SADDLE POINT SHAPES OF NUCLEI

Dorin Poenaru

OUTLINE

- Nuclear shapes and mass asymmetry
- Equilibrium and scission configurations
- Examples of shapes (given parametrization)
- David L. Hill's method
- Integro-differential equation (no param.)
- Phenomenological shell effects
- Fission into 2, 3, and 4 fragments
- Experimental attempts
- Conclusions

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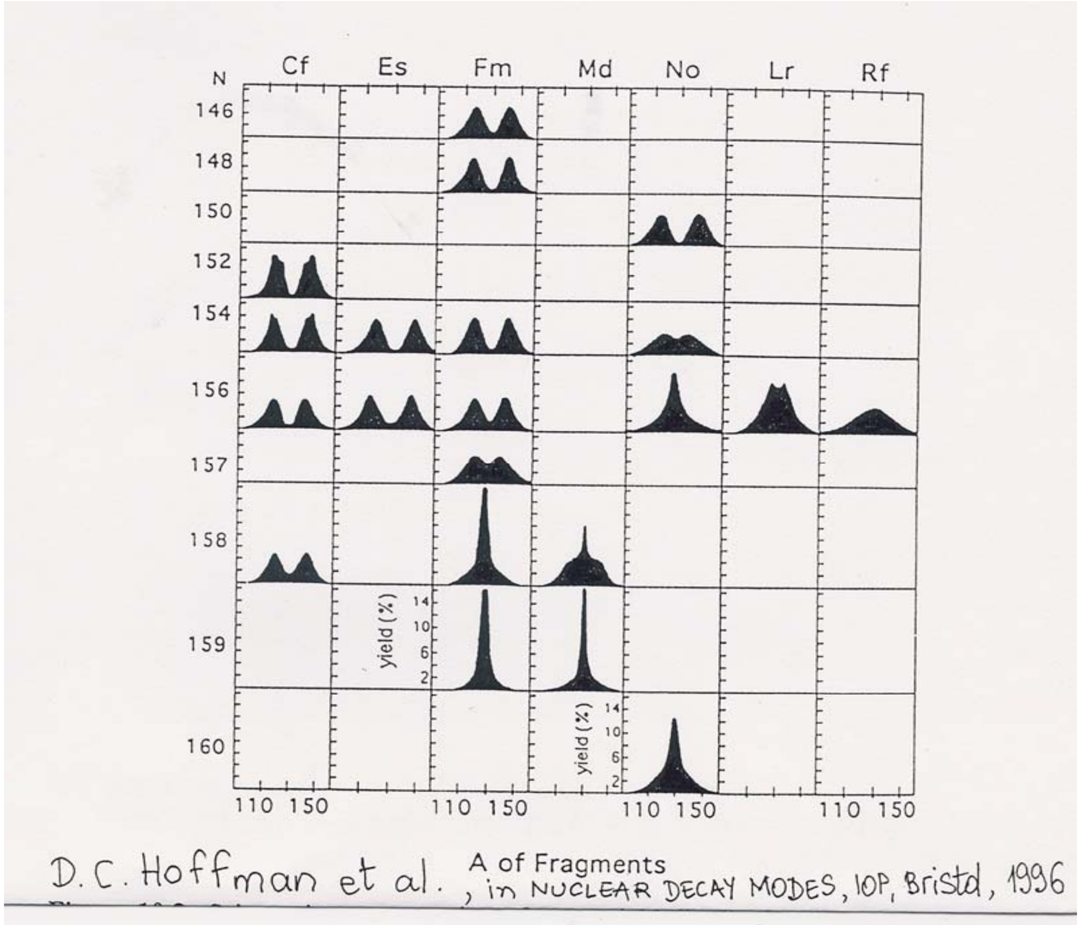
HYSTORICAL MILE-STONES

- 1929 Fine structure in α–decay
- 1939 Induced fission
- 1940 Spontaneous fission
- 1946 Ternary (particle-accomp.) fission
- 1980 Prediction of cluster Radioactivity
- 1981 Cold binary fission
- 1984 cR experimentally confirmed
- 1998 Cold α and Be accompanied fission

FISSION FRAGMENT MASS ASYMMETRY

Symmetry for some Fm, Md, No isotopes when the most probable fragments are very close to the doubly magic ^{132}Sn



EQUILIBRIUM CONFIGURATIONS

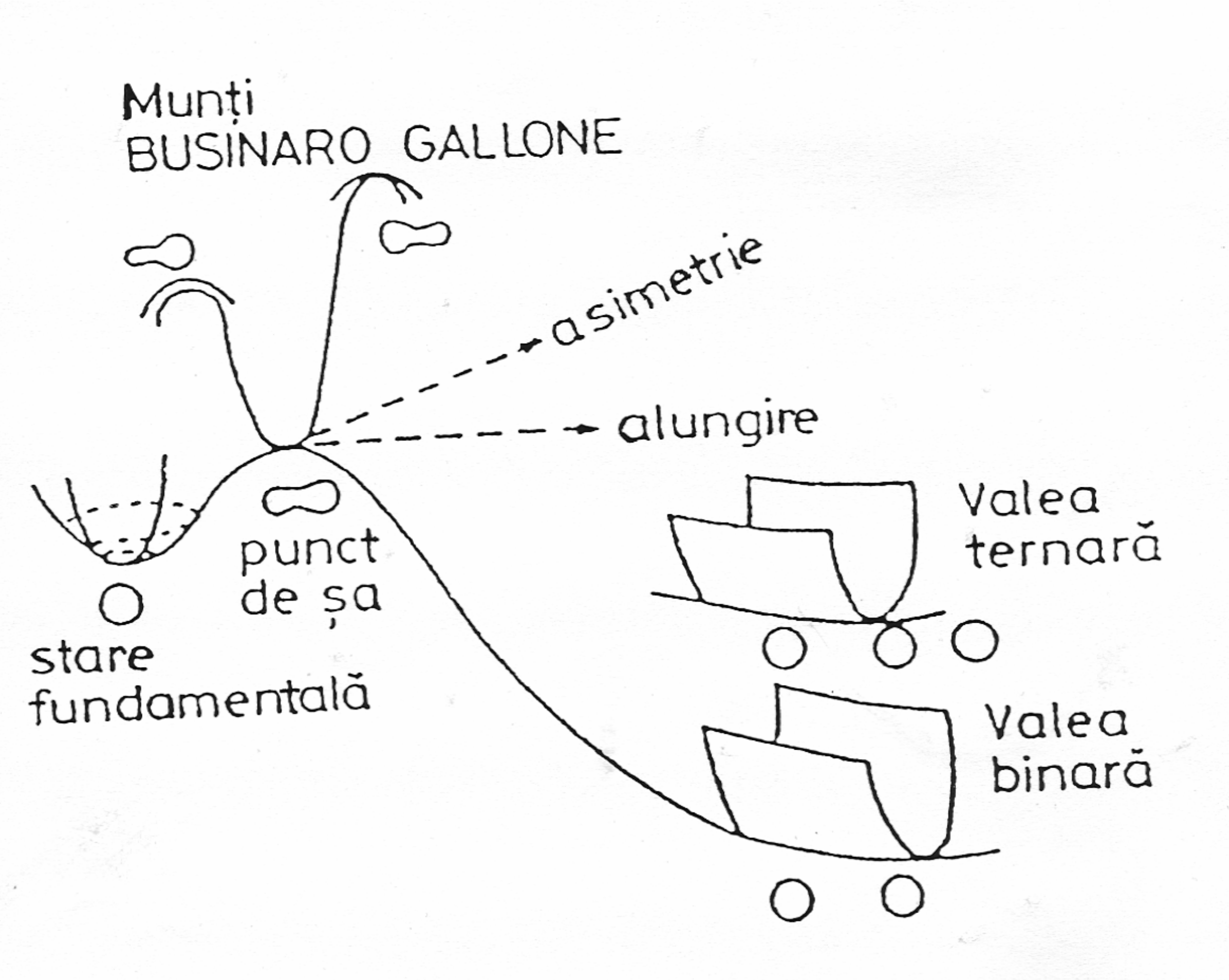
Nuclear gs deformation and fission fragment mass asymmetry are not explained within LDM. One should add shell corrections to calculate Potential Energy Surfaces, PES eg $E = E(q_1, q_2)$

Equilibrium: $$\frac{\partial E}{\partial q_1} = \frac{\partial E}{\partial q_2} = 0$$

- **Minima (ground state, shape isomer)**
- **Sadde-points: maxima on fission valley (min.)** $$\frac{\partial E}{\partial q_1} = \frac{\partial E}{\partial q_2} = 0$$

$$\begin{vmatrix} \frac{\partial^2 E}{\partial q_1^2} & \frac{\partial^2 E}{\partial q_1 \partial q_2} \\ \frac{\partial^2 E}{\partial q_2 \partial q_1} & \frac{\partial^2 E}{\partial q_2^2} \end{vmatrix} < 0$$

G STATE, VALLEYS, SADDLE POINTS



POTENTIAL ENERGY SURFACE of ^{264}Fm

Symmetrical mass distributions

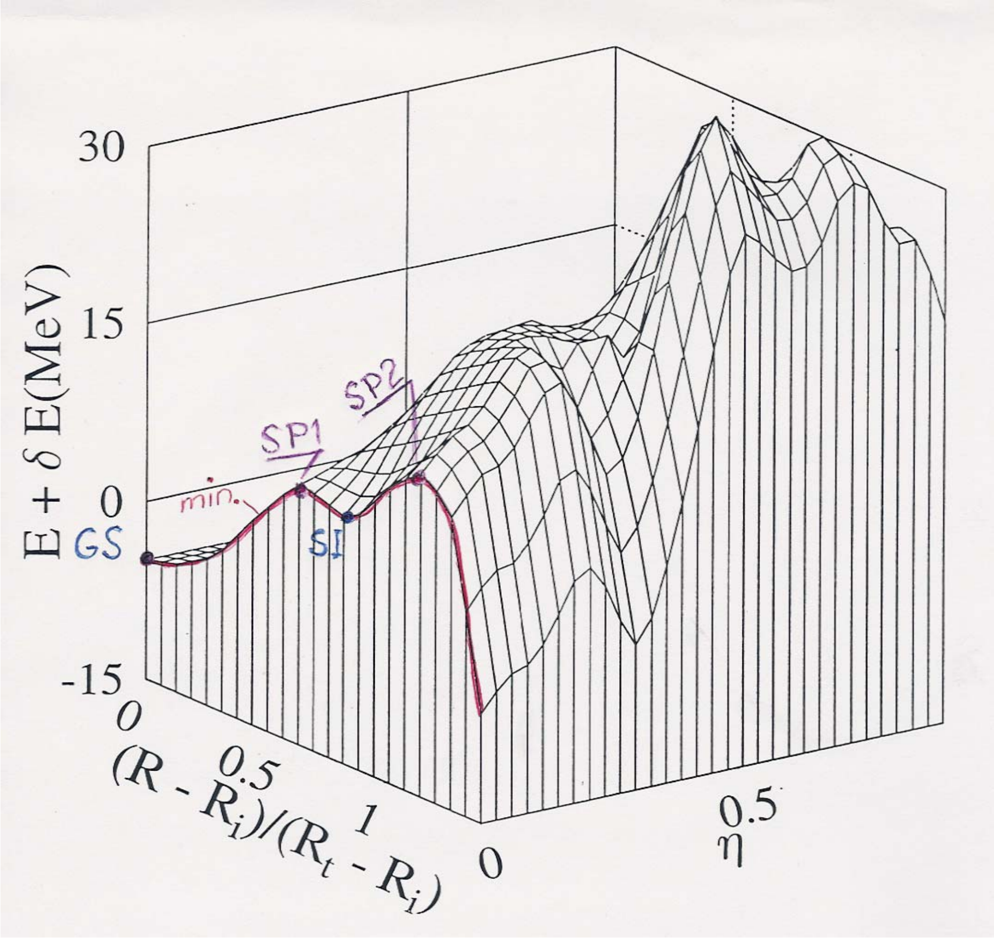
GS=ground state

SI=shape isomer

SP1=saddle point

SP2=saddle point

— Fission valley at $\eta = 0$



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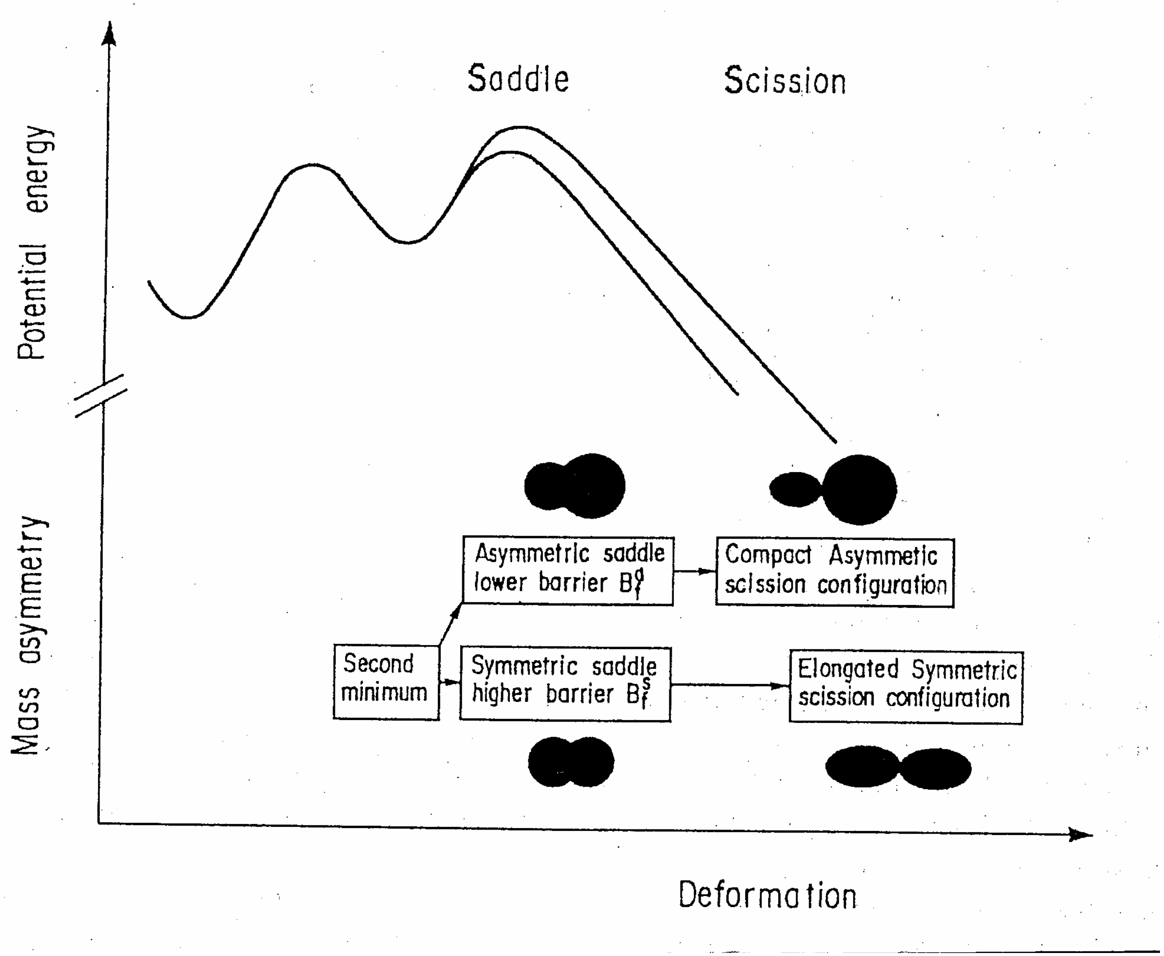
TWO PATHS FROM SADDLE TO SCISSION

Y. Nagame *et al.*

J. Radioanalyt. and Nucl. Chemistry **239** (1999) 97.



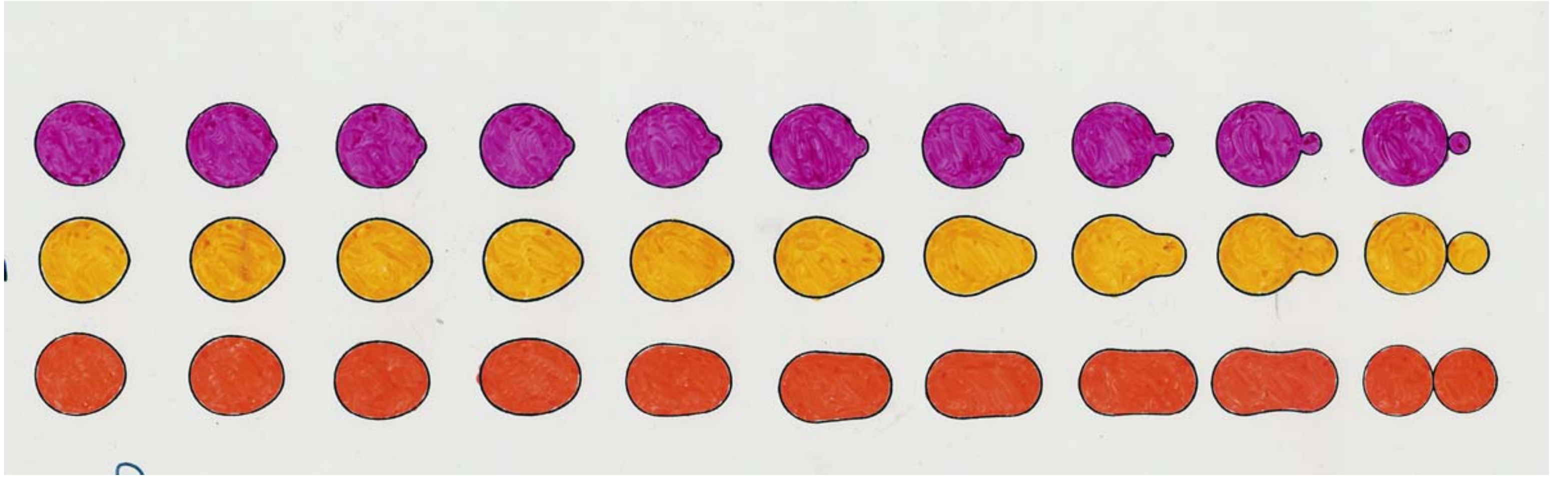
THREE DECAY MODES OF ^{234}U
SHAPES ALONG FISS. PATH

LIGHT FRAGM.

α

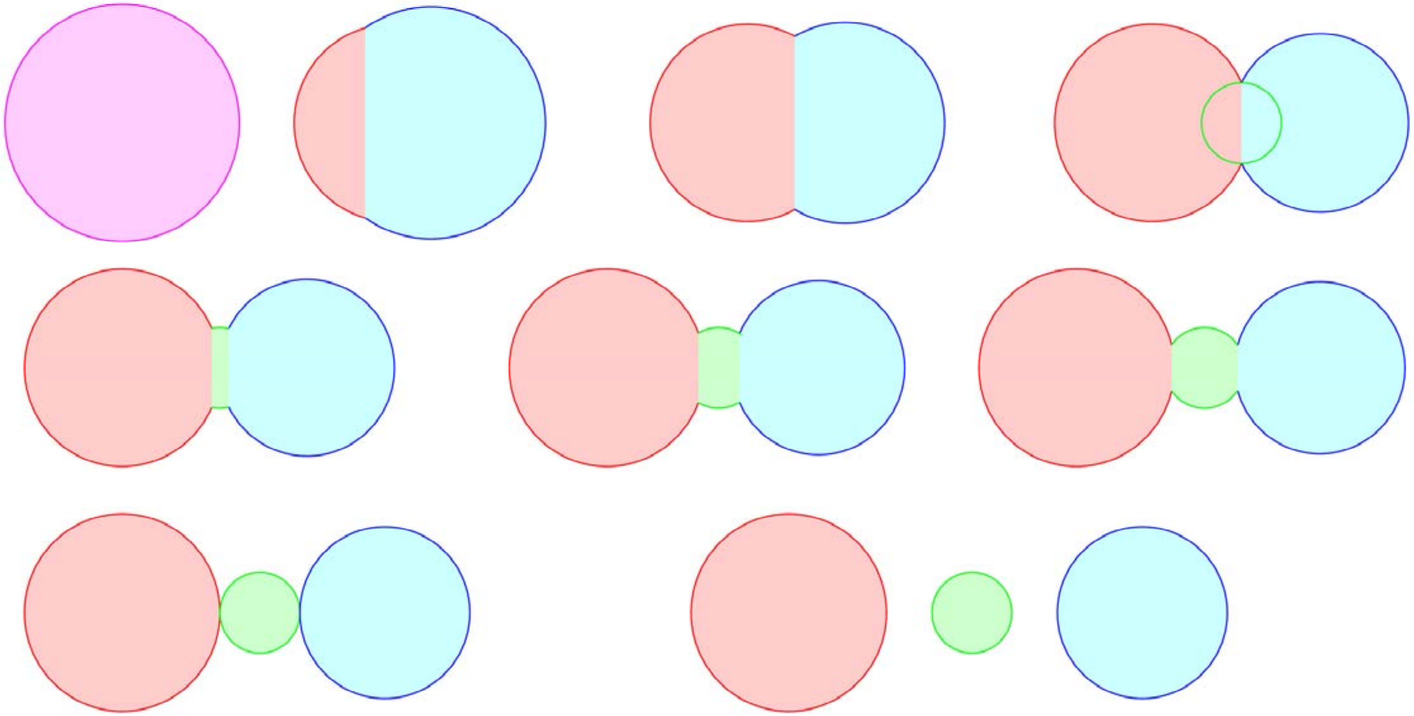
^{28}Mg

^{100}Zr



$^{252}Cf \rightarrow ^{146}Ba + ^{96}Sr + ^{10}Be$

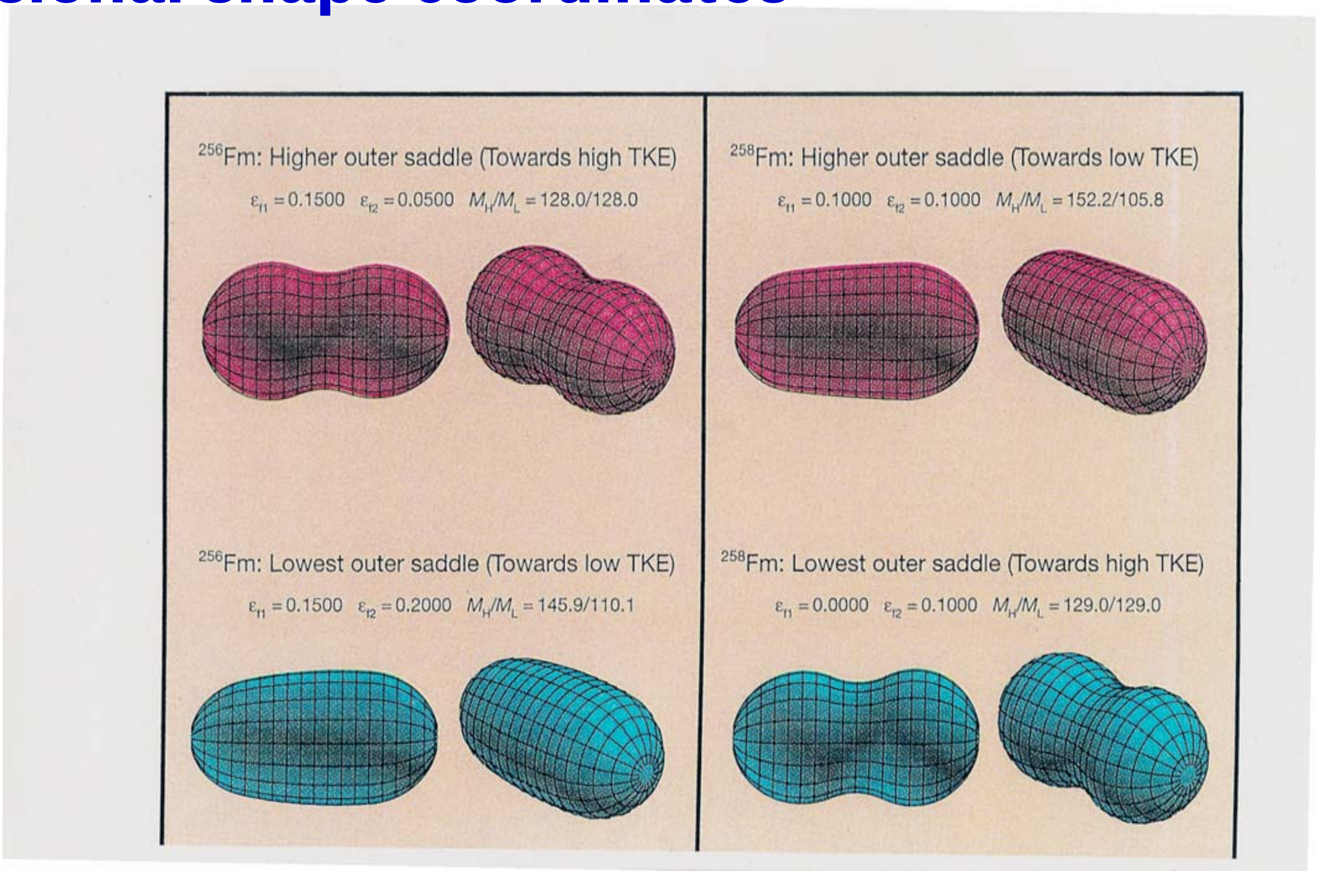
Given parametrization and fission path



GIVEN PARAMETRIZATION

Möller, Madland, Sierk & Iwamoto, *Nature* **409** *(2001)*

5 dimensional shape coordinates



DAVID HILL'S METHOD (I)

Thesis (Princeton 1951) – large amplitude motion of an incompressible fluid with irrotational flow

$$\nabla \vec{v} = 0, \qquad \nabla \times \vec{v} = 0$$

mass density μ **under bulk force** $\vec{F} = -\nabla U$

of electrostatic nature and a pressure distr. p_1

$$\mu \frac{d\vec{v}}{dt} = \vec{\chi} - \nabla p_1 \cong -\nabla p \qquad \nabla^2 H = 0$$

$$H = U + p_1 + \frac{1}{2}\mu v^2 \qquad p(z) = U(z) + \sigma K(z)$$

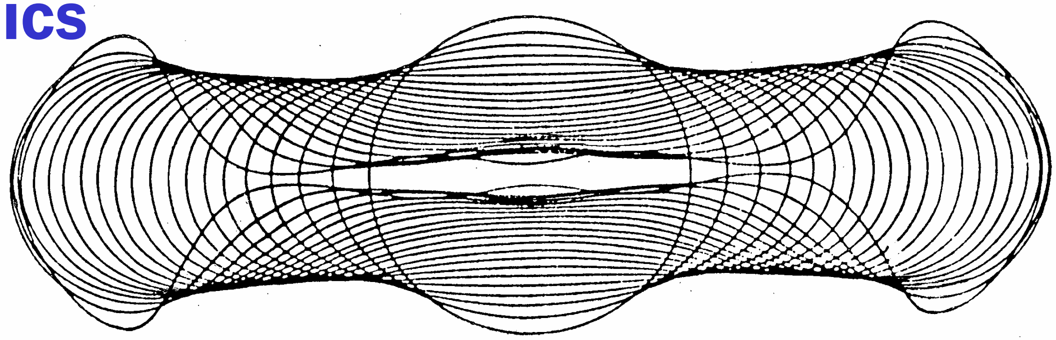
Pseudopressure p, surf. tens. σ, **and curvature K**

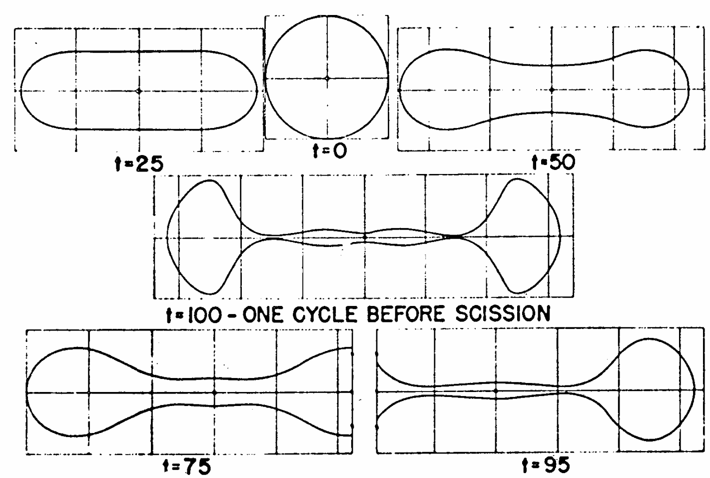
DAVID HILL'S METHOD (II)

D.L. Hill: The Dynamics

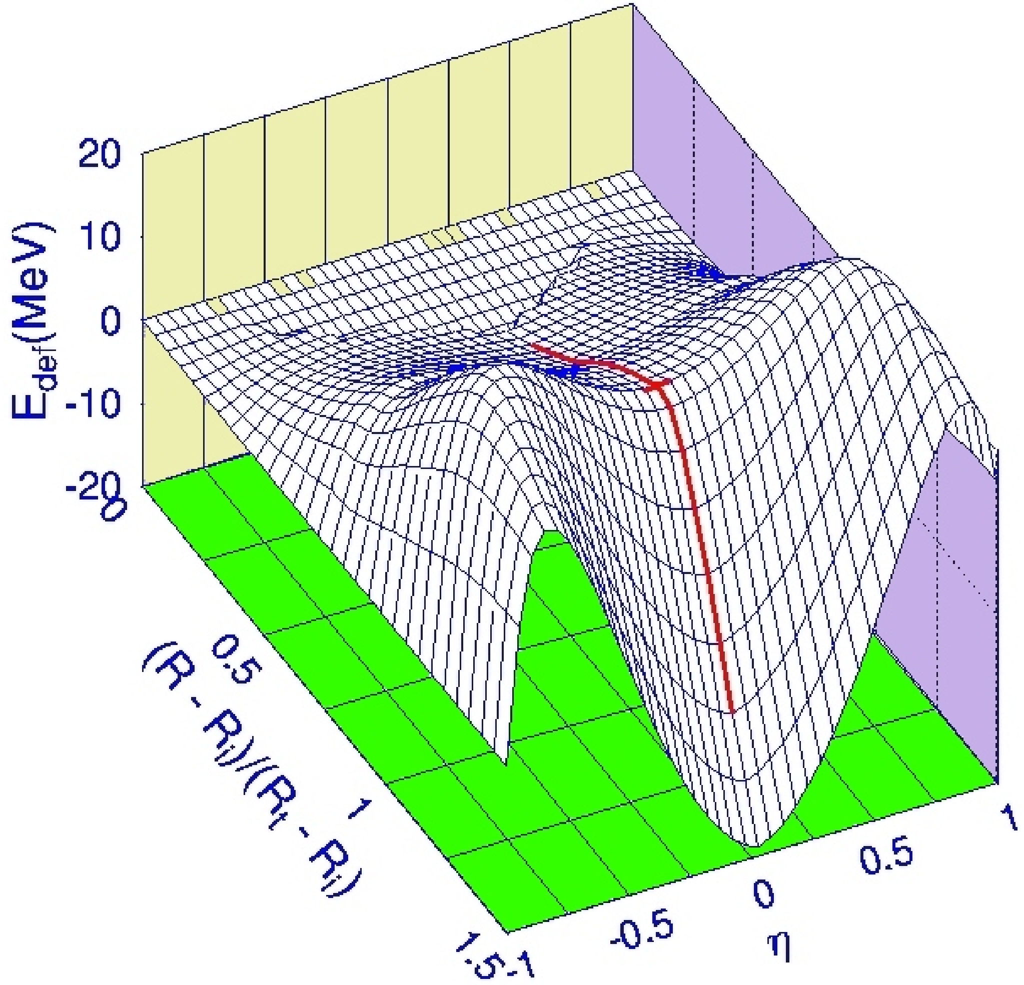
of Nuclear Fission,

Proc. of the 2nd United Nations International Conference on the Peaceful Uses of Atomic Energy, Vol. 15,Geneva, 1958





STATIC PATH ON PES



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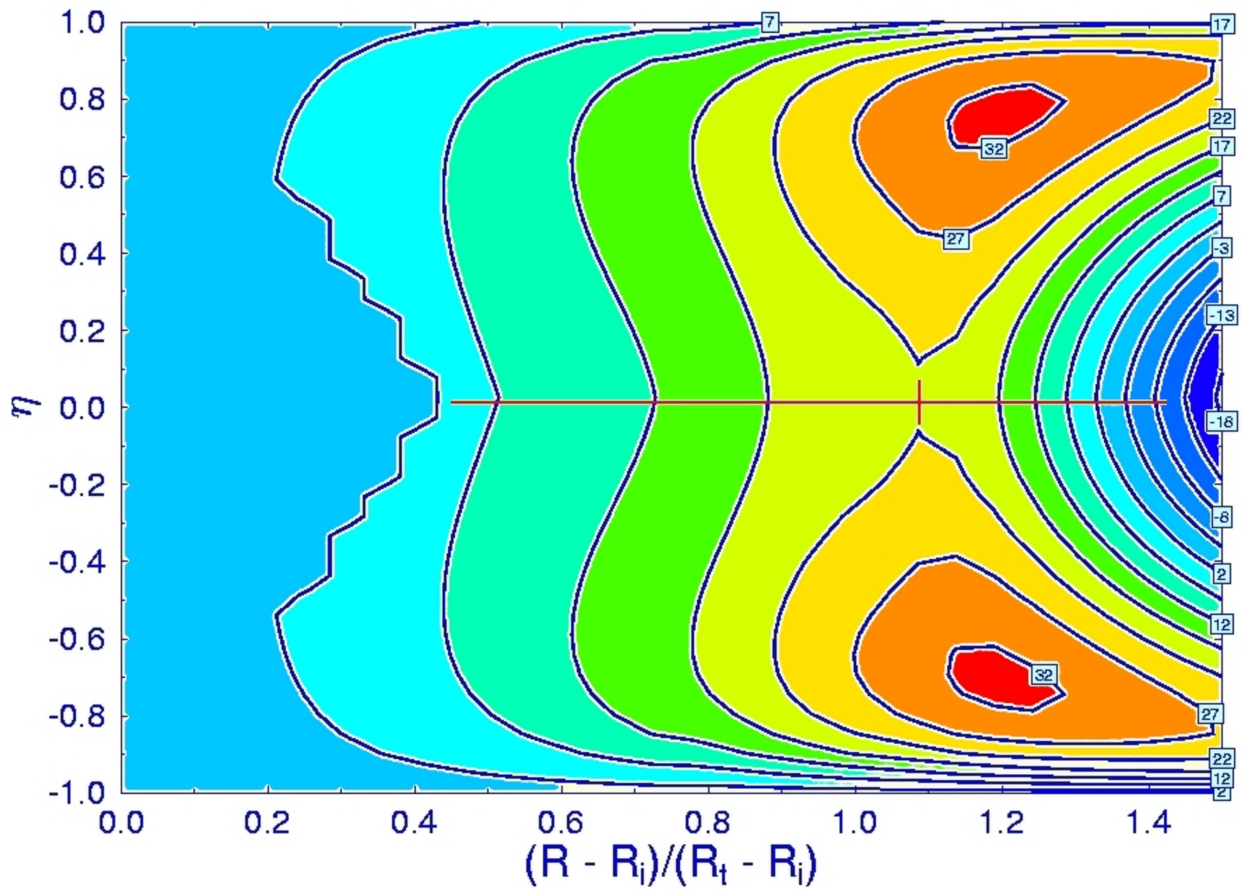
STATIC PATH ON CONTOUR PLOT



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INTEGRO-DIFFERENTIAL EQUATION (I)

$$K(z) = \frac{1}{\Re_a} + \frac{1}{\Re_b} = \frac{1}{\rho(1+\rho'^2)^{1/2}} - \frac{\rho''}{(1+\rho'^2)^{3/2}}$$

ρ = ρ (z) is the surface equation (cylindrical symm.)

minimizing the deformation energy

$$E_{def}(\alpha) = E_{LDM} + \delta E = E_s + E_C + \delta E$$

with constraints: const. deformation α and

mass asymmetry η

$$\alpha = |z_L^c| + |z_R^c| \qquad \eta = \frac{A_1 - A_2}{A_1 + A_2}$$

INTEGRO-DIFFERENTIAL EQUATION (II)

The pressure at the surface $\rho_e V_s(\vec{r}) + 2\sigma K(\vec{r})$

is in equilibrium. V_s **is the electrostatic potential**

ρ_e **is the charge density**

$$\rho\rho'' - \rho'^2 - [\lambda_1 + \lambda_2|z| + \Im(\rho, z)]\rho(1+\rho'^2)^{3/2} - 1 = 0$$

$\Im = 10XV_s(z, \rho)$ **Lagrange multipliers** $\lambda_{1,2}$

X = fissility V_s **is expressed by a double integral**

Conditions $\rho(z_1) = \rho(z_2) = 0$ $\quad \dfrac{d\rho(z_{1,2})}{dz} = \pm\infty$

INTEGRO-DIFFERENTIAL EQUATION (III)

$z_{1,2}$ are the intercepts with z axis at the 2 tips.

Symmetry: $z_2 = z_p = - z_1$. New function

$$u(v) = \Lambda^2 \rho^2 [z(v)]; \quad z(v) = z_p - v / \Lambda$$

$$u'' = 2 + \frac{1}{u}[u'^2 + (v - d + V_{sd})(4u + u'^2)^{3/2}]$$

$$V_{sd} = \frac{5X}{2\Lambda} V_s - a - vb; \quad u(0) = u'(v_{pn}) = 0; \quad u'(0) = 1/d;$$

d **and** ***n*** **are input parameters related to elongation & number of necks. Runge-Kutta numerical meth.**

REFLECTION ASYMMETRICAL SHAPES

Equations for left hand side (L) and right h.s. (R).

Matching $\rho_L(0) = \rho_R(0)$ $\quad u_L^{1/2}(v_p)/\Lambda_L = u_R^{1/2}(v_p)/\Lambda_R$

$$M_L = \frac{2\pi}{3}(1+\eta) = \pi\Lambda_L^{-3}\int_0^{v_p} u_L(v)dv; \quad \Lambda_{L0} = \left\{\frac{3}{2}\int_0^{v_p} u_L(v)dv\right\}^{1/3}$$

And similarly for right hand side.

Alternatively: $\quad n_L \neq n_R, \quad$ **and continuity of $\rho''(0)$**

$$(d_L - v_{pL})u_L^{1/2}(v_{pL}) = (d_R - v_{pR})u_R^{1/2}(v_{pR})$$

DEFORMATION ENERGY

Liquid Drop Model (LDM)

$$E_{LDM} = E - E^0 = E_s^0(B_s - 1) + E_C^0(B_C - 1)$$

$$X = E_C^0/(2E_s^0)$$ **- fissility**

$$E_s^0 = a_s(1-\kappa I^2)A^{2/3};\ I = (N-Z)/A;\ E_C^0 = a_C Z^2 A^{-1/3}$$

Deformation dependent surface energy

y(x) = surface equation with (–1,+1) intercepts on x axis

$$B_s = \frac{d^2}{2}\int_{-1}^{+1}\left[y^2 + \frac{1}{4}\left(\frac{dy^2}{dx}\right)^2\right]^{1/2} dx;\ d = \frac{z_2 - z_1}{2R_0}$$

COULOMB ENERGY

Assume uniform charge density $\rho_{0e} = \rho_{1e} = \rho_{2e}$

$$B_c = \frac{5d^5}{8\pi}\int_{-1}^{+1} dx \int_{-1}^{+1} dx' F(x,x'); \qquad F(x,x') = \{yy_1[(K-2D)/3] \bullet$$

$$\left[2(y^2+y_1^2)-(x-x')^2+\frac{3}{2}(x-x')\left(\frac{dy_1^2}{dx'}-\frac{dy^2}{dx}\right)\right]+$$

$$K\left\{y^2y_1^2/3+\left[y^2-\frac{x-x'}{2}\frac{dy^2}{dx}\right]\left[y_1^2-\frac{x-x'}{2}\frac{dy_1^2}{dx'}\right]\right\}a_\rho^{-1}; \; D=\frac{K-K'}{k^2}$$

K, K' **are complete elliptic integrals of 1st and 2nd kind.   After scission** $E_C = \sum_{i\neq j} e^2 Z_i Z_j / R_{ij}$

PHENOMENOLOGICAL SHELL CORRECTIONS

Adapted after Myers & Swiatecki. Number of nucleons Z, *N* proportional to the volume.

$$\delta E = \sum_i (\delta E_{pi} + \delta E_{ni}) = C \sum_i [s(Z_i) + s(N_i)]; \qquad s(Z) = Z^{-2/3} F(Z) - cZ^{1/3}$$

$$F(n) = \frac{3}{5}\left[\frac{N_i^{5/3} - N_{i-1}^{5/3}}{N_i - N_{i-1}}(n - N_{i-1}) - n^{5/3} + N_{i-1}^{5/3}\right]; \; n \in (N_{i-1}, N_i) \text{ is } Z \text{ or } N$$

N_i are magic numbers. *C=6.2* MeV, *c=0.2* from fit to exp. masses and def. Variation with deformation:

$$\delta E = \frac{C}{2}\left\{\sum_i [s(N_i) + s(Z_i)] \frac{L_i}{R_i}\right\}$$

L_i are the lengths of fragments.

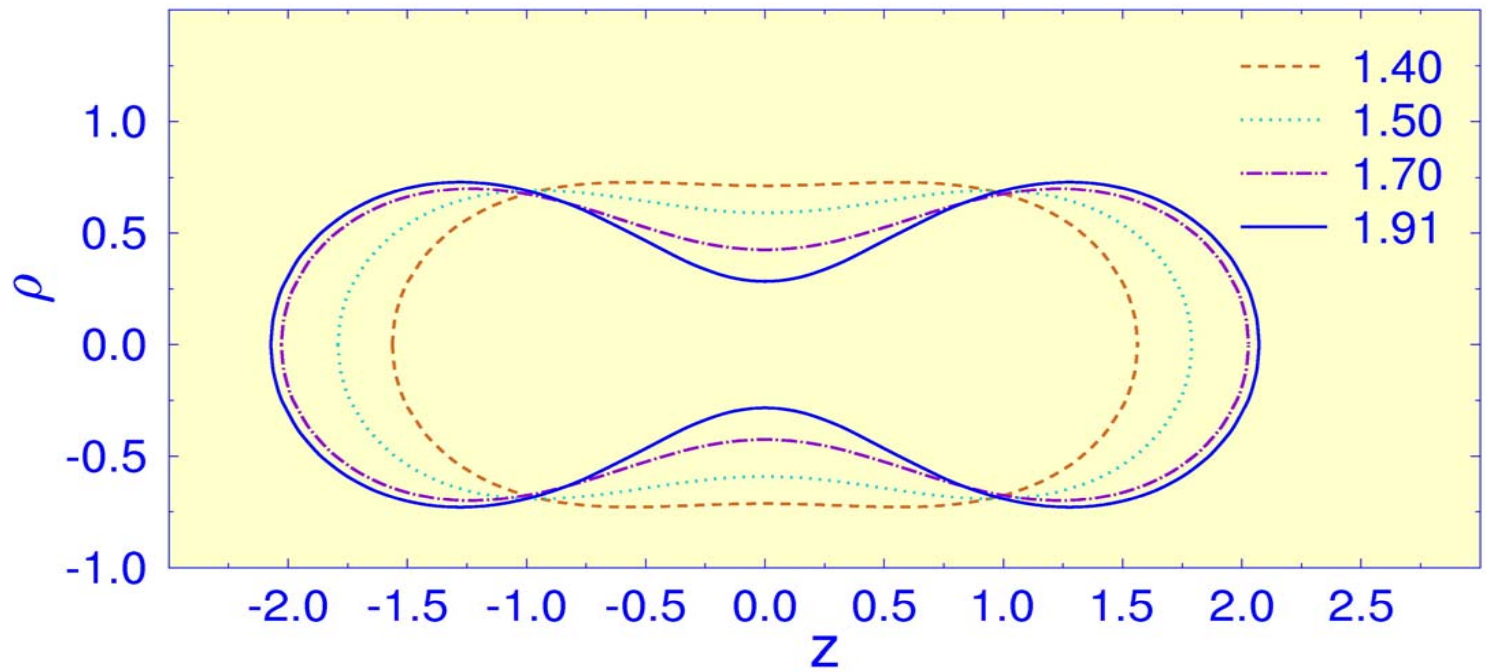
For magic nb. δE is minimum.

CALCULATED SHAPES for X = 0.60

X = 0.60 (*e.g.* ^{170}Yb) **η = 0** **$n_L = n_R = 2$**

$d_L = d_R$ = 1.40, 1.50, 1.70, 1.91(SP) **input parameters**

α / R_0 = 1.31, 1.64, 2.10, 2.30 **deformation**

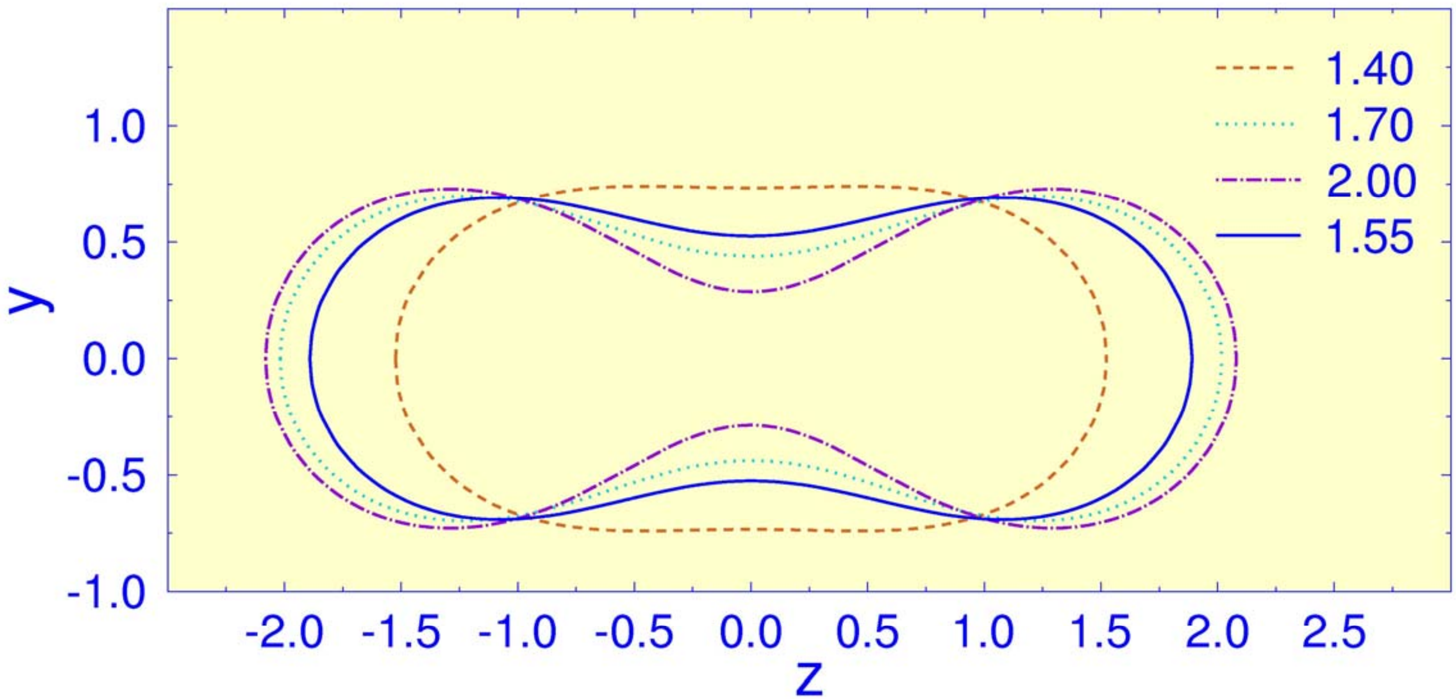


CALCULATED SHAPES for X = 0.70

X = 0.70 (*e.g.* ^{204}Pb) **η = 0** **$n_L = n_R = 2$**

$d_L = d_R$ = 1.40, 1.70, 2.00 , 1.55(SP) input parameters

$(\alpha / R_0)_{SP}$ = 1.82 **SP deformation**



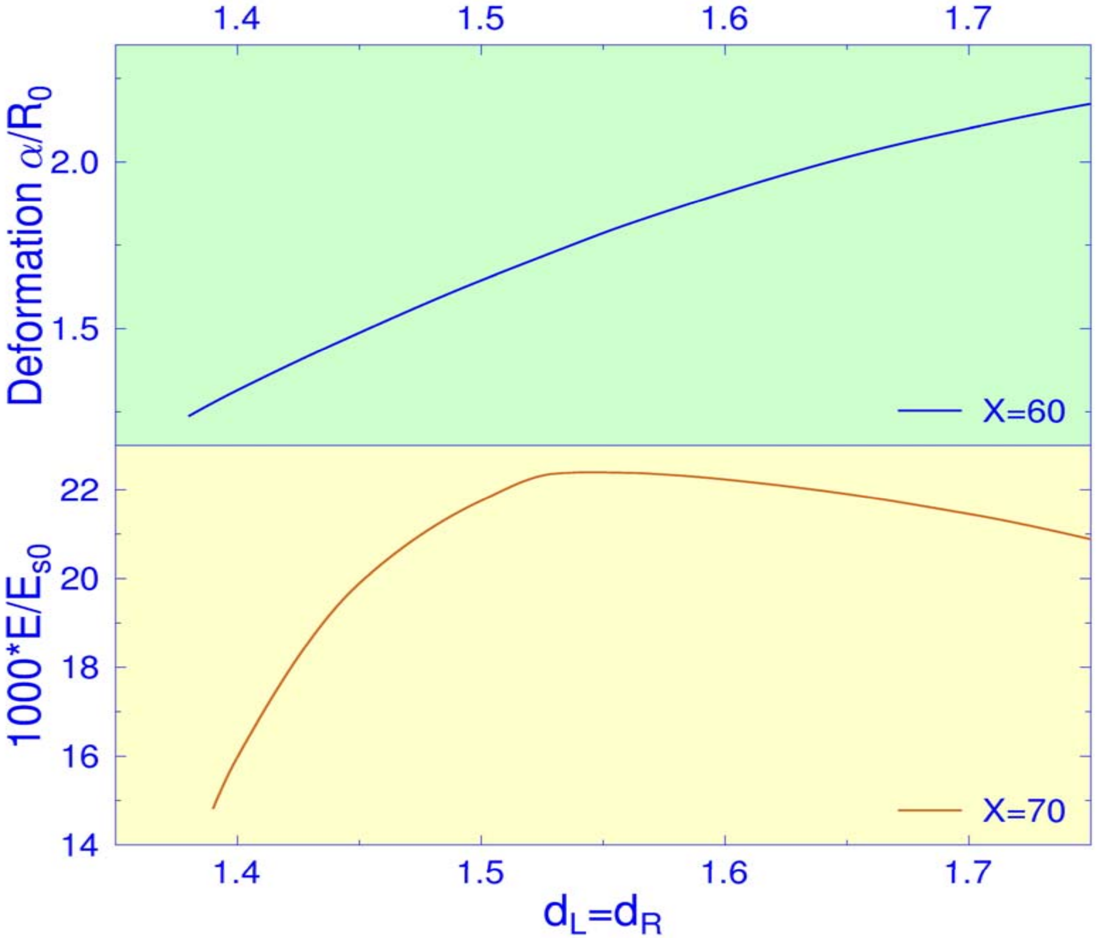
DEFORMATION and ENERGY vs input parameter

X = 0.60 (*e.g.* ^{170}Yb)

and 0.70 (*e.g.* ^{204}Pb)
η = 0

$n_L = n_R = 2$

Maximum energy at the Saddle Point



CALCULATED SADDLE POINT SHAPES

$\eta = 0 \quad n_L = n_R = 2 \quad d_L = d_R = d$

X = 0.60, *e.g.* **^{170}Yb**

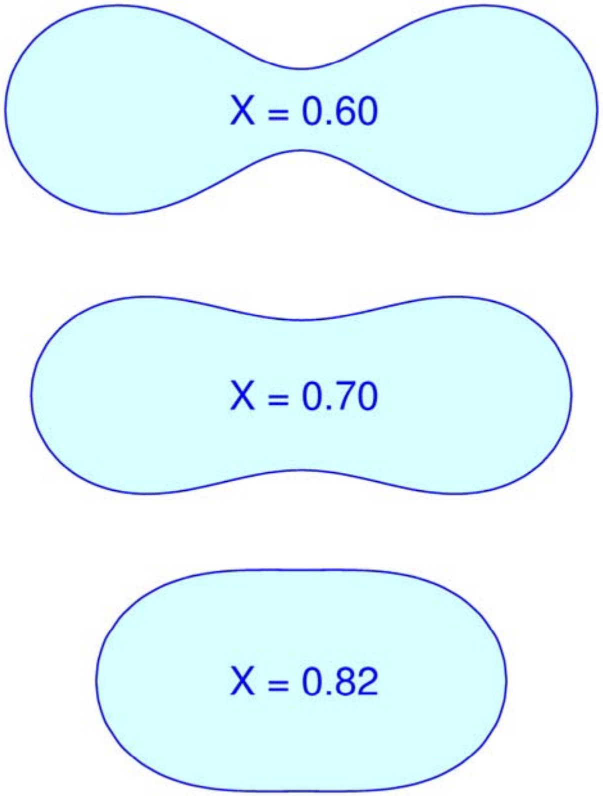
d=1.91 α**/R_0 = 2.30**

X = 0.70, *e.g.* **^{204}Pb**

d=1.55 α**/R_0 = 1.82**

X = 0.82, *e.g.* **^{252}Cf**

d=1.38 α**/R_0 = 1.16**



ASYMMETRICAL SHAPES (I)

$\eta \neq 0$

Two ways to obtain: (1) $n_L = n_R = n$ $d_L \neq d_L$

(2) $n_L \neq n_R$ $d_L \neq d_L$

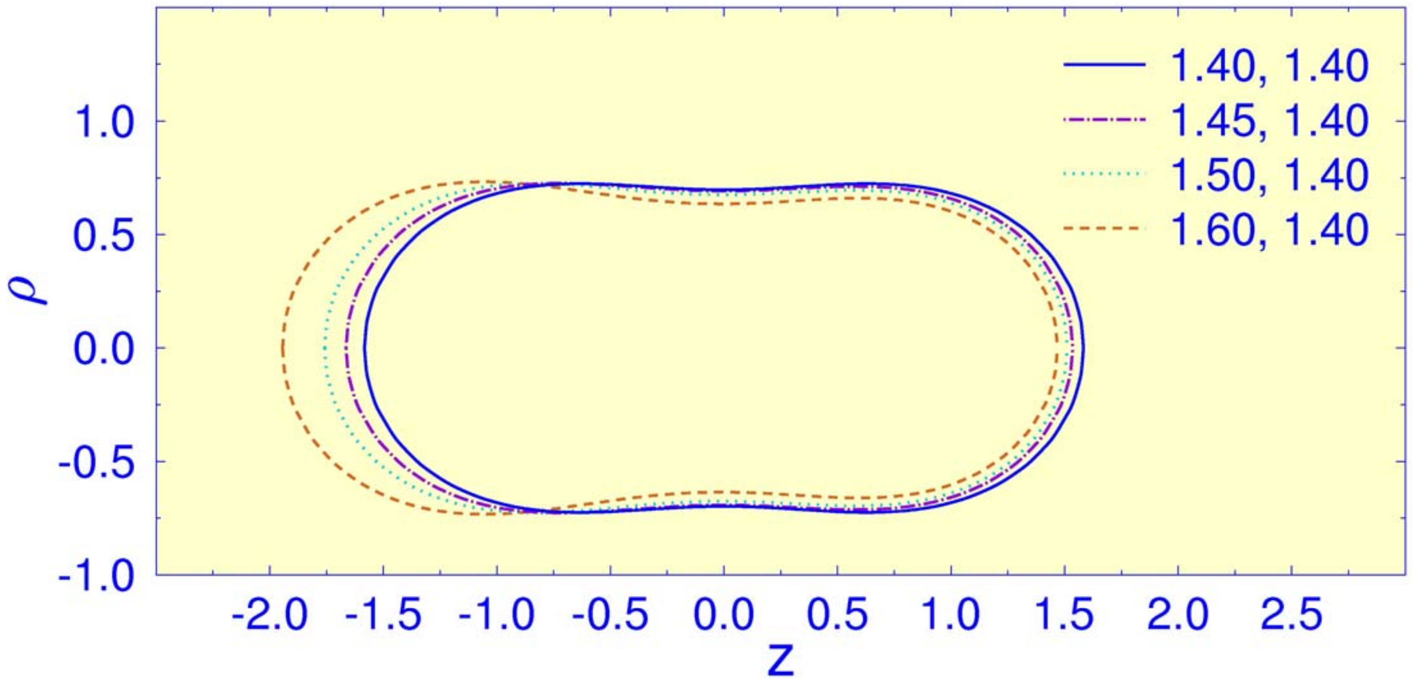
(1) n=2

X = 0.77

e.g. ^{238}**U**

d_R=1.4

d_L=1.4, 1.45, 1.5, 1.6

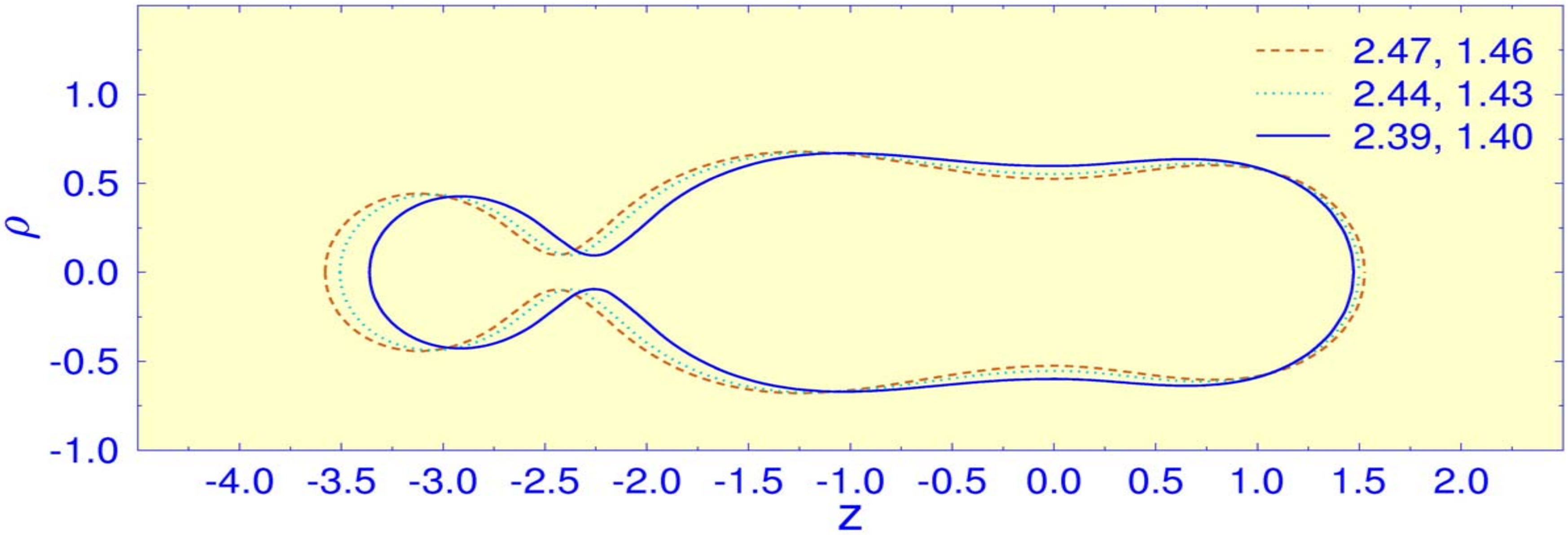


ASYMMETRICAL SHAPES (II)

$\eta \neq 0$ **(2)** $n_L = 4$, $n_R = 2$, $d_L = 2.47, 2.44, 2.39$

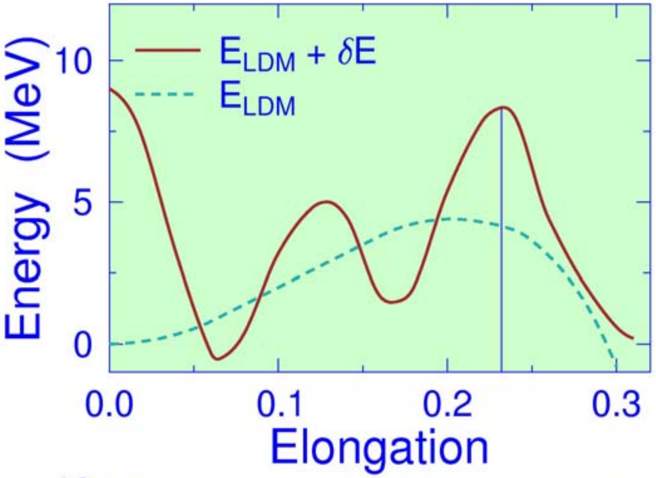
$X = 0.60$, *e.g.* ^{170}Yb $d_R = 1.46, 1.43, 1.40$

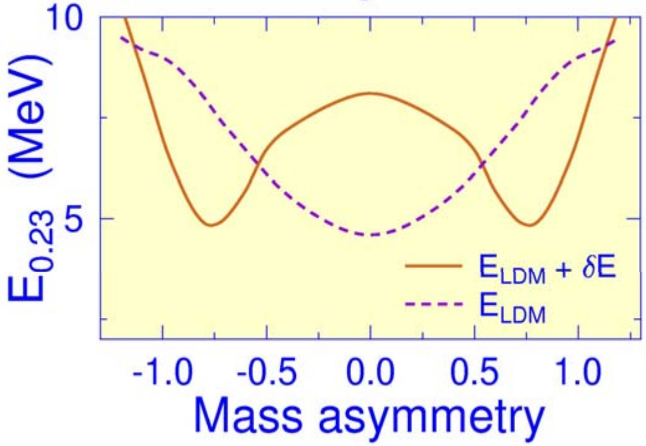
$\alpha / R_0 = 2.06, 1.98, 1.84$



MASS ASYMMETRY AT THE 2nd SADDLE POINT

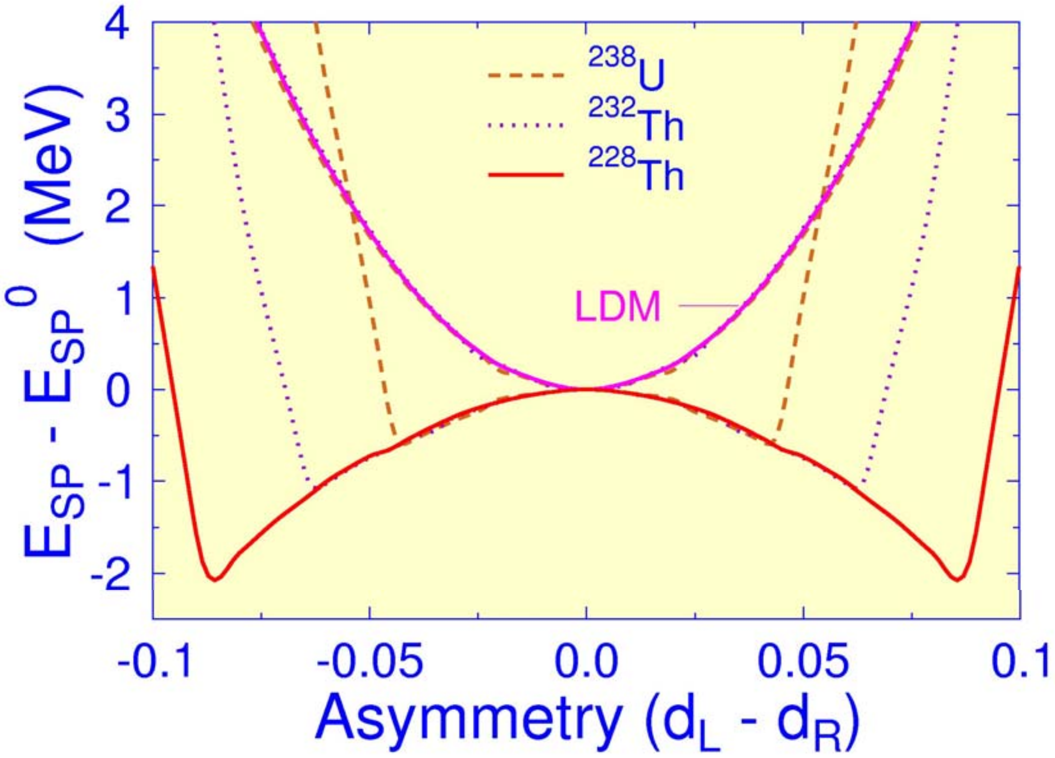
The 2nd barrier height is lower at some finite mass asymmetry.





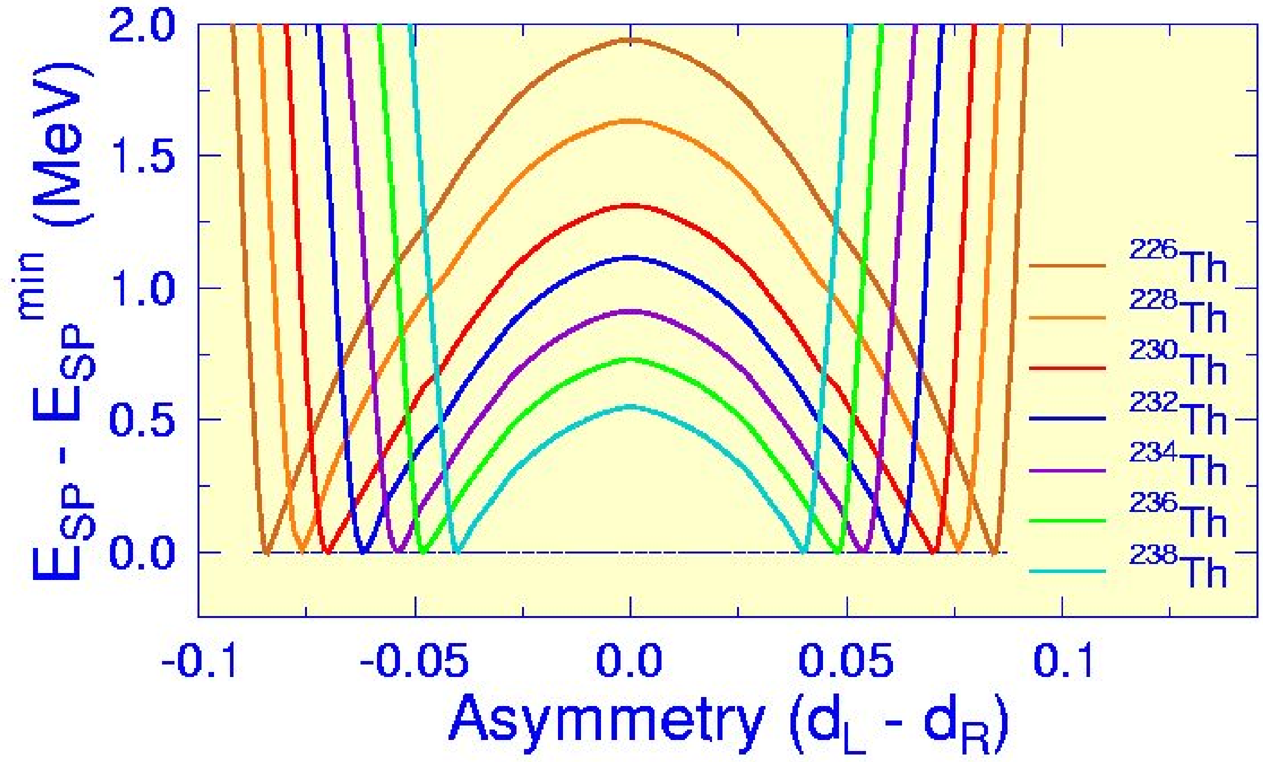
SP MINIMUM DUE TO SHELL EFFECTS (I)

Binary cold fission. Saddle point energy vs mass asymmetry with and without shell effects.



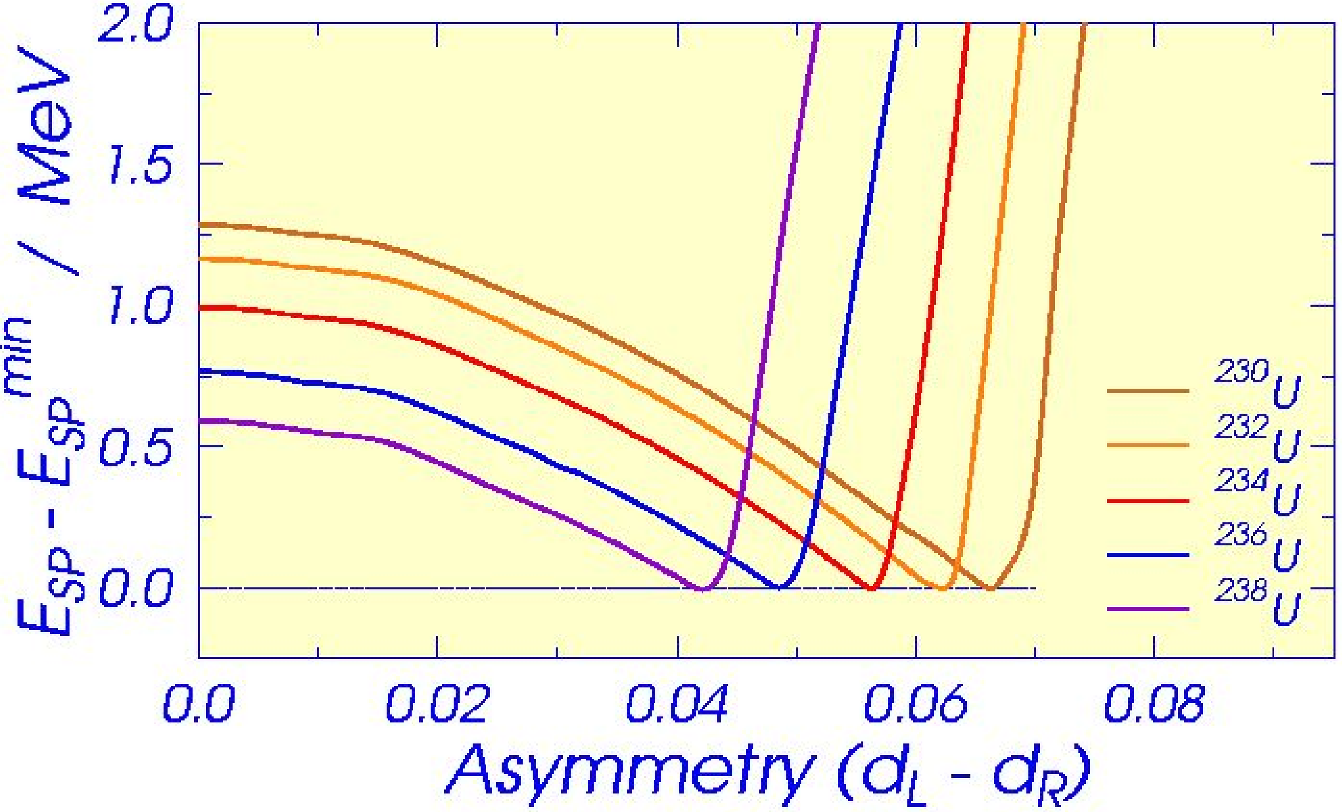
SP MINIMUM DUE TO SHELL EFFECTS (II)

Binary cold fission. Saddle point energy vs mass asymmetry for Th isotopes

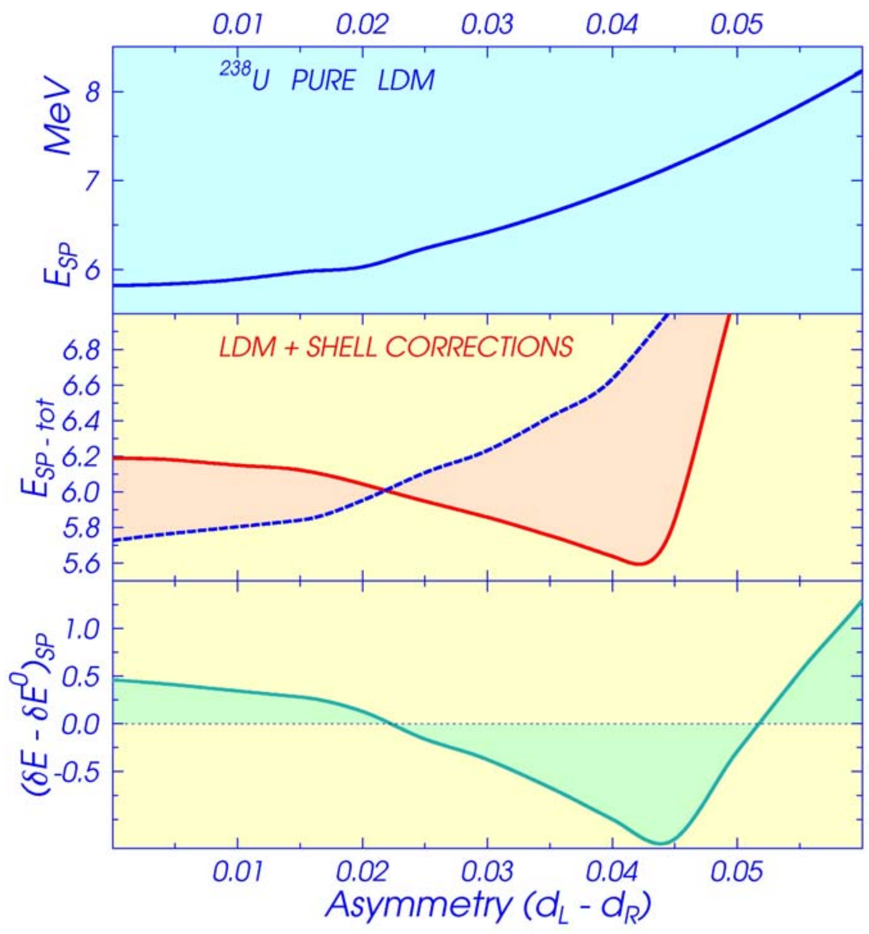


SP MINIMUM DUE TO SHELL EFFECTS (III)

Binary cold fission. Saddle point energy vs mass asymmetry for U isotopes



CONTRIBUTION OF SHELL EFFECTS



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MASS NUMBER OF THE HEAVY FRAGMENT

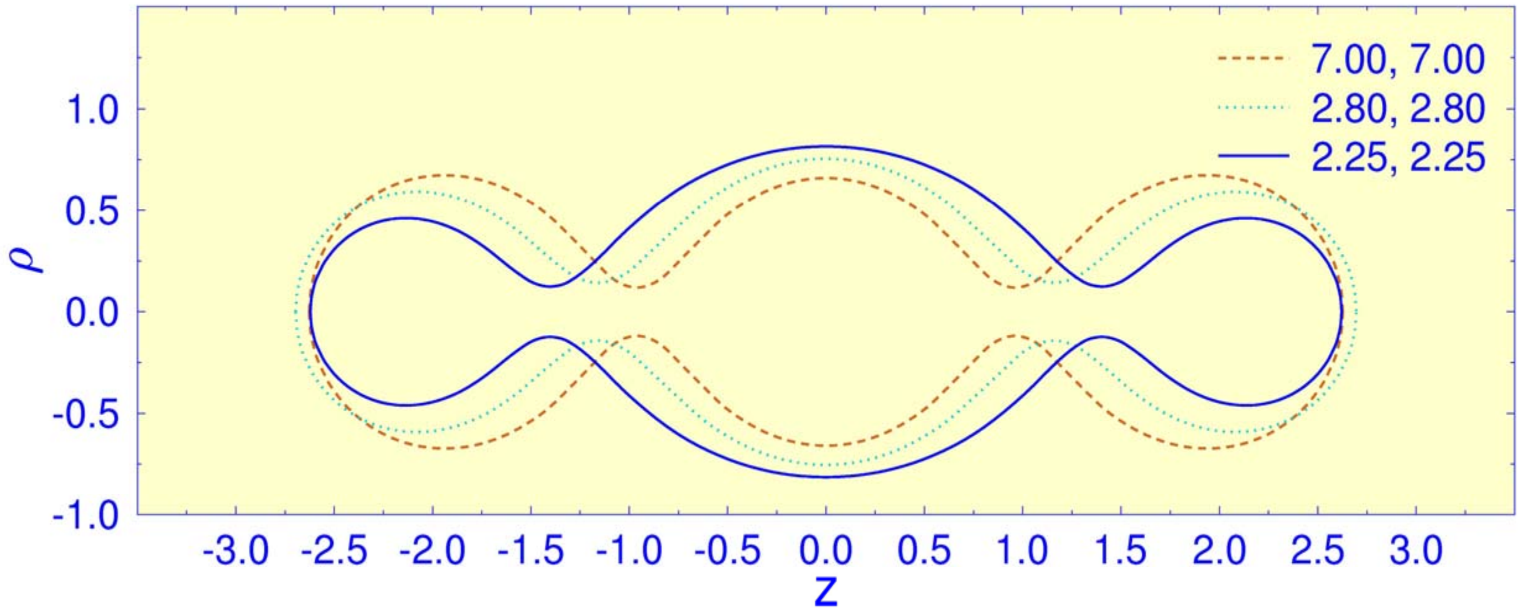
Mass number of the heavy fragment corresponding to the minimum of the saddle point energy

Nuclei	X	$d_L - d_R$	η	A_1
^{238}U	0.769	0.04	.04988	124.93
^{232}Th	0.754	0.06	.07517	124.72
^{228}Th	0.758	0.08	.09512	124.84

TERNARY FISSION (I)

Binary fissility X = 0.60 *e.g.* **^{170}Yb $n_L = n_R = 3$**

$d_L = d_R$ = 2.25, 2.80, 7.00 α / R_0 = 1.65, 2.31, 2.73



TERNARY FISSION (II)

Config. d = 7, $E/E_s^0 = 0.134$ is not far from a

true ternary fission $^{170}Yb \rightarrow {}^{56}V + {}^{56}V + {}^{58}Cr$,

Q=83.64 MeV, $(E_t-Q)/E_s^0 = 0.239$ for spherical shapes in touch

For $^{170}Yb \rightarrow {}^{10}Be + {}^{80}As + {}^{80}As$

Q=70.86 MeV, $(E_t-Q)/E_s^0 = 0.147$

For $^{170}Yb \rightarrow {}^{4}He + {}^{83}Se + {}^{83}Se$

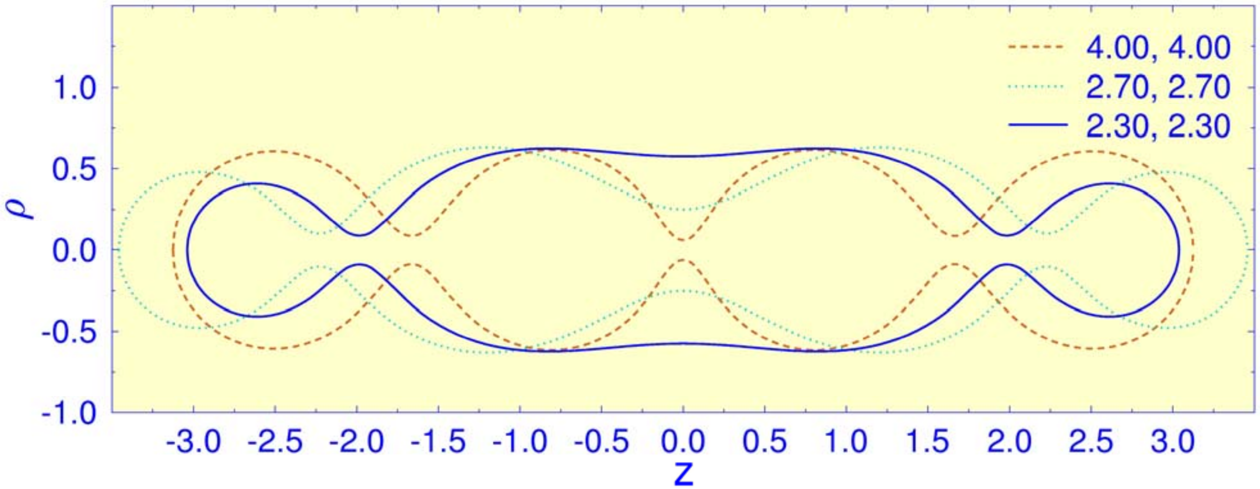
Q=87.48 MeV, $(E_t-Q)/E_s^0 = 0.103$ — higher Q-value

and lower fission barrier

QUATERNARY FISSION

Binary fissility X = 0.60 *e.g.* ^{170}Yb $n_L = n_R = 4$

$d_L = d_R$ = 2.3, 2.7, 4.00 α / R_0 = 2.14, 3.14, 3.23



Config. d = 4, E/E_s^0 = 0.214   is not far from a

true quaternary fission $^{170}Yb \rightarrow {}^{42}Cl + {}^{42}Cl + {}^{43}Ar + {}^{43}Ar$
Q=53.17 MeV, $(E_t - Q) / E_s^0$ = 0.324 for touching spheres

EXPERIMENTAL ATTEMPTS

- Theory: R. D. Present (*Phys. Rev.* 59 (1941) 466): Uranium tripartition would release about 20 MeV more energy than binary
- Exp. Rosen & Hudson (1950): $^{235}U + n_{th}$. Triple gas filled ionization chamber. Yield of 6.7 per 10^6 binary fission acts
- Iyer & Coble (1968): ^{235}U + He ions of intermediate energy. Also optimistic.
- Schall, Heeg, Mutterer & Theobald (1987): spontaneous fission of ^{252}Cf. Triple coincidences with detectors at 120^o Pessimistic: yield under 10^{-8} per binary.

CONCLUSIONS

- **One can obtain saddle-point shapes by solving an integro-differential equation**
- **There is no limitation imposed by the space of deformation coordinates (no parametrization)**
- **Fission barriers for true ternary and quaternary fission are lower than for aligned spherical fragments in touch**
- **Mass-asymmetry in binary fission is explained by adding shell corrections to the LDM energy**
- **Experimental confirmation of true ternary fission still needed**