 0.00303$ if $n_s = 0.9670 \pm 0.0037$; ($N = 60$) upper limit on r : Starobinsky: ($n_s \approx 0.968$)

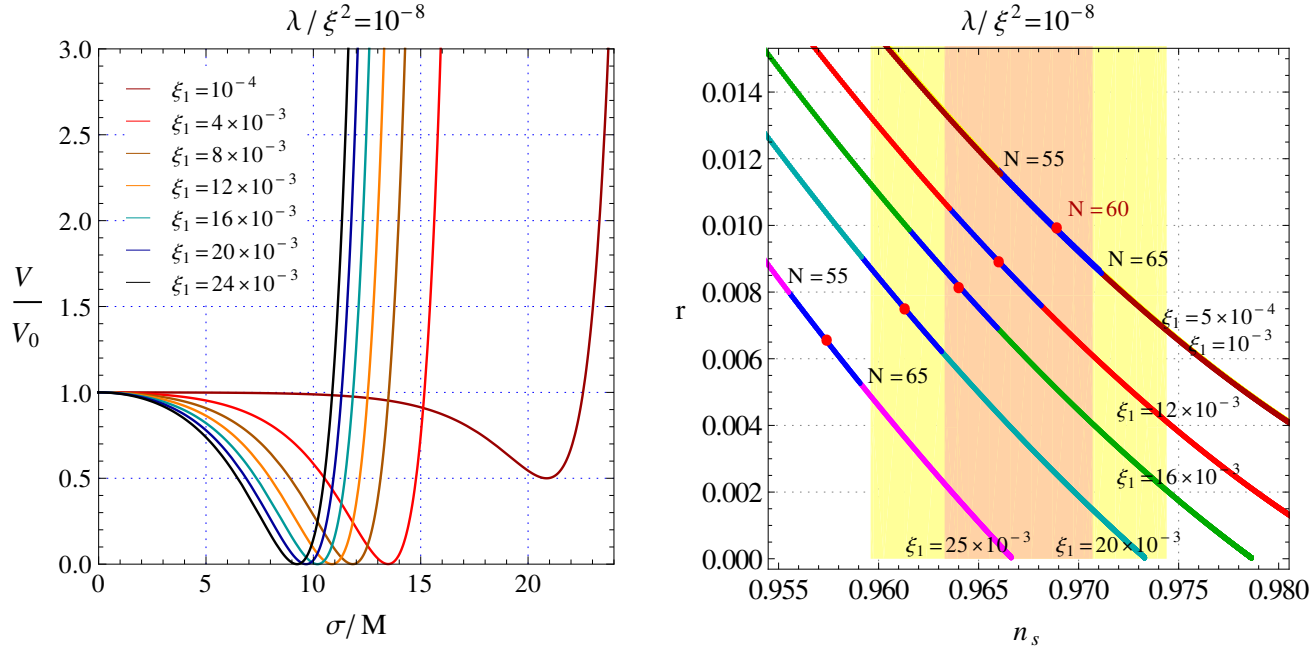
- Starobinsky: $R^2 + M_p R$. Weyl: $\tilde{R}^2 + h^2 \tilde{R} \Rightarrow$ similarity of $r(n_s)$.

[D.G. arxiv:2007.14733, 1906.11572; G. Ross, C. Hill, P. Ferreira, J. Noller 1906.03415]

FRW: $\omega_\mu(t) = (\omega_0(t), 0, 0, 0)$.

• Palatini R^2 -Inflation ($\theta = 4$)

$$V = V_0 \left\{ \left[1 - \theta \xi_1 \sinh^2 \frac{\sigma}{2 M_p \sqrt{6 \theta}} \right]^2 + (\lambda / \xi^2) \theta^2 \sinh^4 \frac{\sigma}{2 M_p \sqrt{6 \theta}} \right\}$$



$$\lambda/\xi^2 \ll \xi_1^2 \ll 1 : \quad \epsilon = \frac{M_p^2 V'^2}{2 V^2} = \frac{\xi_1^2}{3} \theta \sinh^2 \frac{2\sigma}{M_p \sqrt{6\theta}} + \mathcal{O}(\xi_1^3); \quad \eta = M_p^2 \frac{V''}{V} = -\frac{2\xi_1}{3} \cosh \frac{2\sigma}{M_p \sqrt{6\theta}} + \mathcal{O}(\xi_1^2)$$

$$\Rightarrow \quad r = 3\theta (1 - n_s)^2 + \mathcal{O}(\xi_1^2)$$

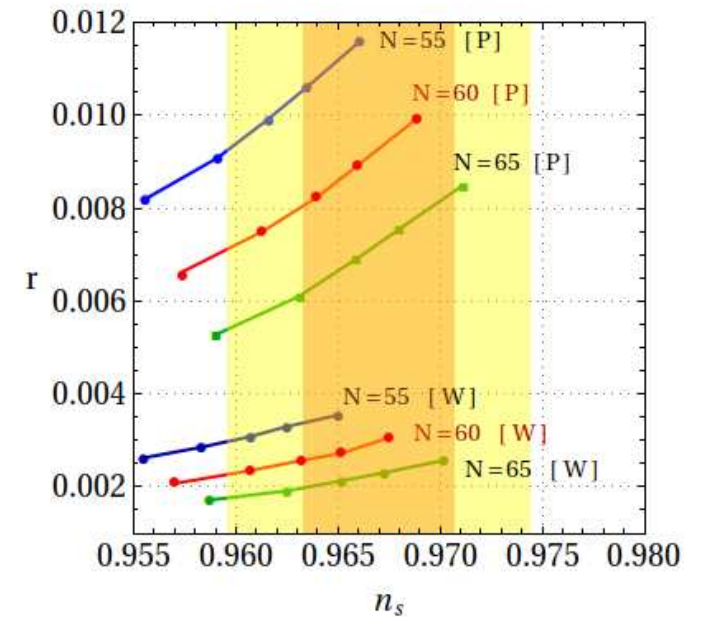
$$0.00794 \leq r \leq 0.01002 \quad \text{if } n_s = 0.9670 \pm 0.0037; (N = 60)$$

[D.G. arxiv:2007.14733, 2003.08516]

- Weyl versus Palatini: R^2 -inflation predictions

[D.G. arXiv:2007.14733]

- tensor-to-scalar ratio r versus spectral index n_s with orange (yellow) values of n_s at 68% (95%) CL.
 - the difference (θ) due to different **non-metricity** of these theories.
 - such values of r reachable by future CMB experiments (0.0005 precision; LiteBIRD, CMB-S4).
- ⇒ One will be able test and discriminate Weyl vs Palatini model



- **Conclusions:**

- Weyl quadratic gravity: **broken via Stueckelberg** to Einstein-Proca action for ω_μ .
- All mass scales: Planck mass, m_ω , Λ have **(non-metric) geometric origin** (no matter).
- **Einstein action** recovered below m_ω . Einstein's critique avoided (ω massive). **$m_\omega > \text{TeV}$** .
- SM in Weyl geometry: both geometry (connection) **and** action have this symmetry.
- Higgs couples to ω : **geometric origin**: $\omega_\mu\omega_\mu \rightarrow hh$.
- Hierarchy problem: Weyl gauge symmetry can be a solution for light $m_\omega(?)$
- Tests? a) Inflation: predictions close to those in Starobinsky, testable, **$0.00257 \leq r \leq 0.00303$** .
 - b) Higgs physics related to non-metricity (m_ω).
 - c) Gravitational waves? Dark matter?
- SM in Weyl's quadratic gravity: viable **gauge theory** of scale invariance, includes Einstein gravity & SM!

- After a century of Weyl geometry, a word from Weyl (1919):

“ The action [...] is [...] a linear combination of \tilde{R}^2 and $F_{\mu\nu}^2$. I believe that one can assert that this action principle implies everything that Einstein’s theory has implied up to now, but in the more far-reaching questions of cosmology and the constitution of matter, it exhibits a clear superiority. Nevertheless, I do not believe that the laws of nature that are [...] applicable in reality are resolved by it. “

H. Weyl: Ann. Phys. (Leipzig) (4) 59 (1919), 101-133

- Weyl “photon” - photon mixing?: adding $U(1)_Y$ to \mathcal{L}_0 ; [source: SM fermions action]

$$\mathcal{L}_0 \rightarrow \mathcal{L}_1 = \sqrt{g} \left\{ \frac{1}{4! \xi^2} \tilde{R}^2 - \frac{1}{4} \left[F_{\mu\nu}^2 + 2 \sin \chi F_{\mu\nu} F_y^{\mu\nu} + F_{y\mu\nu}^2 \right] \right\}.$$

Re-do calculation, diagonalize mixing by:

$$\hat{\omega}_\mu = \gamma \omega'_\mu \sec \chi, \quad \hat{B}_\mu = B'_\mu - \omega'_\mu \tan \chi,$$

Then

$$\mathcal{L}_1 = \sqrt{\hat{g}} \left\{ -\frac{1}{2} M_p^2 \hat{R} + \frac{3}{4} M_p^2 \alpha^2 \gamma^2 \sec^2 \chi \omega'_\mu \omega'^{\mu} - \frac{1}{4} (F_{\mu\nu}'^2 + F_{y\mu\nu}'^2) \right\},$$

- the photon after EWSB:

$$A_\mu = B'_\mu \cos \theta_w + A_\mu^3 \sin \theta_w = [\hat{B}_\mu + \hat{\omega}_\mu \sin \chi] \cos \theta_w + \sin \theta_w A_\mu^3.$$

- gauge kinetic mixing \rightarrow photon includes a small ‘piece’ of ω_μ , suppressed by $\sin \chi$
- Weyl was not completely wrong in trying to relate ω_μ to the photon - they can mix;
- mixing not forbidden by Coleman-Mandula: symmetry **direct product** $U(1)_Y \times D(1)$; both broken spontaneously.

- Higgs sector in Weyl geometry:

$$\tilde{D}_\mu H = [\partial_\mu - i\mathcal{A}_\mu - (1/2)\alpha\omega_\mu] H,$$

$$\mathcal{L}_H = \sqrt{g} \left\{ \frac{\tilde{R}^2}{4! \xi^2} - \frac{\xi_1}{6} |H|^2 \tilde{R} + |\tilde{D}_\mu H|^2 - \lambda |H|^4 - \frac{1}{4} \left[F_{\mu\nu}^2 + 2 \sin \chi F_{\mu\nu} F_y^{\mu\nu} + F_{y\mu\nu}^2 \right] \right\}.$$

where $\mathcal{A}_\mu = (g/2) \vec{\sigma} \cdot \vec{A}_\mu + (g'/2) B_\mu$; \vec{A}_μ is the $SU(2)_L$ boson, B_μ is the $U(1)_Y$ boson.

- Potential:

$$\begin{aligned} \hat{V}(\sigma) &= V_0 \left\{ 6\lambda \sinh^4 \frac{\sigma}{M_p \sqrt{6}} + \xi^2 \left[1 - \xi_1 \sinh^2 \frac{\sigma}{M_p \sqrt{6}} \right]^2 \right\}, \quad V_0 \equiv (3/4) M_p^4. \\ &= \frac{1}{4} \left[\lambda - \frac{1}{9} \xi_1 \xi^2 + \frac{1}{6} \xi_1^2 \xi^2 \right] \sigma^4 - \frac{1}{2} \xi_1 \xi^2 M_p^2 \sigma^2 + \frac{3}{2} \xi^2 M_p^4 + \mathcal{O}(\sigma^6/M_p^2). \end{aligned}$$

$$\text{if } \xi_1 \xi^2 \ll 1: \quad \langle \sigma \rangle^2 = (\xi_1 \xi^2 / \lambda) M_p^2, \quad m_\sigma^2 = 2 \xi_1 \xi^2 M_p^2,$$

- Hierarchy using $\xi \sqrt{\xi_1} \sim 3.5 \times 10^{-17}$, $\lambda \sim 0.12$ (SM). Hierarchy controlled by ξ of \tilde{R}^2 term!

$$m_Z^2 = \frac{1}{4} (g^2 + g'^2) \langle \sigma \rangle^2 \left\{ 1 + \frac{\langle \sigma \rangle^2}{18 M_p^2} \left[1 - \frac{3 g'^2}{\alpha^2} \sin^2 \chi \right] + \mathcal{O}(\langle \sigma \rangle^4 / M_p^4) \right\}.$$

\Rightarrow Part of Z mass due to Weyl geometry (mixing with ω_μ), beyond Einstein gravity/Riemannian geometry

- Precision constraints (Z mass):

$$\varepsilon \equiv \frac{\Delta m_Z}{m_{Z^0}} = -\frac{g'^2 \langle \sigma \rangle^2}{12 M_p^2} \frac{\sin^2 \chi}{\alpha^2} + \mathcal{O}\left(\frac{\langle \sigma \rangle^4}{M_p^4}\right) = -\frac{1}{8} \left(\frac{\langle \sigma \rangle}{m_\omega}\right)^2 (g' \tan \chi)^2 + \mathcal{O}\left(\frac{\langle \sigma \rangle^4}{m_\omega^4}\right).$$

- $\langle \sigma \rangle = 246.22$ GeV; at 68% CL, $\varepsilon = 2.3 \times 10^{-5}$, then: $\alpha \geq 2.17 \times 10^{-15} \sin \chi$.

- in terms of the mass: $\frac{m_\omega}{\text{TeV}} \geq 6.35 \times \tan \chi$.

- current bound on non-metricity scale $m_\omega \sim$ few TeV, then: $\tan \chi \leq 0.16$

\Rightarrow the constraint from Z-mass is v. strong: e.g. effect of ω_μ to Δa_μ of muon magnetic moment:

$$\Delta a_\mu \sim \frac{1}{12\pi^2} \frac{m_\mu^2}{m_\omega^2} (g' \tan \chi)^2 = 2.56 \times 10^{-13},$$

which is very small (cannot match the ongoing discrepancy).

• From Weyl to Palatini:

[D.G. arxiv:2003.08516; 2007.14733]

- Palatini approach to gravity due to Einstein: $\tilde{\Gamma}$ **unknown**, fixed by eqs of motion.
- $\tilde{\Gamma}$ independent of $g_{\mu\nu} \Rightarrow$ invariant of (*); define $\omega_\mu = (1/2)(\tilde{\Gamma}_\mu - \Gamma_\mu)$.
- Minimal Palatini action with WGS as before, but with $\tilde{R} = R(\tilde{\Gamma}, g)$, $\tilde{\Gamma}$ Palatini.

$$L_2 = \sqrt{g} \left\{ \frac{1}{4! \xi^2} \tilde{R}^2(\tilde{\Gamma}, g) - \frac{1}{4q^2} F_{\mu\nu}^2(\tilde{\Gamma}) - \frac{1}{12} \xi_1 \phi^2 \tilde{R}(\tilde{\Gamma}, g) + \frac{1}{2} (\tilde{D}_\mu \phi)^2 - \frac{\lambda}{4!} \phi^4 \right\},$$

Solve for $\tilde{\Gamma}$ (difficult!) $\Rightarrow \tilde{\nabla}_\lambda g_{\mu\nu} = (-2)(g_{\mu\nu} \omega_\lambda - g_{\mu\lambda} \omega_\nu - g_{\nu\lambda} \omega_\mu)$ non-metricity \neq Weyl geometry.

\Rightarrow **Onshell $\tilde{\Gamma}$** : Stueckelberg breaking, same steps as before, etc:

$$L_2 = \sqrt{g} \left\{ -\frac{1}{2} \left[\rho^2 R + 6 (\partial_\mu \rho)^2 \right] + \frac{3}{4} \theta \rho^2 (\omega_\mu - \partial_\mu \ln \rho^2)^2 - \frac{1}{4q^2} F_{\mu\nu}^2 + \frac{1}{2} (\tilde{D}_\mu \phi)^2 - V(\phi, \rho) \right\}.$$

\Rightarrow onshell, gauge fixing: again Einstein-Proca action, similar to Weyl theory but $\theta=4$ (Weyl: $\theta=1$).

\Rightarrow similar structure of V , θ different

\Rightarrow In Palatini quadratic gravity: additional SI operators exist [Percacci 0910.5167 Bastero-Gill, Borunda, Janssen 0804.4440]