Cranking Inertia Tensor

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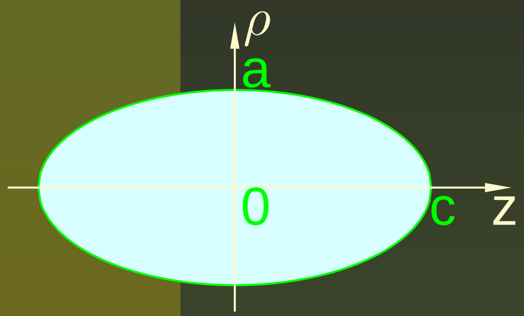
OUTLINE

- Nuclear shape parametrization
- Spheroidal harmonic oscillator
- Pairing interaction
- Deformation dependent quantities
- Nuclear inertia
- Correction term due to var. of Δ and E_F with def.
- Hydrodynamical formulae
- Results for ^{240}Pu and ^{264}Fm
- Conclusions

Nuclear shape parametrization



- Spheroid

Lengths in units of $R_0 = 1.2249 A^{1/3}$ fm. Vol. conserv. $\omega_\perp^2 \omega_z = (\omega_0^0)^3$

$\hbar\omega_0^0 = 41 A^{-1/3}$ MeV

$\hbar^2/M \approx 41.5$ MeV·fm^2

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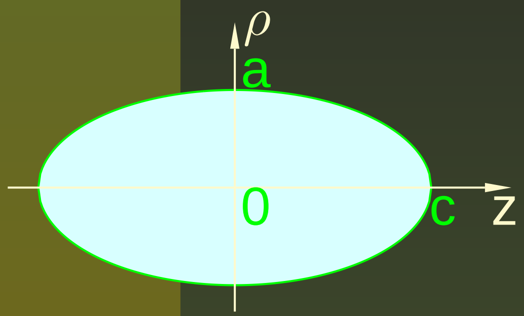

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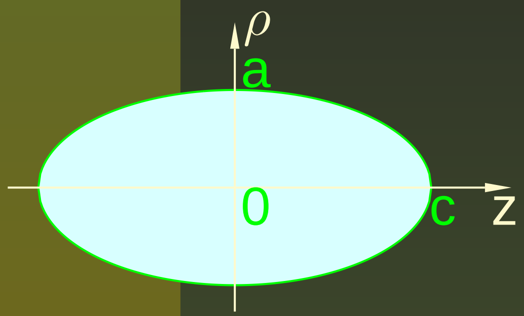

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- Spheroid
- Quadrupolar deformation: $\varepsilon = \frac{3(c-a)}{(2c+a)}$
- Harmonic oscillator frequencies

  $\omega_\perp(\varepsilon) = \omega_0 \left(1 + \frac{\varepsilon}{3}\right)$; $\omega_z(\varepsilon) = \omega_0 \left(1 - \frac{2\varepsilon}{3}\right)$

  S. G. Nilsson 1955

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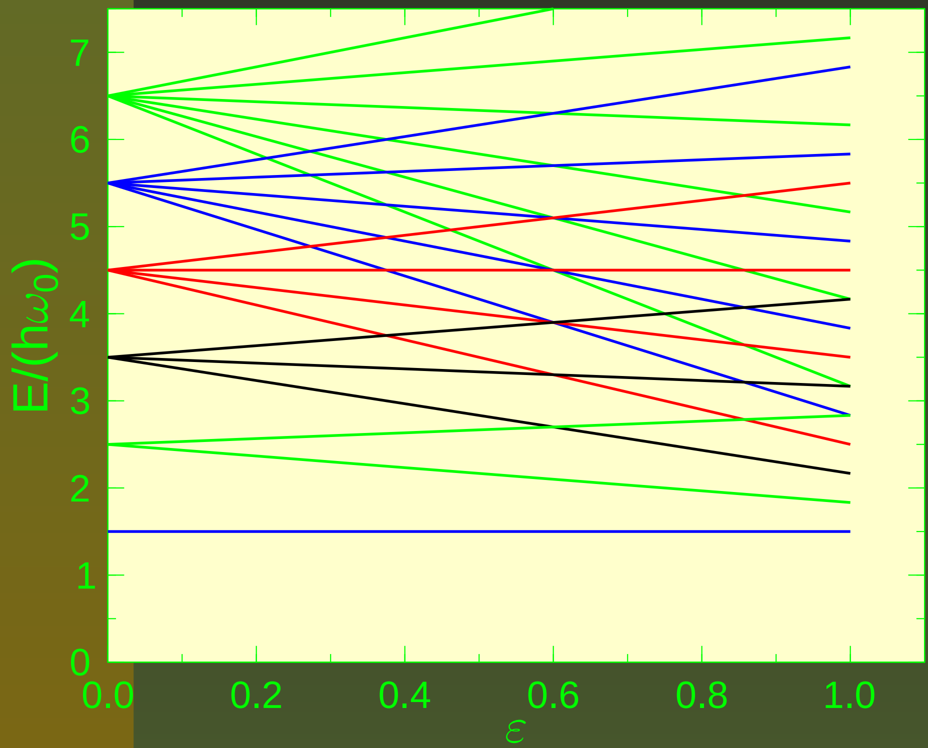
Harmonic oscillator. Energy levels



$V = M(\omega_\perp^2 \rho^2 + \omega_z^2 z^2)/2$
$N = n_\perp + n_z$ main quantum number
$n_\perp = 0, 1, 2, ..., N$
$(N+1)(N+2)$ nucleons in a closed shell (degeneracy).

$$E/(\hbar\omega_0) = N + 3/2 + \varepsilon(n_\perp - 2N/3)$$

$$\epsilon_i = E_i/\hbar\omega_0^0 = [E/(\hbar\omega_0)][1 - \varepsilon^2(1/3 + 2\varepsilon/27)]^{-1/3}$$

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Harmonic oscillator. Wave functions

$$\Psi = |n_r m n_z \Sigma\rangle = \psi_{n_r}^m(\rho)\Phi_m(\varphi)\psi_{n_z}(z)\chi(\Sigma)$$

$$n_r = 0, 1, 2, ..., n_\perp \quad n_z = N - n_\perp \quad m = n_\perp - 2n_r$$

$$\psi_{n_r}^m(\rho) = \frac{\sqrt{2}}{\alpha_\perp} N_{n_r}^m \eta^{\frac{|m|}{2}} e^{-\frac{\eta}{2}} L_{n_r}^{|m|}(\eta) = \frac{\sqrt{2}}{\alpha_\perp}\psi_{n_r}^m(\eta)$$

Laguerre and Hermite polynomials. $\Phi_m(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi}$

$$\psi_{n_z}(z) = \frac{1}{\sqrt{\alpha_z}} N_{n_z} e^{-\frac{\xi^2}{2}} H_{n_z}(\xi) = \frac{1}{\sqrt{\alpha_z}}\psi_{n_z}(\xi)$$

Dimension-less variables $\eta = \rho^2/\alpha_\perp^2 \quad \xi = z/\alpha_z$

$$\alpha_\perp = \sqrt{\hbar/(M\omega_\perp)} \quad \alpha_z = \sqrt{\hbar/(M\omega_z)}$$

Norm. ct. $(N_{n_r}^m)^2 = \frac{n_r!}{(n_r+|m|)!} \quad (N_{n_z})^2 = \frac{1}{\sqrt{\pi}2^{n_z} n_z!}$

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Pairing of protons

Doubly degenerate levels $\{\epsilon_i\}$ in units of $\hbar\omega_0^0$. $Z/2$ levels are occupied. n levels below & n' above Fermi energy contribute to pairing, $n = n' = \Omega\tilde{g}_s/2$. Cutoff energy, $\Omega \simeq 1 \gg \tilde{\Delta} = 12/\sqrt{A}\hbar\omega_0^0$. The gap Δ and Fermi energy λ are solutions of the BCS eqs:

$$0 = \sum_{k_i}^{k_f} \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}} \quad ; \quad \frac{2}{G} = \sum_{k_i}^{k_f} \frac{1}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}$$

$k_i = Z/2 - n + 1, \quad k_f = Z/2 + n', \quad \frac{2}{G} \simeq 2\tilde{g}(\tilde{\lambda}) \ln\left(\frac{2\Omega}{\tilde{\Delta}}\right)$ Occup. probab. qp. (u_k) or hole (v_k) $v_k^2 = \left[1 - (\epsilon_k - \lambda)/E_k\right]/2$; $u_k^2 = 1 - v_k^2$

Qp. en. $E_\nu = \sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}$

Nuclear inertia tensor B_{ij}

The PES in a hyperspace of def. $\beta_1, \beta_2, ..., \beta_n$ gives the forces acting on a nucleus. $\{B_{ij}\}$ shows how the system reacts. The kinetic enery is determined by the shape change

$$E_k = \frac{1}{2} \sum_{i,j=1}^{n} B_{ij}(\beta) \frac{d\beta_i}{dt} \frac{d\beta_j}{dt}$$

$$B_{ij} = 2\hbar^2 \sum_{\nu\mu} \frac{\langle \nu | \partial H / \partial \beta_i | \mu \rangle \langle \mu | \partial H / \partial \beta_j | \nu \rangle}{(E_\nu + E_\mu)^3} (u_\nu v_\mu + u_\mu v_\nu)^2 + P_{ij}$$

summation over states ν, μ considered in pairing.

Nuclear inertia scalar B_ε

For one deformation ε the inertia tensor becomes a scalar

$$B_\varepsilon = 2\hbar^2 \sum_{\nu\mu} \frac{\langle \nu | \partial V / \partial \varepsilon | \mu \rangle \langle \mu | \partial V / \partial \varepsilon | \nu \rangle}{(E_\nu + E_\mu)^3} (u_\nu v_\mu + u_\mu v_\nu)^2 + P_\varepsilon$$

summation over states ν, μ considered in pairing.

Potential function of deformation

Assume: $\partial H/\partial \varepsilon = \partial V/\partial \varepsilon$ and no spin-orbit coupling

$2V(\eta, \xi; \varepsilon) = \hbar\omega_\perp \eta + \hbar\omega_z \xi^2$

$$\frac{1}{\hbar\omega_0^0}\frac{dV}{d\varepsilon} = \frac{3}{2}\left[f_1(\varepsilon)\eta + f_2(\varepsilon)\xi^2\right]$$

where

$f_1 = [\varepsilon(\varepsilon+6)+9]/[27-\varepsilon^2(9+2\varepsilon)]^{4/3}$

$f_2 = 2[\varepsilon(2\varepsilon+3)-9]/[27-\varepsilon^2(9+2\varepsilon)]^{4/3}$

Effective mass vs deformation

$$\frac{\hbar\omega_0^0}{\hbar^2}B_\varepsilon = \frac{9}{2}\sum_{\nu\mu}\frac{\langle\nu|f_1\eta + f_2\xi^2|\mu\rangle\langle\mu|f_1\eta + f_2\xi^2|\nu\rangle}{(E_\nu + E_\mu)^3}(u_\nu v_\mu + u_\mu v_\nu)^2$$

Matrix elements are calculated by performing some integrations

$$\langle n_z' n_r' m'|f_1(\varepsilon)\eta + f_2(\varepsilon)\xi^2|n_z n_r m\rangle = \delta_{m'm}N_{n_r'}^m N_{n_r}^m N_{n_z'} N_{n_z} \cdot [$$
$$f_1\int_0^\infty d\eta\eta^{|m|+1}e^{-\eta}L_{n_r'}^{|m|}(\eta)L_{n_r}^{|m|}(\eta)\int_{-\infty}^\infty d\xi e^{-\xi^2}H_{n_z'}(\xi)H_{n_z}(\xi) +$$
$$f_2\int_0^\infty d\eta\eta^{|m|}e^{-\eta}L_{n_r'}^{|m|}(\eta)L_{n_r}^{|m|}(\eta)\int_{-\infty}^\infty d\xi\xi^2 e^{-\xi^2}H_{n_z'}(\xi)H_{n_z}(\xi)]$$

Analytical relationships

One diagonal contribution and two nondiagonal terms

$$\frac{9}{4}\delta_{n'_r n_r}\delta_{m'm}\sum_{\nu=k_i}^{k_f}\left[f_1(2n_r+|m|+1)+f_2(n_z+1/2)\right]^2\frac{(u_\nu v_\nu)^2}{E_\nu^3}\delta_{n'_z n_z}$$

$$\frac{\hbar\omega_0^0}{\hbar^2}B_{\varepsilon 2}=\frac{9}{4}\delta_{n'_r n_r}\delta_{m'm}\sum_{\nu\neq\mu}\frac{f_2^2}{2}(n_z+1)(n_z+2)\frac{(u_\nu v_\mu+u_\mu v_\nu)^2}{(E_\nu+E_\mu)^3}\delta_{n'_z n_z+2}$$

$$\frac{\hbar\omega_0^0}{\hbar^2}B_{\varepsilon 3}=\frac{9}{4}\delta_{n'_r n_r}\delta_{m'm}\sum_{\nu\neq\mu}\frac{f_2^2}{2}(n_z-1)n_z\frac{(u_\nu v_\mu+u_\mu v_\nu)^2}{(E_\nu+E_\mu)^3}\delta_{n'_z n_z-2}$$

Correction term in general

$$
\begin{aligned}
P_{ij} = \frac{\hbar^2}{4}\sum_{\nu}\frac{1}{E_\nu^5}\Bigg[& \Delta^2\frac{\partial\lambda}{\partial\beta_i}\frac{\partial\lambda}{\partial\beta_j} + \\
& (\epsilon_\nu-\lambda)^2\frac{\partial\Delta}{\partial\beta_i}\frac{\partial\Delta}{\partial\beta_j} + \Delta(\epsilon_\nu-\lambda)\left(\frac{\partial\lambda}{\partial\beta_i}\frac{\partial\Delta}{\partial\beta_j}+\frac{\partial\lambda}{\partial\beta_j}\frac{\partial\Delta}{\partial\beta_i}\right) \\
& -\Delta^2\left(\frac{\partial\lambda}{\partial\beta_i}\langle\nu|\partial H/\partial\beta_j|\nu\rangle + \frac{\partial\lambda}{\partial\beta_j}\langle\nu|\partial H/\partial\beta_i|\nu\rangle\right) \\
& -\Delta(\epsilon_\nu-\lambda)\left(\frac{\partial\Delta}{\partial\beta_i}\langle\nu|\partial H/\partial\beta_j|\nu\rangle + \frac{\partial\Delta}{\partial\beta_j}\langle\nu|\partial H/\partial\beta_i|\nu\rangle\right)\Bigg]
\end{aligned}
$$

Attn.: In “Funny Hills (M. Brack et al. *Rev. Mod. Phys.* **44** (1972) 320),” at the page 388, eq. (IX.41b), there is a misprint which was not observed by some authors. In the 2nd term one should write $(\epsilon_\nu - \lambda)^2$ instead of $(\epsilon_\nu - \lambda)$.

Correction term. Var. Δ, E_F with ε

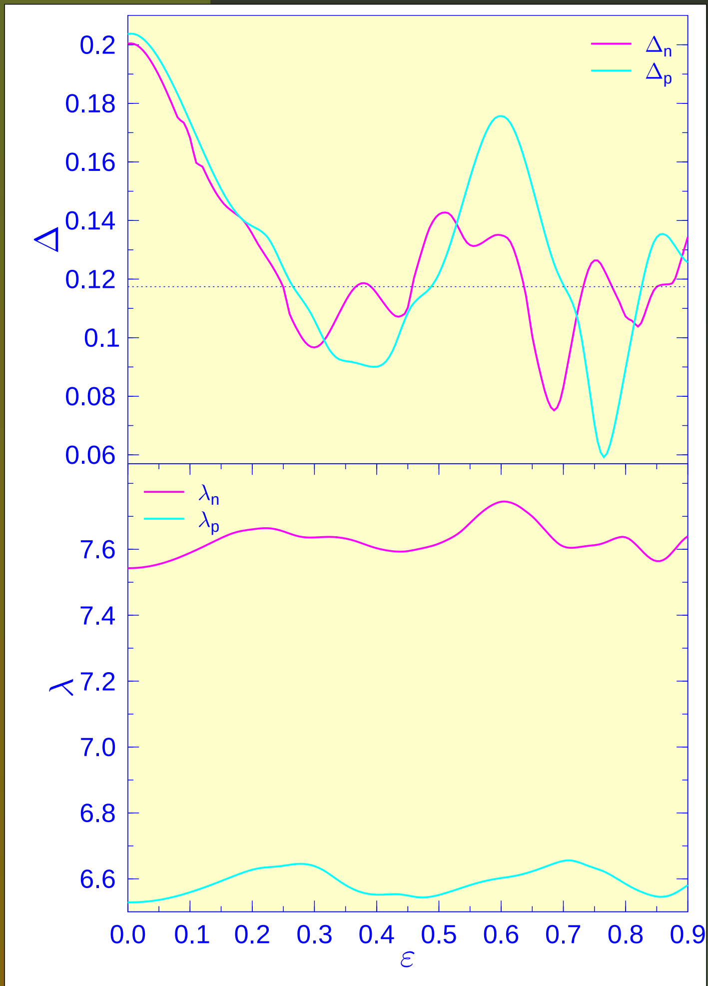
$$P_\varepsilon = \frac{2\hbar^2}{8} \sum_\nu \frac{1}{E_\nu^5} \left[\left(\Delta \frac{d\lambda}{d\varepsilon} \right)^2 + \right.$$

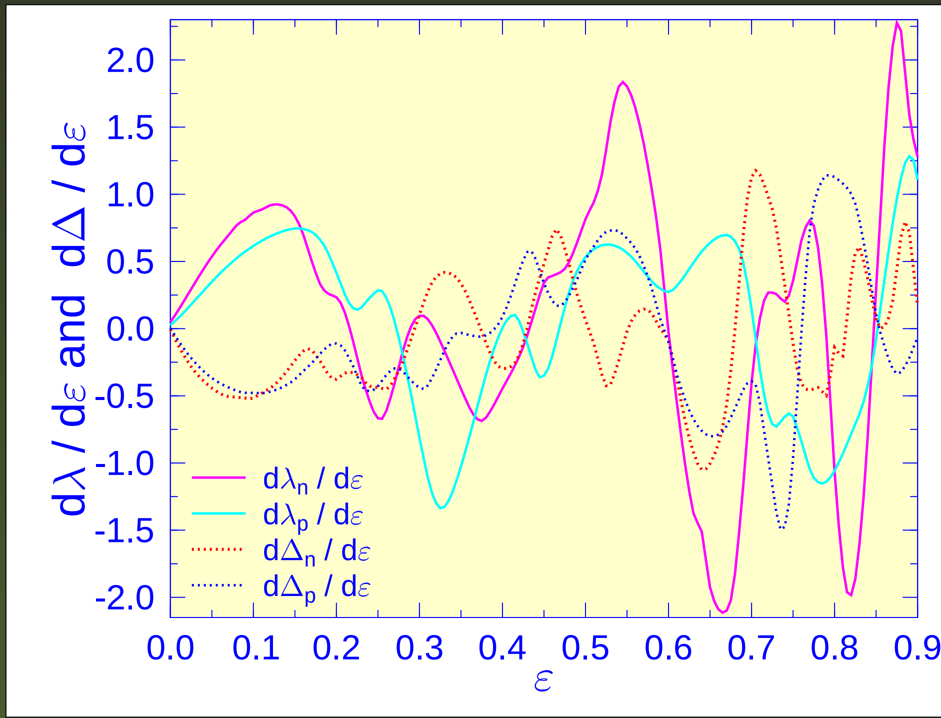
$$(\epsilon_\nu - \lambda)^2 \left(\frac{d\Delta}{d\varepsilon} \right)^2 + 2\Delta(\epsilon_\nu - \lambda) \frac{d\lambda}{d\varepsilon} \frac{d\Delta}{d\varepsilon}$$

$$-2\Delta^2 \frac{d\lambda}{d\varepsilon} \langle \nu | dV/d\varepsilon | \nu \rangle$$

$$\left. -2\Delta(\epsilon_\nu - \lambda) \frac{d\Delta}{d\varepsilon} \langle \nu | dV/d\varepsilon | \nu \rangle \right]$$

Variation of Δ and λ with ε





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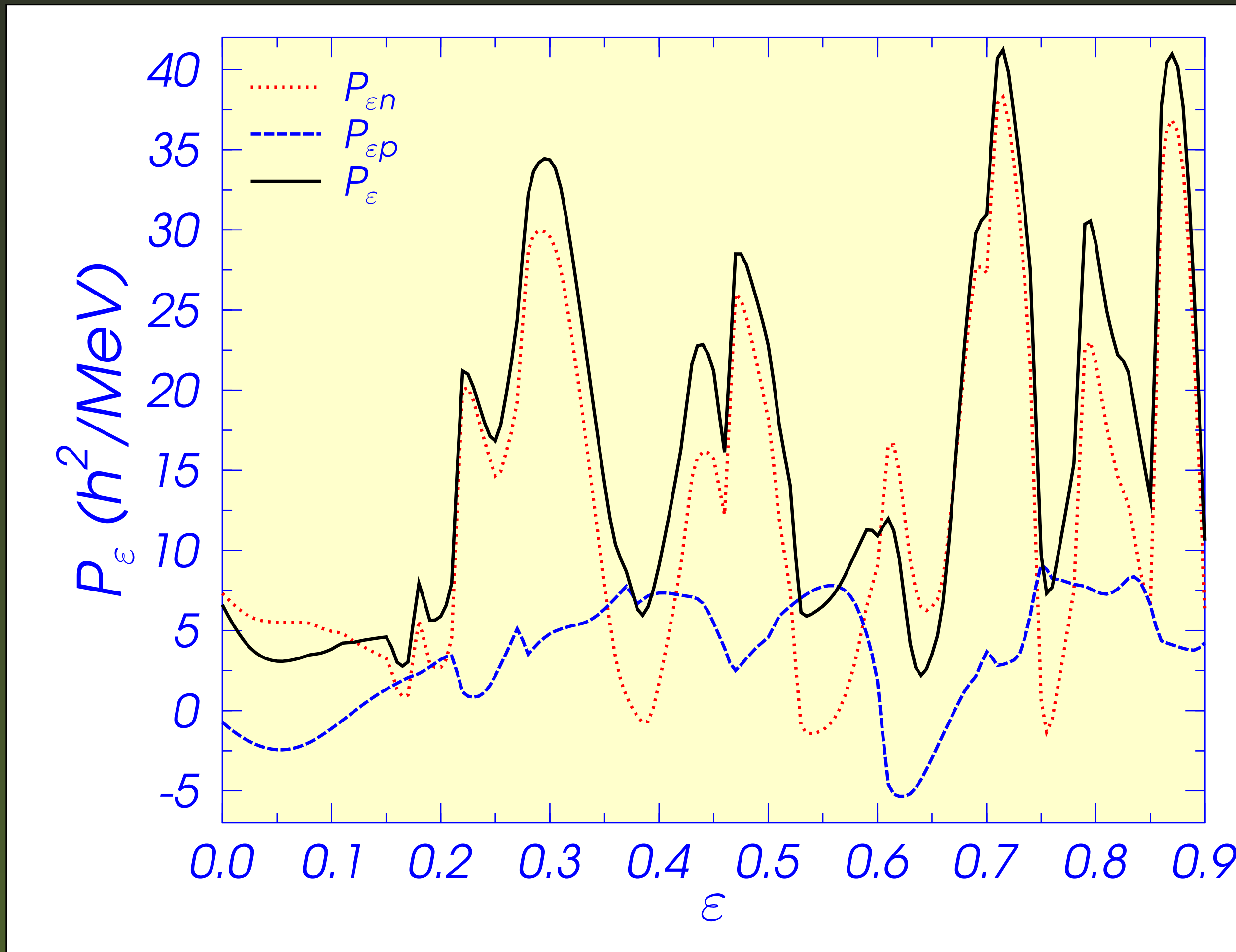
Correction term versus deformation



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Hydrodynamical formulae

For a spherical liquid drop

$$B_{irr}(0) = \frac{2}{15} M A R_0^2 = 0.0048205 A^{5/3} \ \frac{\hbar^2}{\mathrm{MeV}}$$

For a spheroid

$$B_{\varepsilon}^{ir}(\varepsilon) = B_{irr}(0) \frac{81}{[27 - \varepsilon^2(9 + 2\varepsilon)]^{4/3}} \frac{9 + 2\varepsilon^2}{(3 - 2\varepsilon)^2}$$

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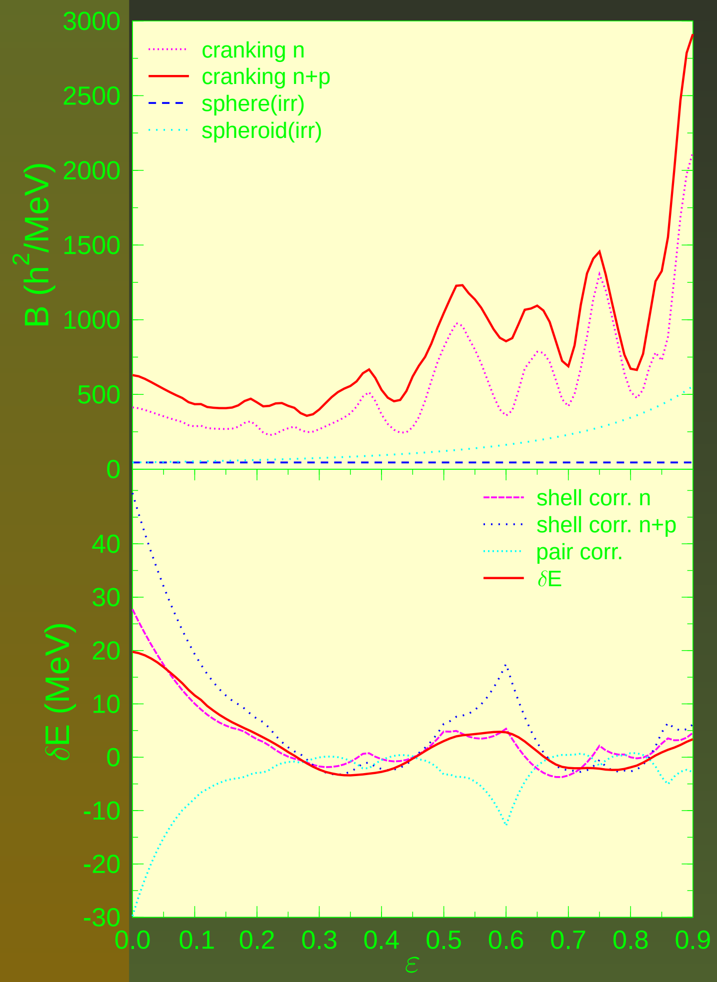
Results for ^{240}Pu and ^{264}Fm



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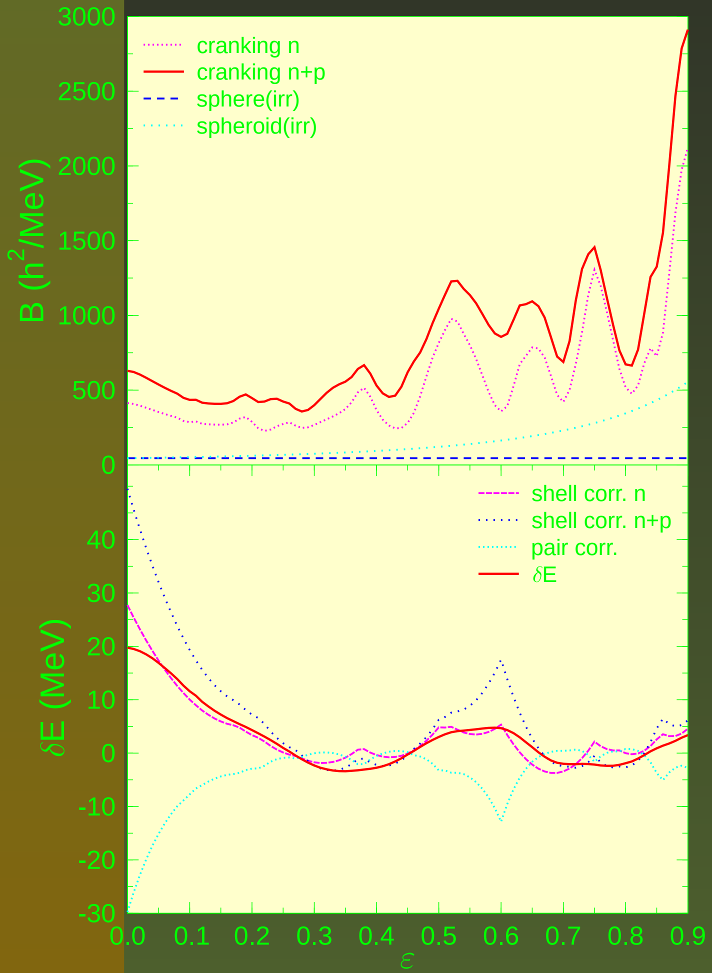

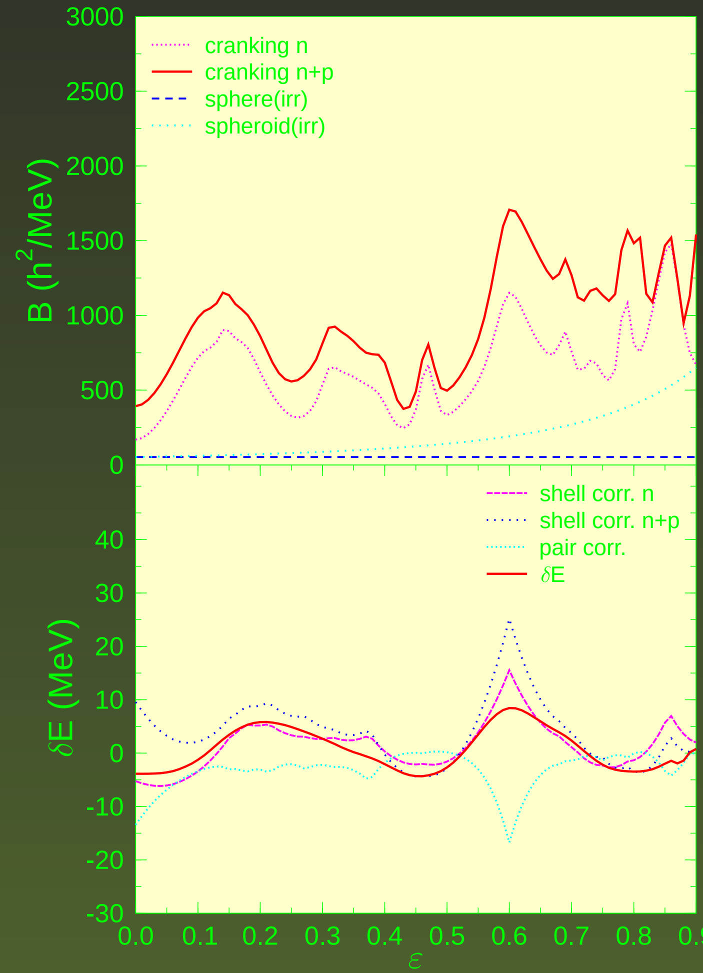

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- The correction term P_{ij} shoud not be ignored.